FOREWORD

PROFESSIONAL EXAMINATION INTERMEDIATE

PAPER 4

COSTING AND QUANTITATIVE TECHNIQUES
FOREWORD

The remarkable success recorded through publication of the first edition of this study pack coupled with the positive impact the initiative had on the performances of professional examination candidates, and other users of the study pack, encouraged the Institute to produce this second edition.

Through the review of the Institute’s professional examination syllabus, which occurs every five years, several innovations were introduced into the curriculum.

The advent of the new 15 - subjects (as against the old 19 - subjects) syllabus provided an auspicious moment for the Institute to review and update the existing study packs in the various subjects while new ones were produced for newly introduced subjects. In this connection, the Institute has produced study packs for the following subjects:

- Fundamentals of Financial Accounting
- Costing and Quantitative Techniques
- Taxation
- Audit and Assurance
- Business Communication and Research Methodology
- Information Technology
- Management Accounting
- Financial Accounting
- Advanced Audit and Assurance
- Public Sector Accounting and Finance
- Financial Reporting and Ethics
- Strategic Financial Management
- Advanced Taxation

In its effort to produce the study packs, the Council of the Institute approached several publishers to take up the challenge of producing them. While others declined the offer, the VI Publishers took up the challenge and commenced the task in earnest by commissioning some writers and reviewers to prepare study pack for each of the thirteen subjects. The writers and reviewers comprised eminent scholars and practitioners with tremendous experiences in their areas of specialization. They have written books, journals, seminar papers, etc. on the relevant subjects and distinguished themselves in the accountancy profession. The output of the writers and reviewers were subjected to further comprehensive review by an editorial board.
Though, the study packs have been specially designed to assist candidates preparing for the Institute’s professional examinations, other professional bodies and tertiary institutions, who may have cause to use the study packs, are advised to use them in conjunction with other materials listed in the bibliography and recommended readings.

OGUNJUBOUN, F. I.
CHAIRMAN, SYLLABUS IMPLEMENTATION AND STUDY PACK REVIEW COMMITTEE
PREFACE

INTRODUCTION

The maiden edition of this study pack was produced in 2006 by The Institute of Chartered Accountants of Nigeria (ICAN) to address the dearth of good study materials for students of the Professional examinations. The need to continue to provide students with up-to-date information and good reading materials prompted the Council of the Institute to carry out a review of the syllabus and consequently, the study pack on this subject.

It should be noted that where new subjects have been introduced in the new syllabus as a result of the review, efforts have been made to prepare new study packs for them.

The Institute committed enormous resources in support of this project by engaging a team of very experienced and seasoned practitioners and academicians to review existing study packs and where necessary produce new ones. It is envisaged that remarkable improvement will begin to manifest in the performances of the professional examination candidates and other users of the study packs.

READERSHIP

These study packs will benefit the following categories of readers:
♦ Candidates of the Institute’s professional examinations;
♦ Candidates of other professional institutes offering the same or similar subjects to those of ICAN;
♦ Students of tertiary institutions;
♦ Lecturers in tertiary institutions and training centres;
♦ Professional Accountants; and
♦ Practitioners of other fields requiring good working knowledge of these subjects.

CONTENTS

CHAPTER 1  INTRODUCTION TO COST ACCOUNTING

This chapter deals with the meaning of cost accounting. It explains the scientific ways of determining total cost of a cost unit and the qualities expected of good cost accounting information.
CHAPTER 2  MATERIAL ACCOUNTING AND CONTROL
Chapter 2 relates to the procedures involve in the procurement, storage, usage and accounting for material as a cost element. It emphasizes on the strategies for materials control and the various methods of valuing closing stock in for reflection in the balance sheet. The concept of Just-In-Time (JIT) as a modern purchasing and production method is also discussed.

CHAPTER 3  LABOUR ACCOUNTING AND CONTROL
This chapter covers the various methods of remuneration, general features of incentive schemes, determination of labour cost and accounting treatment of wages cost. In addition, it discusses the control of labour cost, labour turnover, causes and suggestions for the removal of labour turnover, job evaluation and merit rating.

CHAPTER 4  OVERHEAD COST ACCOUNTING AND CONTROL
Chapter 4 treats the nature of expenses normally regarded as overheads including apportionment of service department costs. It also covers the absorption of overheads into product costs, various overhead recovery rates and choice of appropriate rates, and the effect of activity based costing on overhead allotment.

CHAPTER 5  COST ANALYSIS
Chapter 5 relates the various ways of classifying costs, the cost behavioural patterns, cost estimation or suggestion processes and the importance of cost analysis in managerial decision making process.

CHAPTER 6  COST METHODS
This chapter explains the distinction between costing methods and costing techniques, the various branches of specific order costing, the various branches of unit (or average) costing, the various methods of job costing, contract costing and service or operating costing.

CHAPTER 7  PROCESS COST AND PRODUCT COSTING
This chapter explains the meaning of process costing, the characteristics of process costing and the determination of equivalent units and the meaning of its concept. In addition, it treats the account for spoilages, cost of production report,
joint costs and joint products, various methods of apportioning joint costs to joint products, the accounting treatment of by-products and the determination of total costs of producing joint products.

CHAPTER 8  BUDGETING AND BUDGETARY
This chapter explains the meaning and significance of budget, budgeting process, the various types of budgets, using activity, time and quantitative perspectives. It deals with budgetary control process, including comparison between budgeted and actual result, the significance of budget committee, budget manual, budget officer, etc. and the use of budgetary improvement techniques like incremental budgeting, zero base budgeting, planning, programming, budgeting system, rolling budgeting and the preparation of functional and master budgets of a business organisations.

CHAPTER 9  STANDARD COSTING
Chapter 9 treats the objectives of standard costing, types of standards, applications of standard costing, setting of standards, variance analysis, advantages of standard costing and control ratios.

CHAPTER 10  COST DATA FOR SHORT-RUN TACTICAL DECISION MAKING
This chapter deals with the fundamental differences between absorption and marginal costing techniques, nature of management decision making with emphasis on marginal costing. In addition, the chapter discusses the concept of CVP analysis and necessity of cost accounting data in short-term tactical decision making.

CHAPTER 11  COST CONTROL
This chapter explains the difference between cost control and cost reduction, the mechanisms for controlling costs, the various mechanisms for reducing costs and the recent development in cost accounting.

CHAPTER 12  COST LEDGER ACCOUNTS
This chapter provides understanding of the interlocking system of accounts, integral or integrated system of account, reconciliation or cost and financial accounting profits, recreation of the cost records and the treatment of notional charges.
CHAPTER 13  THEORY OF INDEX NUMBERS
This chapter treats the theory of index numbers with emphasis on simple relative index and aggregate index numbers.

CHAPTER 14  BASIC PROBABILITY CONCEPTS
This chapter contains elementary treatment of probability including the addition and multiplication laws of probability. The treatment is extended to conditional probability and independence, the concept of random variables and their distributions, and some commonly used distributions including Binominal, Poisson and Normal.

CHAPTER 15  SET THEORY
This chapter explains the application of set theory in business set, types and their presentation using the Venn Diagram approach.

CHAPTER 16  INTRODUCTION TO MATRICES
This chapter discusses the term “functions” and their use in solving real life problems. The functions include: linear functions, quadratic functions, exponential function, as well as polynomials and logarithm functions.

CHAPTER 17  BASIC CONCEPTS OF DIFFERENTIATION
The chapter explains the concept of derivates, function differentiation, application of the principle to economic and business problems and the concept of simple partial differentiation of first order.

CHAPTER 18  BASIC LINEAR PROGRAMMING
This chapter explains the concept of linear programming, “its basic assumptions, problem formulation, methods of solving LP problems, interpretation of results and the concept of duality and shadow cost.

CHAPTER 19  NETWORK ANALYSIS
Chapter 19 discusses the concept and meaning of network analysis, types, terms and methods especially Critical Path Method (CPM) and Programme Evaluation Review Technique (PERT).
CHAPTER 20  REPLACEMENT ANALYSIS

The chapter explains the meaning and purpose of replacement theory, concept and technique, gradual and sudden failure replacement policies and difference between individual and group replacement policies.

CHAPTER 21  TRANSPORTATION MODEL

The transportation problem is introduced as a special class of linear programming problem in which the objective is to transport or distribute a single commodity from various sources to different destinations at a minimum cost. Some basic methods of solving transportation problems are outlined with examples.

CHAPTER 22  COMPUTER ASSISTED COSTING TECHNIQUES

This chapter discusses the application of IT tools for cost accounting activities with emphasis on web conferencing and e-mail.

VI Publishers  May 2009
ACKNOWLEDGEMENT

A number of individuals and Institutions who are very vital in the production of this pack are hereby acknowledged. We are especially grateful to the Syllabus Implementation/Study Pack Committee made up of:

1. Ogunjuboun, F. I. - Chairman
2. Odejaiyi, Muyiwa - Deputy Chairman
3. Adeleke, S. O. - Member
4. Bammke, S. A. - Member
5. Ekungba, J. A. - Member
6. Erobu, U. I. - Member
7. Obisesan, D.O.O. - Member
8. Popoola, Toyin - Member

We also appreciate the following copyright holders for permission to use their intellectual properties:

♦ The Institute of Chartered Accountants of Nigeria (ICAN) for the use of the Institute’s examination materials;

♦ International Federation of Accountants (IFAC) for the use of her various publications;

♦ International Accounting Standards Board (IASB) for the use of International Accounting Standards:

♦ Nigerian Accounting Standards Board (NASB) for the use of Statements of Accounting Standards (SAS’s); and

♦ Owners of Trademarks and Trade names referred to or mentioned in this study packs.

The publisher has made every effort to obtain permission for use of intellectual materials in this study pack from the appropriate sources. If there are any errors or omissions, please contact the publisher who will make suitable acknowledgement in the reprint.

We acknowledge, with thanks, the immense contributions of the writers and reviewers of this study pack, namely: Mrs. L.O. Nkanbia – Davies, Prof. C.O.A.
Awosope, Messrs, M.A. Adelanwa, Doyin Talabi, M.A. Adenola, Dr. I.R. Akintoye, F.I. Ogunjuboun, D.O.O. Obisesan and Toyin Popoola, as well as the Institute’s secretarial staff under the leadership of O.A. Adepate (Registrar/Chief Executive), F.A. Olawuyi, O.E. Babatunde, S.O. Oyewo and other support staff.

This acknowledgement will not be complete if the typesetters, Olufunke Oyeniyi and Ogunbiyi Babatunde Julius are not mentioned.

Our sincere appreciation goes to the members of the following committees of the Institute of Chartered Accountants of Nigeria who contributed their resources to make this project a reality:

♦ Students Affairs’ Committee
♦ Examinations’ Committee
♦ Technical Committee on Syllabus Review
♦ Syllabus Implementation/Study Packs Committee.

Finally, we are indebted to the President, Chief (Dr.) Richard U. Uche, the Council of The Institute of Chartered Accountants of Nigeria for the financial and moral support which gave impetus to the production of this study pack.

VI Publishers

May 2009
# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>American Accounting Association</td>
</tr>
<tr>
<td>ABC</td>
<td>Activity Based Costing</td>
</tr>
<tr>
<td>AMT</td>
<td>Advanced Manufacturing Techniques</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer Aided Design</td>
</tr>
<tr>
<td>CAM</td>
<td>Computer Aided Manufacturing</td>
</tr>
<tr>
<td>CB</td>
<td>Continuous (Rolling) Budgeting</td>
</tr>
<tr>
<td>CPM</td>
<td>Critical Path Method</td>
</tr>
<tr>
<td>CVP</td>
<td>Cost Volume Profit</td>
</tr>
<tr>
<td>DLC</td>
<td>Direct Labour Cost</td>
</tr>
<tr>
<td>DMC</td>
<td>Direct Material Cost</td>
</tr>
<tr>
<td>EOQ</td>
<td>Economic Order Quantity</td>
</tr>
<tr>
<td>FIFO</td>
<td>First In First Out</td>
</tr>
<tr>
<td>GRN</td>
<td>Goods Received Note</td>
</tr>
<tr>
<td>IB</td>
<td>Incremental Budget</td>
</tr>
<tr>
<td>JIT</td>
<td>Just In Time</td>
</tr>
<tr>
<td>LIFO</td>
<td>Last In First Out</td>
</tr>
<tr>
<td>MRF</td>
<td>Materials Requirement Form</td>
</tr>
<tr>
<td>NSITF</td>
<td>National Social Insurance Trust Fund</td>
</tr>
<tr>
<td>NHF</td>
<td>National Housing Fund</td>
</tr>
<tr>
<td>NRV</td>
<td>Net Realizable Value</td>
</tr>
<tr>
<td>PPBS</td>
<td>Planning, Programming, Budgeting System</td>
</tr>
<tr>
<td>PERT</td>
<td>Program Evaluation Review Technique</td>
</tr>
<tr>
<td>TQM</td>
<td>Total Quality Management</td>
</tr>
<tr>
<td>WAP</td>
<td>Weighted Average Price</td>
</tr>
<tr>
<td>WIP</td>
<td>Work In Progress</td>
</tr>
<tr>
<td>ZBB</td>
<td>Zero Base Budgeting</td>
</tr>
</tbody>
</table>
SYLLABUS AND EXAMINATION QUESTIONS FORMAT

INTERMEDIATE

PAPER 4: COSTING AND QUANTITATIVE TECHNIQUES

A. OBJECTIVE

To examine candidates’

♦ Mastery of the concepts and principles of Cost Accounting and their applications in pricing decisions and other control matters in public and private sectors.

♦ Ability to analyse and predict cost behavioural patterns as output and market factor vary and the impact of patterns on profit and loss.

♦ Ability to apply appropriate methods and techniques in collecting, measuring and reporting cost information relevant to business and other activities.

♦ Familiarity with basic concepts and processes in Statistics, Business Mathematics and Operations Research

♦ Ability to employ suitable mathematical models and techniques to solve problems involving rational choice among alternatives.

B. CONTENTS

COMPONENT A COSTING

1. INTRODUCTION TO COST ACCOUNTING (10%)

(a) Definition and purpose of cost accounting.
(b) Classification of cost by nature, functions, elements, responsibility and behaviour.

(c) Materials accounting and control procedures including:
   (i) Stock recording and management
   (ii) Procurement and pricing
   (iii) Methods of Inventory valuation
   (iv) Just-In-Time purchasing and production
   (v) ABC Analysis Technique
   (vi) Inventory Control
(d) Labour accounting and control procedures including:
   (i) Basic methods of remuneration
   (ii) General features of incentive schemes
   (iii) Labour costing and labour cost control.
   (iv) Job evaluation, merit rating, labour turnover and their impact on labour cost.

(e) Overhead Cost Accounting and control procedures including:
   (i) Overhead classification and analysis
   (ii) Overhead allocation, apportionment and absorption

(f) Cost Centre:
   (i) Selection and attributable costs
   (ii) Product and departmental costs

(g) Cost behavioural patterns and cost estimation

2. **COSTING METHODS (10%)**

   (a) Operation cost
   (b) Specific order costing: Job, batch, uniform and contract costing including work-in-progress
   (c) Process costing, including principles of equivalent units, treatment of normal and abnormal losses and gains
   (d) Joint products and by-product costing.
   (e) Service costing.

3. **BUDGETING AND BUDGETARY CONTROL (10%)**

   (a) Forecasting problems and techniques.
   (b) Budgeting process and organisation.
   (c) Preparation of functional budget such as fixed and flexible budgets, cash budget and master budget.
   (d) Behavioural aspects of budgetary control.
   (e) Preparation and reconciliation of budgeted and actual results.

4. **STANDARD COSTING TECHNIQUES (10%)**

   (a) Types and basis of setting standards such as basic, current, ideal and normal standards.
   (b) Methods of determining standard cost and the uses of standard cost.
   (c) Types of variances and their analyses.
5. **OTHER COSTING TECHNIQUES (10%)**

(a) Absorption costing
   (i) Reciprocal services
   (ii) Absorption of overheads by product or service
   (iii) Absorption costing method of income statement preparation

(b) Marginal costing
   Uses of marginal costing in decision making such as production planning, pricing decision, make or buy decision, closure of operation, elimination of product/department/unit, accept or reject, accepting additional order.

(c) Cost- Volume - Profit Analysis
   (i) Limitations of Break-even analysis
   (ii) Types of break-even charts.
   (iii) Methods of calculating break-even point and other levels of activities.
   (iv) Margin of safety and ascertainment of angle of incidence.
   (v) Profit volume ratio.
   (vi) Prove of various levels of activities through marginal costing income statement.

(d) Activity Based Costing Technique

6. **COST CONTROL (5%)**

Cost control and reduction techniques such as
(a) value analysis
(b) work study
(c) method study
(d) quality control techniques
(e) merit rating
(f) job evaluation
(g) work measurement and cost audit.

7. **INTEGRATED ACCOUNTS (5%)**

(a) Book-keeping entries for costing systems, integrated and interlocking systems
(b) Reconciliation of financial and cost accounting profits
(c) Budget Classifications and Chart of Accounts (BC&COA).
(d) Computer Assisted Costing Techniques.
 COMPONENT B: QUANTITATIVE TECHNIQUES

8. STATISTICS (10%)

(a) Index Numbers
(i) Meaning and purpose
(ii) Problems associated with the construction of index numbers
(iii) Unweighted indices: simple aggregate index, price relative
(iv) Unweighted indices: Laspeyre, Paasche, Fisher and Marshall Edgeworth
(v) Applications of index numbers to business analysis.

(b) Probability
(i) Concept and meaning: random experiment, sample space, sample point, events, etc.
(ii) Measurement/determination of simple probability directly from sample
(iii) Additive law of probability/mutual exclusive events
(iv) Multiplication law and conditional probability/independent events

9. BUSINESS MATHEMATICS (15%)

(a) Set Theory
(i) Concept and definition
(ii) Types of sets: null, subset, finite, infinite, universal and equal sets.
(iii) Cardinality: number of elements in a set and number of all possible sub-sets in a set.
(iv) Operations: union, intersection, difference and complement.
(v) Euler-Venn diagram: using Euler-Venn diagram in set theory.
(vi) Applications of set theory to solving business related problem

(b) Matrix
(i) Array of numbers
(ii) Meaning of a matrix
(iii) Types of matrices: identity, null, square, diagonal, symmetric and skew symmetric, triangular matrix (upper and lower).
(iv) Basic operations with matrix: Addition, subtraction, multiplication of matrices and conditions under which these are possible; transpose of a matrix.
(v) Meaning of determinant and its evaluation
(vi) Minor, cofactor, matrix of cofactors and adjoint matrix.
(vii) Inverse of a square matrix: its definition and determination (other methods in addition to adjoint approach may be used. Also, limited to at most 3 by 3 matrix).
(viii) Solution of systems of linear equations including Cramer’s rule.
(ix) Applications of matrices to solving business related problems.

(c) **Differentiation**
(i) Concept and Meaning: Measures of rate of change as derivative or slope measure of marginal.
(ii) Rules for differentiating the following functions: polynomial, product, quotient, function of function, implicit function, exponential and logarithmic functions.
(iii) Second-order derivative.
(iv) Application of differentiation: finding marginal, break-even points, elasticity, maximum and minimum values.
(v) Simple partial differentiation of first order.

10. **OPERATIONS RESEARCH (15%)**

(a) **Basic Linear Programming Techniques**
(i) Concept and meaning (as a resource allocation tool).
(ii) Underlying basic assumptions
(iii) Problem formulation in linear programming.
(iv) Methods of solution
   ♦ graphical methods (for two decision variables only)
   ♦ simplex method (limited to three decision variables)

(b) **Network analysis**
(i) Concept and meaning
(ii) Network diagram – Activity –on-Node Network diagram - Activity –on –arrow Network diagram
(iii) Concept of the following terms:
   ♦ floats, slack, earliest time, latest time, etc.
(iv) Methods of network analysis
   ♦ Critical Path Method (CPM): using floats and without using floats
meaning of critical path and how to determine it
Programme Evaluation Review Technique (PERT)

(c) Replacement analysis
(i) Meaning and purpose
(ii) Replacement of items that wear gradually.
(iii) Replacement of items that fail suddenly.

(d) Transportation Model
(i) Nature of transportation models
(ii) Special linear programming problems.
(iii) Concept of balanced and unbalanced transportation problems.
(iv) Methods of providing basic initial solutions to transportation problems: Northwest Corner rule, Least-cost, Vogel’s approximation method.
(v) Test for optimality of the solution using the Stepping-Stone algorithm method.
(vi) Solving assignment problem as a special transportation model.

C. RECOMMENDED TEXTS:

- Biggs, W.W., Cost Accounts, Macdonald and Evans
- Dandago, K.I. and Tijani, B. Cost and Management Accounting, Lagos: Malthouse Press
- Marsland, M.W., Quantitative Techniques for Business, Polytechnic
TABLE OF CONTENTS

FOREWORD iii
PREFACE v
ACKNOWLEDGEMENT xi
LIST OF ABBREVIATIONS xiii
SYLLABUS AND EXAMINATION QUESTIONS FORMAT xv

TABLE OF CONTENTS xxv

CHAPTER 1 INTRODUCTION TO COST ACCOUNTING 1
1.0 Learning Objectives 1
1.1 Introduction 1
1.2 Cost Defined 2
1.3 Accounting as a Concept 2
1.4 Cost Accounting 3
1.5 Cost Terminologies 3
  1.5.1 Cost Unit 3
  1.5.2 Cost Centre 4
  1.5.3 Cost Control 5
  1.5.4 Cost Elements 5
1.6 Cost Accounting Sense 6
1.7 Qualities of Good Cost Accounting Information 10
1.8 Summary and Conclusions 11
1.9 Revision Questions 11
  1.9.1 Multiple Choice Questions 11
  1.9.2 Short Answer Questions 12

CHAPTER 2 MATERIALS ACCOUNTING AND CONTROL 13
2.0 Learning Objectives 13
2.1 Introduction 13
2.2 Materials 13
2.3 Materials Acquisition 14
  2.3.1 Purchase Requisition 14
  2.3.2 The Procurement Department 15
  2.3.3 The Purchase Order Form (POF) 16
  2.3.4 The Goods Receiving Department 17
  2.3.5 Debit and Credit Note 18
  2.3.6 The Accounts Department 19
2.4 Materials Planning and Control 19
  2.4.1 Requirements for a Good System of Materials Planning and Control 20
2.4.2 Documents for Material Planning and Control 21
   2.4.2.1 Material Requisition Form (MRF) 21
   2.4.2.2 Materials Return Note (MRN) 22
   2.4.2.3 The Bin Card (BC) 23
   2.4.2.4 Stores Ledger Card 25

2.5 Materials Pricing 26
   2.5.1 First-In-First-Out (FIFO) 27
   2.5.2 Last-In-First-Out (LIFO) 29
   2.5.3 Weighted Average Price (WAP) 30

2.6 Just In Time (JIT) Method of Purchasing and Production 32
   2.6.1 JIT Techniques 32
   2.6.2 JIT Philosophy 32
   2.6.3 Value Added 33
   2.6.4 Problems Associated With JIT 34

2.7 ABC Analysis Techniques (ABC Plan) On Inventory Control 34

2.8 Case Study 36

2.9 Summary and Conclusions 38

2.10 Review Questions 39
   2.10.1 Multiple Choice Questions 39
   2.10.2 Short Answer Questions 40

CHAPTER 3  LABOUR ACCOUNTING AND CONTROL  41
3.0 Learning Objectives 41
3.1 Introduction 41
3.2 Methods of Remuneration 41
   3.2.1 Time-Based Methods 42
   3.2.2 Overtime 42
   3.2.3 Output-Based Methods 43
   3.2.4 Bonus Incentive Schemes 45

3.3 Determination of Labour Cost 48
   3.3.1 The National Social Insurance Trust Fund (NSITF) 48
   3.3.2 The National Housing Fund 48
   3.3.3 Gratuity Scheme 48

3.4 Accounting Treatment of Labour Cost 49
3.5 Control of Labour Cost 49
3.6 Job Evaluation and Merit Rating 51
3.7 Summary and Conclusions 51
3.8 Revision Questions 52
   3.8.1 Multiple Choice Questions 52
   3.8.2 Short Answer Questions 52

CHAPTER 4  OVERHEAD COST ACCOUNTING AND CONTROL  55
4.0 Learning Objectives 55
4.1 Introduction 55
4.2 Important Concepts and Terminologies 56
   4.2.1 Overheads 56
   4.2.2 Direct Expenses 57
### 6.7.1 Peculiar Features of Contract Costing System 96
### 6.7.2 Accounting for Contracts 97
### 6.8 Summary and Conclusions 104
### 6.9 Revision Questions 104
#### 6.9.1 Multiple Choice Questions 104
#### 6.9.2 Short Answer Questions 105

<table>
<thead>
<tr>
<th><strong>CHAPTER 7</strong></th>
<th><strong>PROCESS COSTING AND JOINT PRODUCTS COSTING</strong> 107</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7.0</strong></td>
<td>Learning Objectives 107</td>
</tr>
<tr>
<td><strong>7.1</strong></td>
<td>Introduction 107</td>
</tr>
<tr>
<td><strong>7.2</strong></td>
<td>Characteristics of Process Costing 108</td>
</tr>
<tr>
<td><strong>7.3</strong></td>
<td>Product Flow 109</td>
</tr>
<tr>
<td><strong>7.4</strong></td>
<td>Equivalent Units 111</td>
</tr>
<tr>
<td><strong>7.5</strong></td>
<td>Transferred In 113</td>
</tr>
<tr>
<td><strong>7.6</strong></td>
<td>Accounting Treatment of Spoilages 113</td>
</tr>
<tr>
<td><strong>7.7</strong></td>
<td>Valuation Process for Cost Statement 115</td>
</tr>
<tr>
<td><strong>7.8</strong></td>
<td>Cost of Production Report 116</td>
</tr>
<tr>
<td><strong>7.9</strong></td>
<td>Joint and By-Products Costing 126</td>
</tr>
<tr>
<td><strong>7.10</strong></td>
<td>By-Product and Its Accounting Treatment 127</td>
</tr>
<tr>
<td><strong>7.11</strong></td>
<td>Accounting Treatment of Joint Cost 127</td>
</tr>
<tr>
<td><strong>7.11.1</strong></td>
<td>Physical Units Basis 128</td>
</tr>
<tr>
<td><strong>7.11.2</strong></td>
<td>Sales Value (At The Point Of Separation) 129</td>
</tr>
<tr>
<td><strong>7.11.3</strong></td>
<td>Net Realizable Value Basis 130</td>
</tr>
<tr>
<td><strong>7.12</strong></td>
<td>Total Cost per Unit Determination Using NRV Method 131</td>
</tr>
<tr>
<td><strong>7.13</strong></td>
<td>Summary and Conclusions 131</td>
</tr>
<tr>
<td><strong>7.14</strong></td>
<td>Revision Questions 132</td>
</tr>
<tr>
<td><strong>7.14.1</strong></td>
<td>Multiple Choice Questions 132</td>
</tr>
<tr>
<td><strong>7.14.2</strong></td>
<td>Short Answer Questions 133</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>CHAPTER 8</strong></th>
<th><strong>BUDGETING AND BUDGETARY CONTROL</strong> 135</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8.0</strong></td>
<td>Learning Objectives 135</td>
</tr>
<tr>
<td><strong>8.1</strong></td>
<td>Introduction 135</td>
</tr>
<tr>
<td><strong>8.2</strong></td>
<td>What Is A Budget? 136</td>
</tr>
<tr>
<td><strong>8.3</strong></td>
<td>Budgeting Framework (Process) 136</td>
</tr>
<tr>
<td><strong>8.3.1</strong></td>
<td>Goal(s) Identification 137</td>
</tr>
<tr>
<td><strong>8.3.2</strong></td>
<td>Collection and Analysis of Data 137</td>
</tr>
<tr>
<td><strong>8.3.3</strong></td>
<td>Choose Decision Rules 138</td>
</tr>
<tr>
<td><strong>8.3.4</strong></td>
<td>Ranking of Alternative Courses of Action 138</td>
</tr>
<tr>
<td><strong>8.3.5</strong></td>
<td>Make a Decision and State Expected Outcome 138</td>
</tr>
<tr>
<td><strong>8.3.6</strong></td>
<td>Monitor Actions to Ensure Results 138</td>
</tr>
<tr>
<td><strong>8.3.7</strong></td>
<td>Measurement and Reporting Of Actual Results 139</td>
</tr>
<tr>
<td><strong>8.4</strong></td>
<td>Budget Limiting Factors 139</td>
</tr>
<tr>
<td><strong>8.5</strong></td>
<td>General Purposes of Budget 139</td>
</tr>
<tr>
<td><strong>8.5.1</strong></td>
<td>Accountability and Control 139</td>
</tr>
</tbody>
</table>
8.5.2 Management
8.5.3 Planning
8.5.4 Economic Policies

8.6 Budgetary Control Process
8.6.1 Establishment of Objectives
8.6.2 Budget Centres
8.6.3 Budget Co-Ordination
8.6.4 Budgets Approval
8.6.5 Measurement of Actual Performance
8.6.6 Feedback Actions

8.7 Budget Committee and Budget Officer
8.7.1 Duties of Budget Committee
8.7.2 Budget Officer

8.8 Types of Budget
8.8.1 Time Perspective of Types of Budget
8.8.2 Activity Perspective of Types of Budget
8.8.3 Quantitative Perspective of Types of Budget

8.9 Budgetary Control and Responsibility Accounting

8.10 Budgetary Improvement Techniques
8.10.1 Incremental Budgeting (IB)
8.10.2 Zero Based Budgeting (ZBB)
8.10.3 Continuous (Rolling) Budgeting (CB)
8.10.4 Planning, Programming, Budgeting System (PPBS)

8.11 Preparation of Functional and Master Budgets
8.11.1 Master Budgets

8.12 Flexible Budget

8.13 Summary and Conclusions

8.14 Revision Questions
8.14.1 Multiple Choice Questions
8.14.2 Short Answer Questions

CHAPTER 9 STANDARD COSTING
9.0 Learning Objectives
9.1 Introduction
9.2 Objectives of Standard Costing
9.2.1 Definitions
9.2.2 Setting Standards
9.2.3 Types of Standard
9.2.4 Correlation of Quantity & Prices

9.3 Variance Analysis
9.3.1 Possible Causes of Variances
9.3.2 The Pyramid of Variances
9.3.3 Inter-Relationship of Variances

9.4 Calculation of Variances

9.5 Material Cost Variance
9.5.1 Material Price Variance
9.5.2 Material Usage Variance
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5.3</td>
<td>Further Analysis of the Materials Usage Variance</td>
<td>189</td>
</tr>
<tr>
<td>9.6</td>
<td>Labour Cost Variance</td>
<td>189</td>
</tr>
<tr>
<td>9.7</td>
<td>Material Mix and Yield Variances</td>
<td>191</td>
</tr>
<tr>
<td>9.7.1</td>
<td>Material Yield Variances</td>
<td>192</td>
</tr>
<tr>
<td>9.8</td>
<td>Alternative Basis for Material Price Variance</td>
<td>194</td>
</tr>
<tr>
<td>9.9</td>
<td>Overhead Variances</td>
<td>194</td>
</tr>
<tr>
<td>9.9.1</td>
<td>Fixed Overhead Cost Variance</td>
<td>196</td>
</tr>
<tr>
<td>9.9.2</td>
<td>Fixed Overhead Expenditure Variance</td>
<td>197</td>
</tr>
<tr>
<td>9.9.3</td>
<td>Fixed Overhead Volume Variance</td>
<td>197</td>
</tr>
<tr>
<td>9.10</td>
<td>Control Ratios</td>
<td>204</td>
</tr>
<tr>
<td>9.10.1</td>
<td>The Standard Hour</td>
<td>204</td>
</tr>
<tr>
<td>9.11</td>
<td>Summary and Conclusions</td>
<td>205</td>
</tr>
<tr>
<td>9.12</td>
<td>Revision Questions</td>
<td>206</td>
</tr>
<tr>
<td>9.12.1</td>
<td>Multiple Choice Questions</td>
<td>206</td>
</tr>
<tr>
<td>9.12.2</td>
<td>Short Answer Questions</td>
<td>207</td>
</tr>
</tbody>
</table>

**CHAPTER 10  COST DATA FOR SHORT-RUN TACTICAL DECISION MAKING** | 209
| 10.0 | Learning Objectives | 209 |
| 10.1 | Introduction | 209 |
| 10.2 | Nature of Managerial Decision Making | 210 |
| 10.3 | The Concept of Marginal Costing | 210 |
| 10.4 | Comparing Absorption and Marginal Costing | 212 |
| 10.5 | The Concept of Contribution | 215 |
| 10.6 | Other Managerial Areas of Marginal Costing | 218 |
| 10.6.1 | Make or Buy Decisions | 218 |
| 10.6.2 | Mix of Sales | 221 |
| 10.6.3 | Acceptance/Rejection of Special Order | 224 |
| 10.6.4 | Price Determination Based On Capacity to Sale | 225 |
| 10.6.5 | Allocation of Scarce Resources | 227 |
| 10.7 | Importance of Marginal Costing | 229 |
| 10.8 | Cost-Volume-Profit Analysis | 230 |
| 10.9 | Presentation of Break-Even Analysis | 232 |
| 10.10 | Summary and Conclusions | 243 |
| 10.11 | Revision Questions | 244 |
| 10.11.1 | Multiple Choice Questions | 244 |
| 10.11.2 | Short Answer Questions | 245 |

**CHAPTER 11  COST CONTROL AND COST REDUCTION** | 247
| 11.0 | Learning Objectives | 247 |
| 11.1 | Introduction | 247 |
| 11.2 | Cost Control | 247 |
| 11.2.1 | Tools for Cost Control | 248 |
| 11.3 | Cost Reduction | 249 |
| 11.3.1 | Tools for Cost Reduction | 249 |
| 11.4 | Developments in Cost Accounting | 251 |
| 11.5 | Summary and Conclusions | 253 |
| 11.6 | Revision Questions | 253 |
CHAPTER 15  SET THEORY  307
15.0 Learning Objectives  307
15.1 Introduction  307
   15.1.1 Definition and Concept of Sets  307
15.2 Types of Sets  308
15.3 Set Operations  311
15.4 An Euler-Venn Diagram  312
15.5 Cardinality of a Set and the Power of a Set  314
   15.5.1 Definitions  314
15.6 Applications  315
15.7 Summary and Conclusions  318
15.8 Revision Questions  319
   15.9.1 Multiple Choice Questions  319
   15.8.2 Short Answer Questions  319
CHAPTER 16  INTRODUCTION TO MATRICES  321
  16.0 Learning Objectives  321
  16.1 Introduction  321
    16.1.1 Basic Concepts  321
  16.2 Types of Matrices  322
  16.3 Basic Operations on Matrices  325
  16.4 Determinants  328
  16.5 Minor, Cofactor And Adjoint Of A Matrix  332
  16.6 Systems of Linear Equations  340
  16.7 Business Application  348
  16.8 Summary and Conclusions  350
  16.9 Revision Questions  351
    16.9.1 Multiple Choice Questions  351
    16.9.2 Short Answer Questions  351

CHAPTER 17  BASIC CONCEPTS OF DIFFERENTIATION  353
  17.0 Learning Objectives  353
  17.1 Introduction  353
  17.2 Second – Order Derivative  364
  17.3 Application of Differentiation  365
  17.4 Partial Differentiation of First Order  371
  17.5 Summary and Conclusions  372
  17.6 Revision Questions  373
    17.6.1 Multiple Choice Questions  373
    17.6.2 Short Answer Questions  373

CHAPTER 18  BASIC LINEAR PROGRAMMING  375
  18.0 Learning Objectives  375
  18.1 Introduction  375
  18.2 Concept and Meaning of Linear Programming  375
  18.3 Basic Assumptions of Linear Programming  377
  18.4 Problem Formulation in Linear Programming  378
  18.5 Methods Of Solving Linear Programming Problems  381
  18.6 Use of Artificial Variable/Methods of Handling It in L.P. Problem  396
  18.7 Duality Concept in L. P. Problem  404
  18.8 Summary and Conclusions  407
  18.9 Revision Questions  390
    18.9.1 Multiple Choice Questions  408
    18.9.2 Short Answer Questions  408

CHAPTER 19  NETWORK ANALYSIS  411
  19.0 Learning Objectives  411
  19.1 Introduction: Concept and Meaning of Network Analysis  411
  19.2 Some Basic Terms in Network  412
  19.3 Network Diagram  413
    19.3.1 Comparison of AOA and AON Diagrams  415
  19.4 Methods of Network Analysis  416
19.4.1 CPM Procedure 417
19.4.2 Alternative Methods of Obtaining Critical Path 420
19.4.3 Pert Procedure 421
19.5 Summary and Conclusions 424
19.6 Revision Questions 424
  19.6.1 Multiple Choice Questions 424
  19.6.2 Short Answer Questions 425

CHAPTER 20 REPLACEMENT ANALYSIS 427
20.0 Learning Objectives 427
20.1 Introduction 427
20.2 Replacement of Equipments/Items That Wear-Out Gradually 428
20.3 Replacement Of Equipments/Items That Fail Suddenly 433
20.4 Summary and Conclusions 436
20.5 Revision Questions 437
  20.5.1 Multiple Choice Questions 437
  20.5.2 Short Answer Questions 437

CHAPTER 21 TRANSPORTATION MODEL 439
21.0 Learning Objectives 439
21.1 Introduction 439
  21.1.1 Nature of Transportation Model 439
21.2 Methods For Solving Transportation Problem 441
21.3 Optimality Test 451
  21.3.1 Stepping Stone Algorithm Method 451
21.4 Profit Maximization Problem (In Transportation Model) 454
21.5 Assignment Problem as a Special Class of the Transportation Model 455
  21.5.1 Solution to Assignment Model 455
21.6 Maximization Problem 458
21.7 Summary and Conclusions 458
21.8 Revision Questions 459
  21.8.1 Multiple Choice Questions 459
  21.8.2 Short Answer Questions 459

CHAPTER 22 COMPUTERS ASSISTED COSTING TECHNIQUES 461
22.0 Learning Objectives 461
22.1 Introduction 461
22.2 Electronic Spreadsheet – An Introduction 462
22.3 What-If Analysis 465
22.4 Scenario 466
22.5 Depreciation Calculation 468
22.6 Charting 470
22.7 Summary and Conclusions 478
22.8 Revision Question 478
  22.8.1 Multiple Choice Questions 478
22.8.2 Short Answer Questions 479

APPENDICES 481

APPENDIX I - SUGGESTED SOLUTION TO REVISION QUESTIONS 483
APPENDIX II - COMPREHENSIVE QUESTIONS AND ANSWERS 501
APPENDIX III - NORMAL DISTRIBUTION TABLE 557
APPENDIX IV - GLOSSARY OF TERMS 579
APPENDIX V - BIBLIOGRAPHY 589
APPENDIX VI - STUDY AND EXAMINATION TECHNIQUES 591
INDEX 595
INTRODUCTION TO COST ACCOUNTING

1.0 LEARNING OBJECTIVES

After studying this chapter, readers should be able to understand:

♦ The meaning of cost, accounting and cost accounting;
♦ The classification of costs;
♦ An objective / scientific ways of determining total cost of a product / service unit; and
♦ The qualities of a good cost accounting system.

1.1 INTRODUCTION

Business of all sizes - micro, small and medium and large scale and of all kind; profit and not-for-profit making whether in the private or public sector needs information relating to the cost of goods and services they produce and render. Cost accounting is relevant to these businesses and organisations because it is one of the sources from which information relating to cost can be obtained and effectively managed.

Cost Accounting, a subject that provides management with information relating to costs, is clearly a service to management. Management of a manufacturing or service organisation, private sector or public sector organisation, profit-making or non-profit-making organisation, military or civilian organisation would ensure that cost accounting is accorded the necessary attention it deserves. The management of an organisation ensures the creation of a department, section or unit to take care of the supply of cost accounting information for managerial decision making.
1.2 **COST DEFINED**

In accounting, cost is defined as sacrifice rendered for benefit derived, while in economics it is defined as alternative foregone. Therefore to economists, cost is what must be given up in order to obtain something. If, for example, you part with ten naira (₦10) to obtain a biro, the cost of that biro is the ten naira (₦10) you are parting with. When you sacrifice some resources, for example, money, machines and materials in order to execute a project, the cost of the completed project is the total monetary value of the resources sacrificed for the project.

Accountants, extending the perspective of economists, view cost as the value of economic resources used in the production of goods, services, income or profit. This shows that the accountant is always linking the concept of cost to production, as cost is being incurred for the production of goods or services, the generation of revenue (income) or the making of profit (negative, positive or zero). Cost elements, that is, materials, labour and expenses, are, therefore, to be considered and aggregated before arriving at the ‘cost’ of production. ‘Cost’ elements can be classified into various ways and is to be carefully identified in relation to various products or services, short term or long-term projects, etc.

1.3 **ACCOUNTING AS A CONCEPT**

Accounting is a term derived from ‘account’ that is simply defined as an expression of transactions. Accounting is, therefore, about giving account of various transactions, that is, explaining the transactions to the satisfaction of interested parties.

Accounting has been defined by different experts and different institutions, in different ways. The definition that might probably encompass all other attempts at defining Accounting is the one given by the American Accounting Association (AAA). The Association defines Accounting as ‘the process of identifying, measuring and communicating economic information to permit informed judgments and decisions by users of the information”. The AAA definition highlights the need to:

(a) Identify users of accounting information and their various needs;
(b) Communicate reliable information to users; and
(c) Make the information relevant to the decisions they want to make.
The definition also shows that Accounting has many purposes. The most important of which is to avail various users of economic information with relevant, reliable, timely and understandable information that would assist them in taking economic, political, social, legal or technological decisions.

1.4 COST ACCOUNTING

Cost Accounting, is about ‘accounting’ for ‘cost’ – two terms that have been explained above. Cost information, predetermined or actual, is usually of interest to the producers of goods, services, income or profit. The consumer is always requesting for the ‘price’ of goods or services. It is, therefore, the managers of organisations (public or private) that would concern themselves about costs of activities to be undertaken for different purposes. The owners of organisations are principally interested in having their ‘funds’ well managed by the managers and in having steady inflow of income in the form of dividends and capital gains. Cost accounting, therefore, concerns managers of the organisations.

Cost Accounting can be defined as the process of identifying, analysing, computing and reporting cost to management. This shows that, unlike accounting as a whole (of which cost accounting is a branch), cost accounting has only the management of an organisation as the user of its information. The fact that cost accounting is about cost identification, analysis, computation and reporting should be enough to show how important that branch of accounting is to propel management of big or small organisations; manufacturing, distributive or service organisations; and public-sector or private-sector organisations, as they all need information related to cost of products, services, operations, functions or projects, both budgeted and actual.

1.5 COST TERMINOLOGIES

Some important cost terminologies that students will come across in this study pack and other texts on Cost Accounting are explained below:

1.5.1 Cost Unit

This is a unit of quantity of product or service in relation to which costs may be ascertained or expressed. Some examples are:
(a) Units of product: Contracts, tons of cement, litres of liquid, books, pairs of shoes, caps, tables, etc.

(b) Units of service: Kilowatt-hours, cinema seats, passenger-kilometre, hospital operations, consulting hours, etc.

In relation to each of the products or services mentioned above, costs (of material, labour and expenses) can be ascertained or expressed. The cost accountant should be able to avail management with information about the total cost of a product or service per unit.

1.5.2 Cost Centre

This refers to a location, a person or an item of equipment in relation to which costs may be ascertained and used for the purpose of cost control.

A location may mean a department, store-yard, sales area, or a factory. A person may be sales manager, production manager, personnel manager, finance manager or work engineer. An item of equipment may be machine, delivery vehicles, car, truck and lathe.

As the purpose of cost centre is to aid effective cost control within an organisation, all the costs incurred will have to be charged to the relevant cost centre. It is then that cost control could be exercised. For example, in an academic department of a University, all the costs incurred must be charged to that department as a cost centre, such as repairs and maintenance of vehicles, stationeries, entertainment, telephone, postage, etc. A car, as a cost centre, is to be charged with depreciation, petrol, tyres, maintenance, licence, insurance, etc.

In relation to each of the cost centres mentioned above, allocation of resources can be made at the beginning of an accounting period for cost to be incurred and report is to be submitted at the end of the accounting period as to how the money is spent. Comparison can be made between the amount allocated and the amount spent, bearing in mind the purpose(s) to be achieved. Appropriate action is taken on the variance that is bound to arise.
1.5.3 Cost Control

This refers to the ability of management to monitor and supervise expenditures (re-current and capital) in order, to ensure that things are going according to plan and that actual results (cost incurred) are obtained for comparison against planned results (cost to be incurred) so that appropriate corrective action(s) can be taken on the variance that is bound to arise before it is too late.

Before any cost control can be effected, standards or targets of performance must be set against which actual costs can be measured and compared. This would reveal inefficiencies so that appropriate actions could be taken to guard against such occurrence in the future.

1.5.4 Cost Elements

These are the items of cost making up the total cost of a product or service. These items, to be identified and aggregated, are as follows:

(a) **Prime or Direct Cost:** This is made up of direct material costs, (DMC), direct labour costs (DLC) and direct expenses (DE). Direct materials are those that actually become part of the product or service while direct labour costs (wages) refer to the wages paid to employees for the time they are engaged in working on the direct materials and other production inputs. Direct expenses are expenses incurred specifically for the product or service.

(b) **Factory Cost:** This is made up of prime cost plus the share of fixed production overhead costs chargeable to the product or service. Fixed production costs are incurred in production, that is, overheads incurred within the four walls of the factory. Factory cost is about total manufacturing cost of a cost unit (product or service).

(c) **Total Cost:** This would be given by the sum of factory cost plus the share of selling, administrative and distributive overhead expenses attributable to the product or service. Selling expenses are those incurred in inducing customers to place orders (advert and similar expenses). Distributive expenses are those incurred in getting finished products to reach the customers. They include warehousing,
packaging and transportation expenses. Administrative expenses are those incurred in managing the enterprise. They include management costs, accounts, legal and personnel department costs, audit fees and other general administrative costs.

For the apportionment of fixed production overhead expenses and selling, distributive and administrative expenses, there is need for the adoption of a clear and acceptable method of apportioning overhead costs to departments or products/services.

(d) Selling Price: This is about the total cost, as identified above, plus target profit. Although, selling price is not part of the cost elements of the seller, it is about cost of purchase to the buyer. The target profit is to be projected by the management based on its pricing policy.

1.6 COST ACCOUNTING SENSE

Cost accounting sense refers to the need for organisations to critically and objectively trace all the cost elements related to their products or services before arriving at their selling prices. It involves pricing products or services scientifically based on accurate identification and computation of actual total cost of a product or service. It is necessary if management of an organisation is to be able to assess its performance profitability-wise, liquidity-wise or productivity-wise.

Practical Examples

(a) Restaurants

Le Chic, The Hang-out, and Number One Restaurant all based in Kano, Nigeria are examples of service organisations that have cost accounting sense. Even though, they do not go into detail analysis and accurate computation of total cost of their cost unit, they are mindful about the need to apportion their overhead costs to all the units of services produced. A customer should not be surprised to notice that a plate of food in any of these restaurants costs about six (6) times what a similar plate (of the same quantity, of even better taste) with a road-side food seller will cost. In the restaurant, the customer enjoys a lot of comfort that are missing with the road-side food seller. The restaurant is air-conditioned, carpeted, has seats well arranged for the comfort of the customer, has a Television set with cable satellite
(well stationed for viewing) and the customer is served food by well trained stewards – in most cases beautifully dressed smiling ladies. All these cost the restaurant a lot of money to provide and maintain and the investments have to be recouped. As you take a plate of food, you should bear in mind that you are not paying for the food only; you are also paying for the carpet you walk on, the air-conditioner you enjoy, the smile you appreciate and so on and so forth. With cost accounting sense, one is of the view that the plate of food in the restaurant is cheaper than the plate of food from the roadside food sellers. However, in consideration of the unhygienic environment where the roadside operates, it may swing the tide in favour of the restaurant.

(b) **Supermarkets**

Sahad Stores, Zango Stores, Mainagge (Kaura Merchant) Stores all based in Kano, Nigeria, are examples of distributive business organisations with cost accounting sense. They, like the cited restaurants, might not be very knowledgeable or conscious of details of computational requirements for overhead apportionment but, they are very much mindful about the need for them to build their overhead costs elements into the prices of the items they sell. A customer should not be surprised as to why a shirt, a pair of shoes, a gallon or a packet in a supermarket would cost higher than in the open market like Kwari, Kurmi, Rimi, Sabon Gari, Singer, etc. The reason is just that the cost of the comfort you enjoy (in the form of a rented store, fans or air conditioners, carpet, waiters, etc.,) is being apportioned to all the items sold. Albeit, you are not paying for the items alone, but you are also paying for these comforts. If you are to go to the open market for the shopping, the dust, the sun, the noise and the hazard you would encounter could be very harmful to you so much so that you might go back home with illness like fever, headache, catarrh, etc. that might cost you a lot to cure.

(c) **Banks**

Some banks like First Bank of Nigeria Plc, Zenith International Bank Plc, and Union Bank of Nigeria Plc, operating in the banking industry of the Nigerian economy are examples that have cost accounting sense. They are quite aware that overhead cost constitutes a substantial percentage of the total cost of the services they render. They normally take great care in finding out how their services could absorb the overhead expenses.
Many banks in Nigeria, in the 1990s, had to adopt drastic measures of cutting off all avoidable overhead cost (resulting in the scrapping or merger of some posts, departments and sections) before bouncing back to profitability. Many of those who survived the then harsh economic environment were about going to the grave when their result-oriented management teams used their cost accounting sense to get rid of unnecessary overhead and apportioned the necessary ones to the units of services produced for them to bounce back. These banks are now among the highly rated banks in Nigeria, in terms of profit-making, dividend yielding, liquidity position and employee-orientation. All the banks have the policy of apportioning general overhead costs to all relevant branches, if not to all branches nationwide, and the performance of each branch is assessed on the basis of profitability (return on investment or residual income approach). It is with this policy that all available overhead costs could be absorbed in the course of costing the services being rendered to the banks’ customers.

The above three cases are meant to show that it is not only in the manufacturing establishment that cost accounting sense is to be demonstrated. Even, if total cost per unit of a product or service can not be accurately computed, efforts should be made to come up with an objective estimation of how the total cost should be, taking into consideration all the identified cost elements. Cost accounting sense must be demonstrated before arriving at the accurate and reliable price of a product or service.

**ILLUSTRATION 1-1**

The management of Isiaq Ismail Battery Manufacturing Company Limited is planning to fix a selling price per unit of battery for the coming year. The cost accounting department presents the following figures derived from cost records and estimates made to guide the management in pricing decision:

<table>
<thead>
<tr>
<th>Item</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct material cost per unit</td>
<td>220</td>
</tr>
<tr>
<td>Direct wages per unit</td>
<td>400</td>
</tr>
<tr>
<td>Variable overhead cost per unit</td>
<td>150</td>
</tr>
<tr>
<td>Total fixed production overhead cost</td>
<td>850,000</td>
</tr>
<tr>
<td>Total administrative expenses</td>
<td>1,400,000</td>
</tr>
<tr>
<td>Total selling and distribution expenses</td>
<td>750,000</td>
</tr>
</tbody>
</table>

The company uses per unit basis of apportioning fixed (general) overhead, and the planned production for the year is 5,000 batteries.
The figure represents about 60% of plant capacity utilization; it is the maximum number that can be sold in the market. The company has a policy of charging 70% of total cost as target profit.

**Required**

Compute the selling price per unit of a battery in the company for the coming year.

**SUGGESTED SOLUTION 1-1**

The Prime cost, factory (total production) cost and actual total cost are to be computed before arriving at the selling price. General overheads are to be apportioned using the planned production level to determine cost per unit as follows:

- **Fixed production overhead cost per unit** = \( \frac{₦850,000}{5,000} = ₦170 \)
- **Administrative expenses per unit** = \( \frac{₦1,400,000}{5,000} = ₦280 \)
- **Selling and Distribution expenses per unit** = \( \frac{₦750,000}{5,000} = ₦150 \)

**Total Cost Per Unit and Selling Price Determination**

<table>
<thead>
<tr>
<th>Cost Category</th>
<th>Cost (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct material cost</td>
<td>220</td>
</tr>
<tr>
<td>Direct wages cost</td>
<td>400</td>
</tr>
<tr>
<td>Direct Expenses</td>
<td>150</td>
</tr>
<tr>
<td>Prime Cost</td>
<td>770</td>
</tr>
<tr>
<td>Fixed Production overhead cost</td>
<td>170</td>
</tr>
<tr>
<td>Factory (Total Production) cost</td>
<td>940</td>
</tr>
<tr>
<td>Administrative Expenses</td>
<td>280</td>
</tr>
<tr>
<td>Selling and Distributive Expenses</td>
<td>150</td>
</tr>
<tr>
<td><strong>Total Cost</strong></td>
<td>1,370</td>
</tr>
<tr>
<td><strong>Target Profit (70% of ₦1370)</strong></td>
<td>959</td>
</tr>
<tr>
<td><strong>Selling price</strong></td>
<td>2,329</td>
</tr>
</tbody>
</table>

The selling price above might appear so low that the management might have to raise it up to tally with the price obtaining in the market. It might also appear so high that the management might have to bring it down to what obtains in the market. The management should, however, think of making the price of its product very competitive (a little lower than the price of competitors), bearing in mind the need to establish and maintain high quality of the product.
1.7 **QUALITIES OF GOOD COST ACCOUNTING INFORMATION**

In order to give room for good planning, effective control and visionary decision-making, cost information must be obtained. Cost accounting information is always needed for managerial planning, control and decision making in both private and public sector organisations. The information should bear certain qualities for it to be seen as capable of assisting management towards bringing about positive changes in the affairs of the organisation. The required qualities of good cost accounting information are:

(a) **Relevance:** The information should be relevant to the user(s) and the purpose for which it is intended.

(b) **Timeliness:** The information should be supplied timeously for the purpose for which it is required. It should be made available within the time it is called for. The cost accountant should know when management needs cost accounting information and make it readily available.

(c) **Reliable/Accurate:** The information should be accurate and produced in compliance with relevant statutory regulations, professional requirements or managerial guidelines. It is then that the information could be said to be accurate and reliable. If there is need for checking or certification before it is put to use, that should be done.

(d) **Understandable:** The information should be simple to understand. The cost accountant has to strike a balance between simplicity and complexity while producing the information.

(e) **Completeness:** The information should fully disclose what is required. It is not to be released in a piece-meal manner. Incomplete cost accounting information is dangerous to managerial decision-making, which is about the future. The future, as it should be understood, is full of risks and uncertainties and these are not to be decided upon using incomplete information.

(f) **Objectivity:** The information is not to be subjective or personal to the provider. It should be the type that would assist management towards achieving organisational objectives and goals. Cost accountants should learn to remove all the subjectivity in the information they give management for planning, control and decision-making.
(g) **Comparable:** The information should give room for comparative analysis with information provided in the previous period(s). It should enhance inter-firm, intra-firm and other forms of comparative analysis, for easy performance assessment.

### 1.8 SUMMARY AND CONCLUSIONS

The reader has been taken through various aspects of cost accounting. The meaning, nature and importance of cost, accounting and cost accounting have been discussed. Important cost terminologies such as *cost unit, cost centre, cost control* and *cost elements* have been explained and exemplified. The concept of *Cost Accounting Sense*, is illustrated using the case studies developed on restaurants, supermarkets and banks in Nigeria. Finally, the qualities expected of a good cost accounting information were explained.

*Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.*

### 1.9 REVISION QUESTIONS

#### 1.9.1 MULTIPLE CHOICE QUESTIONS

1. Prime Costs consist of  
   A. Direct Material and Direct Labour  
   B. Direct Labour and Production Overhead  
   C. Fixed Costs and Variable Costs  
   D. Direct Labour only  
   E. Direct Material only

2. Which of the following is not a Product Cost?  
   A. Direct Labour  
   B. Factory Overhead  
   C. Selling and Distribution costs  
   D. Direct Material  
   E. Variable costs

3. A cost that changes in proportion to a change in activity level is a  
   A. Mixed Cost  
   B. Fixed Cost  
   C. Variable Cost  
   D. Step Cost  
   E. Sunk cost

4. Which of the following is a Product Cost?  
   A. Raw Material  
   B. Selling Cost  
   C. Distribution Cost  
   D. Administration Cost  
   E. Finance Interest
5. The ability of management to monitor and supervise expenditure in order to ensure that organizational objectives are achieved may be termed

A. Cost Control
B. Cost Reduction
C. Cost Allocation
D. Cost Absorption
E. Cost Maximization

1.9.2 SHORT ANSWER QUESTIONS

1. What are Cost Elements?
2. What is the Accountant’s view on cost?
3. What are the qualities of good Cost Accounting?
4. What would the polypropylene going into the manufacture of grain sack be described as?
5. What is Cost Accounting Sense?

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
2.0 LEARNING OBJECTIVES
After studying this chapter, readers should be able to understand:

- The procedures involved in the procurement, storage, usage and accounting for material, as a cost element;
- The strategies for materials control;
- The various methods of valuing closing stocks for balance sheet purposes;
- Just-in-time (JIT) as a modern purchasing and production method; and
- Activity based costing (ABC) techniques as it applies to material control.

2.1 INTRODUCTION
In order to ascertain the cost of a product, service, department, ministry or subsidiary, the cost elements of material, labour and expenses have to be taken into consideration. They all need to be identified, analysed critically and objectively computed and reported to management for pricing and other managerial decision making. This chapter discusses materials accounting and control in detail, bearing in mind organisations that would want to put an effective materials cost accounting system in place.

2.2 MATERIALS
The term ‘materials’ covers a wide range of items. Depending on the nature of the activities carried on by an organisation, materials can take the following forms:

(a) Raw materials, which are needed to produce a finished product. For example, flour and sugar for biscuit, or wood for furniture;
(b) Work-in-Process, which refers to semi-produced raw materials at a particular point in time;

(c) Components or ‘piece parts’, for assembly into a finished product;

(d) Finished products for use or sale. For example, packaged food or bag of cement; and

(e) Indirect materials for use by one or more cost centres of an organisation, such as stationery, fuel and lubricants, and cleaning materials.

Whatever the nature of the business of an organisation, materials are required, obtained, stored and used to achieve its objectives. The cost or management accountant must, therefore, provide management with information that would guide their decision in respect of materials acquisition, planning and control, and pricing.

2.3 MATERIALS ACQUISITION

The various steps to be observed by a cost-conscious organisation in respect of purchase and receipt of materials are as follows:

(a) Initiation of the purchase procedure by means of a purchase requisition;

(b) Preparation of the purchase order and its despatch to the supplier;

(c) Receipt of the materials;

(d) Inspection or testing of the materials;

(e) Debit note to the supplier in respect of defects, rejects, etc.;

(f) Passing of the transaction to accounts department for payment; and

(g) Entry in the relevant accounting books.

2.3.1 Purchase Requisition

This is the formal instruction or request to the Procurement Department to purchase goods/materials. It is initiated by the person or department requiring the goods, who must state the quantity of each type of material required, give a description of the goods (including quality, specification, etc), the account to be charged, a suggested name of the supplier and the point of delivery. Usually, two copies of the Purchase Requisition Form (PRF) would be prepared, one for the Procurement Department and the second for retention in the originating cost centre’s file.
The specimen Purchase Requisition Form (PRF) above could be modified to suit the need of an organisation. In order to ensure accountability, it is advisable to put the form into use.

### 2.3.2 The Procurement Department

Purchasing is a specialist activity frequently carried out by a separate department under the control of a procurement officer or manager. It is a specialist activity, in the sense that the purchaser must have bargaining skills and power to purchase at the lowest possible cost. This will have a positive effect on the overall profit of the organisation. It is to be noted that economical procurement should always be the aim of the business; the manager must not allow quality to suffer. He should buy the right quality, at the right time, at the right price and always from dependable sources of supply.
2.3.3 The Purchase Order Form (POF)

This form is used for ordering supplies. The form is prepared based on the specification included in the purchase requisition form as signed by the procurement manager. It contains name and address of the purchasing organisation, name of the supplier, date of order, terms of payment, prices, delivery instructions and delivery date.

The procurement manager should make his organisation’s requirement absolutely clear and refer to the standard specification published by the Nigerian Standards Organisation (NSO), if need be.

The POF is usually raised in five (5) copies and distributed as follows:
(a) Supplier;
(b) Goods Receiving Department;
(c) Accounts Department;
(d) Initiating Department; and
(e) Procurement Department, for reference purpose.

A SPECIMEN PURCHASE ORDER FORM

| KAYAKWA CEMENT PLC | Order No:………………..
|-------------------|-----------------------
|                   | FOR OFFICE USE ONLY    |
|                   | Dept: ……………………………. |
|                   | Purchase Requisition No:………. |

PURCHASE ORDER

To:………………………………………….
………………………………………….
………………………………………….
Date:………………………………

Please supply in accordance with conditions set out at the back of this order

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>Code</th>
<th>Amount (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Please quote Order Number on all advice notes & invoices

To be delivered at…………………………
On………………………… 20………………

FOR KAYALWA CEMENT PLC

………………………………
Procurement Officer
The specimen Purchase Order Form on page 16 could be adjusted to suit the specific needs of an organisation.

2.3.4 The Goods Receiving Department

In any organisation which is well organised, all incoming materials should be received by Goods Receiving Department. This department checks quantity, quality and condition of items against the copy of purchase order form sent to it as well as the supplier’s advice note. After a satisfactory check, a Goods Received Note (GRN) is raised. The GRN shows the date, supplier’s name, purchase order number, quantity and description of the goods, condition on arrival and details of returnable packing material. The GRN is to be signed by the head of the department or any other person authorized to do so.

The number of copies of GRN to be raised depends on the organisation concerned. In a large organisation, five copies are to be raised and distributed as follows:

(a) Purchasing department;
(b) Originating department;
(c) Stores;
(d) Accounts department; and
(e) The goods receiving department for reference purpose.

Where the receiving, approving and storing of goods are responsibilities of the storekeeper, four copies of the GRN are to be raised.

Sometimes, technical or laboratory inspection is necessary and, therefore, the goods have to be passed to a separate Inspection Section or Laboratory Department, which will provide a quality report either on the GRN or on a separate sheet of paper. In either case, the report is to be forwarded to the Procurement Department.
A SPECIMEN GOODS RECEIVED NOTE

GOODS RECEIVED NOTE
To: Inspection Dept, then to Store Dept or Dept receiving goods/materials

<table>
<thead>
<tr>
<th>Received</th>
<th>No of Boxes/Items</th>
<th>Gross Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Description</td>
<td></td>
</tr>
</tbody>
</table>

GRN No:……………………
Date
Received:………………..
Purchase Order No:……………..
Advice Note No:………………..

Quantity Passed………….. rejected………….

Reason(s) for rejection:…………………………
………………………………………………

Received into Store by:………………
Signature:…………………………
Date:…………………………

Signature of Inspector…………
Date:…………………………

The specimen GRN above could be modified to suit the needs of an organisation.

2.3.5 Debit and Credit Note

When the materials received from the supplier are either:

(a) Not of the type ordered,
(b) Not in accordance with the specification in the Purchase Order Form,
(c) Damaged while in transit, or
(d) Incorrect in quantity

It is usual to forward a Debit Note to the supplier, informing him that his account has been debited with the value of the materials concerned.

The Debit Note is to be prepared by the procurement department in three copies and distributed as follows:

(a) The supplier;
(b) The Accounts Department; and
(c) Retained.
The unsatisfactory materials may be returned to the supplier immediately, or they may be held pending his instructions. If the supplier accepts the claim(s) of the buyer, he signifies his acceptance through the issue of a Credit Note. The Credit Note indicates a credit to the company (debtor/buyer) with the value of the materials on which claim for allowance is made.

2.3.6 The Accounts Department

When an invoice is received from a supplier, the invoice must be receipted and numbered. The Procurement Department will check the description of the goods and prices against the Purchase Order Form. The quantities will also be verified from the Goods Received Note and reference will be made to inspection or laboratory test report. At each stage, the appropriate officers must append their names and signatures. The Procurement Department staff will state the quantity, quality and price and an appropriate ledger code number will be entered.

After this processing, the invoice is passed to the Accounts Department for payment. In this department, an accounts clerk will check the calculations in the invoice and, if found correct, the invoice value will be entered in the Purchase book. From there, the supplier’s account will be credited in the Purchase Ledger with the amount payable to him on the goods purchased. The total of the Purchase book is to be debited to Purchase Account in the General Ledger. When payment is made to the supplier, his account will be debited and cash book (cash or bank account) will be credited.

2.4 MATERIALS PLANNING AND CONTROL

Materials control is the system that ensures the provision of the required quantity of material, of the required quality, at the required time and at the least possible cost. If a cost accounting system is to be fully effective, there must be an adequate system for the control of materials from the time an order is placed with a supplier until the material is issued to production or for other use.

Material represents an important asset and is the largest single item of cost in every manufacturing business and in almost every organisation. Usually, more than 40% of the total manufacturing cost represents material cost. Accordingly, the success or failure of a business concern
may depend largely on efficient material procurement, storage, utilization and accounting.

Efficient materials control eliminates losses and other forms of waste that usually pass unnoticed. Theft, breakage, deterioration and the use of excessive floor space can be reduced to the barest minimum through proper materials control.

2.4.1 Requirements for a Good System of Materials Planning and Control

The major requirements of a satisfactory system of materials control are:

(a) Efficient co-ordination between the departments involved in the buying, receiving, inspection and storage of materials and in accounting for them;

(b) Centralisation of purchasing under the authority of the procurement department or officer;

(c) The materials must be well planned and programmed;

(d) Proper classification and, where applicable, coding of materials;

(e) The use of standard forms for orders, requisitions, checking, reporting, etc., upon which written and signed instructions are given;

(f) Regular budget and budgetary control on materials and equipment to be purchased;

(g) Internal check and audit must be introduced and effected in order to ensure that all transactions involving materials and equipment are checked by reliable and independent officials;

(h) Materials must be stored in well planned and properly designed stores, subject to adequate safeguards and supervision;

(i) Work-in-Process materials too must be adequately recorded and controlled, likewise the manufactured stock; and
(j) Regular reports to management on purchases of materials, storage, issues and in particular, obsolete stocks, returns to suppliers, spoilage or wastage.

2.4.2 Documents for Material Planning and Control

The store department must keep records of all goods received into store and of the physical issues of materials from the store. Four main documents that are used for materials control are:

(a) Material Requisition Form
(b) Material Return Note
(c) Bin Card
(d) Store Ledger Card (Account).

2.4.2.1 Material Requisition Form (MRF)

This form is used by a department, section, or unit or request for materials in the store. This is an important document for both storage control and cost ascertainment in virtually all cost accounting systems. This document is usually raised in three copies. One copy would be retained by the originator and the other two copies are sent to store department to allow authorised withdrawal of goods from the store. Of these two copies, the store department would retain one to up-date their own records, and, having completed the quantity issue section, send the other copy to the Store Ledger section. Here, the material would be priced and a copy of the MRF would be used to update the relevant store card before being forwarded to the Cost Account section. It is this section that will charge the relevant department, unit, product, job or process with the value of materials issued, using the stock valuation system of the organisation. The same document will also be used to update the balances in other relevant ledger accounts.
A SPECIMEN MATERIAL REQUISITION FORM

<table>
<thead>
<tr>
<th>Code or Item No.</th>
<th>Description</th>
<th>Quantity</th>
<th>COST OFFICE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Unit Price</td>
</tr>
</tbody>
</table>

Authorized by:.............  Date:.............  Priced by:.............  Date:.............
Storekeeper:.............  Date:.............  Calculation checked by:.............
Received by:.............  Date:.............  Posted to:.............

Bin Card No:......................

The specimen MRF above could be modified to suit the specific needs of an organisation. It is normally the storekeeper’s responsibility to ensure that the requisition is properly authorised and accurately completed, after which the materials are issued and the stores ledger card updated. The stock ledger account is to be credited, that is, transaction entered in the issues column to arrive at a reduced balance and the account of the appropriate job or cost centre is to be debited.

2.4.2.2 Materials Return Note (MRN)

This is a document which records the return to store department of surplus materials no longer required by the cost centre. MRN can be of virtually identical design to the MRF but coloured differently. The various materials records and cost accounts are to be adjusted to ensure that the cost centre concerned receives credit for the materials returned while the store account is debited for the returned materials.
2.4.2.3 **The Bin Card (BC)**

This is a document on which is recorded every transaction in respect of each item of materials, after the physical receipt or issue. A bin card is normally attached to the bin, drawer or shelf in or on which each individual material is stored. It provides a running record of receipts and issues in the simplest possible form. An entry will be made at the time of each receipt or issue and a new balance will be determined. The bin card records the unit or quantity issued or received into store and not the cost value. For each item, on the bin card, the maximum, minimum and re-order levels as well as the order quantity should be reflected. This is more so in organisations where the store department initiates purchase requisitions.

**A SPECIMEN BIN CARD**

<table>
<thead>
<tr>
<th>BIN CARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum stock level:..........................  Material Code:..................</td>
</tr>
<tr>
<td>Minimum stock level:............................</td>
</tr>
<tr>
<td>BC No:.........................................</td>
</tr>
<tr>
<td>Order Level:.................................  Description:........................</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RECEIPTS</th>
<th>ISSUES</th>
<th>BALANCE</th>
<th>AUDIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>GRN No. Quant.</td>
<td>Date</td>
<td>MRF No. Quant.</td>
</tr>
</tbody>
</table>

The specimen bin card above can be adjusted to suit the specific needs of an organisation.
The stock levels in the bin card are meant to ensure effective control of materials in the store and to avoid excessive stock, wastages, work stoppages, etc. The levels could be defined as follows:

(a) **Maximum Stock Level**: This is the upper level of the inventory and the quantity that must not be exceeded without specific authority from the management of the organisation. The level is given by the formula:

\[ \text{Re-order level} + \text{Re-order Quantity} - (\text{Minimum Usage} \times \text{Minimum Re-order period}) \]

(b) **Minimum Stock Level**: This is the lowest level to which the inventory should be allowed to fall. It is given by the formula:

\[ \text{Re-order level} - (\text{Average Usage} \times \text{Average Re-order period}) \]

(c) **Average Stock Level**: This represents the average addition of maximum and minimum stock levels, it is denoted by the formula:

\[ \frac{\text{Maximum Stock Level} + \text{Minimum Stock level}}{2} \]

(d) **Re-order Level or Order Level**: This is the level to which the materials stock level is to be allowed to fall before an order for further supplies is placed. It is given by:

\[ \text{Maximum Usage} \times \text{Maximum Re-order Period} \]

(e) **Re-order Period or Lead Time**: This is the time taken between the placing of an order and the receipt of the materials.

(f) **Re-order Quantity**: This is the economic order quantity (EOQ) or simply the normal order. It is the quantity that minimizes the total cost of order and holding of stock.

**ILLUSTRATION 2-1**

The following data were extracted from the records of Zahari Ismaila Manufacturing Limited, Kano for a period:

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average usage</td>
<td>100 units per day</td>
</tr>
<tr>
<td>Minimum usage</td>
<td>600 units per day</td>
</tr>
<tr>
<td>Maximum usage</td>
<td>1300 units per day</td>
</tr>
<tr>
<td>Economic order quantity (EOQ)</td>
<td>50,000 Units</td>
</tr>
<tr>
<td>Re-order period</td>
<td>25 to 30 days</td>
</tr>
</tbody>
</table>
Required

Determine the following for the company:
(a) Maximum stock level
(b) Minimum stock level
(c) Re-order level
(d) Average stock level

SUGGESTED SOLUTION 2-1

Re-order level must be determined first before any of the first two stock levels.
(a) Maximum Stock Level = 39,000 + 50,000 – (600 x 25) = 74,000 units
(b) Minimum Stock Level = 39,000 – (1000 x 27.5) = 11,500 Units
(c) Re-order level = 1300 x 30 = 39,000 units
(d) Average Stock Level

\[ \frac{74,000 + 11,500}{2} = 42,750 \text{ units} \]

Maximum, minimum and re-order levels are not static. They must be varied to suit changing circumstances. Alterations will take place if the usage of certain materials is increased or decreased. If the re-order period changes or is decided by the management, the overall inventory must be increased or decreased.

2.4.2.4 Stores Ledger Card

This is the key to an effective materials control system as it provides the details necessary to ascertain material values and for the checking of physical stock of materials. For materials control to be efficient and effective, there must be minimum delay between the physical movement of materials and their recording in the card. For the re-ordering routine to operate efficiently, it is necessary to take account of the amount of materials on order and the existing physical materials held.
### 2.5 MATERIALS PRICING

There are a number of factors to be considered in valuing materials issued for use from the store. Where purchase prices are constant over a long period and there is no variation in quantities purchased, there would be little or no difficulty. In practice, prices fluctuate due to a number of reasons. Such reasons are: Inflation; changes in world commodity prices, buying from different sources, differences in quantity discounts, etc. It is clear that there may be a number of identical materials in the store bought at different prices. When one of these materials is issued, it is necessary to determine the price at which it should be charged.

The closing stocks of finished goods, work-in-progress, raw materials, etc need to be valued for reflection in the balance sheet, as a current asset. A material pricing or valuation method has to be used for the determination of the value of closing stock of different forms of materials. Materials pricing is, therefore, not just about the value of materials issued to the store.
There are many methods of pricing or valuing materials. The most popular method are hereby discussed. Statement of Accounting Standards (SAS) No. 4, on Stocks recommends valuation methods like First-In-First Out (FIFO); Weighted Average (WA), Specific Identification (S.I.), Last-In-First-Out (LIFO), Standard Cost (SC), Based Stock (BS), Last Purchase Price (LPP) and Adjusted Selling Price (ASP) or Retail Inventory Method (RIM) for use by businesses.

The standard, however, lays more emphasis on three methods. These are:

(a) First-In-First-out (FIFO);
(b) Last-In-First-Out (LIFO); and,
(c) Weighted Average Price (WAP).

2.5.1 First-In-First-Out (FIFO)

This method assumes that the materials are used up in the same sequence as they are purchased, that is, the older materials must have been used up first.

FIFO is most suitable where the material is of comparatively high value and not in frequent demand. It is also appropriate where there are relatively few changes in purchase prices, or where the ratio of the number of issues to the number of receipts is not large.

During an inflationary period, the cheapest materials will be issued first, whereas during a deflationary period, it is the most expensive material that would be the first to be issued out. This is so because in the inflationary period, the first batch of materials will be underpriced while in the deflationary period, the first batch of materials will be overpriced.

**ILLUSTRATION 2-2**

Taura Company Limited purchased the following articles: 2003

<table>
<thead>
<tr>
<th>Date</th>
<th>Units</th>
<th>Price per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 1</td>
<td>2000</td>
<td>N100</td>
</tr>
<tr>
<td>Feb. 4</td>
<td>1500</td>
<td>N125</td>
</tr>
<tr>
<td>April 21</td>
<td>3000</td>
<td>N150</td>
</tr>
</tbody>
</table>

Issues were made from the store as follows:

<table>
<thead>
<tr>
<th>Date</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 10</td>
<td>800</td>
</tr>
<tr>
<td>Feb. 15</td>
<td>1000</td>
</tr>
<tr>
<td>March 30</td>
<td>600</td>
</tr>
</tbody>
</table>
You are required to prepare the Stores Ledger Account of the company, using FIFO Method of Stock Valuation to determine the closing stock as at 30 April, 2003.

**SUGGESTED SOLUTION 2-2**

**TAURA COMPANY LIMITED**

**STORES LEDGER ACCOUNT (FIFO METHOD)**

<table>
<thead>
<tr>
<th>DATE</th>
<th>RECEIPTS</th>
<th>ISSUES</th>
<th>BALANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Qty</td>
<td>Price</td>
<td>Value</td>
</tr>
<tr>
<td>2003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan. 1 10</td>
<td>2000</td>
<td>100</td>
<td>200,000</td>
</tr>
<tr>
<td>Feb. 4 15</td>
<td>1500</td>
<td>125</td>
<td>187,500</td>
</tr>
<tr>
<td>Mar. 30</td>
<td>600</td>
<td>200 x 100 + 400 x 125</td>
<td>70,000</td>
</tr>
<tr>
<td>Apr. 21 30</td>
<td>3000</td>
<td>150</td>
<td>450,000</td>
</tr>
</tbody>
</table>

Using the FIFO method, and under perpetual inventory system, the closing stock as at 30 April, 1993 was 2100 units and its value was ₦315,000. The total value of materials issued to different cost centres within the four months period was ₦522,500.

Perpetual inventory system is about recording of purchases and issues continuously up to the end of a specified period. This is against the periodic inventory system, which is about the determination of stock value at the end of a certain period only.

Using the FIFO method, and under periodic inventory system, the value of closing stock = 2100 x ₦150 (as the ending units come to the last batch) = ₦315,000. The value of materials used = Total value of materials purchased in the period minus the value of closing stock (₦837,500 – ₦315,000 = ₦522,500).
FIFO method has the following advantages to its users:
(a) It is easy to operate.
(b) The value is based on cost and so no profits or losses will arise.
(c) The stock balance will represent a fair commercial valuation of stock.
(d) The method ensures that older materials are issued out first.
(e) The method is accepted for accountancy and taxation purposes.

The method, however, has the following disadvantages:
(a) It is burdensome, at times, to apply.
(b) The price of issued materials may not reflect current economic value.

2.5.2 Last-In-First-Out (LIFO)

This method ensures that the most recently purchased materials will be the first to be issued and are always issued at actual cost. The method is used to relate cost, as closely as possible, to current price levels.

### ILLUSTRATION 2-3

Based on the information in illustration 2-2, prepare the Store Ledger Account of the Company using LIFO Method of Stock Valuation and determine the value of closing stock as at 30 April, 2003.

### SUGGESTED SOLUTION 2-3

#### STORES LEDGER ACCOUNT (LIFO METHOD)

<table>
<thead>
<tr>
<th>DATE</th>
<th>RECEIPTS</th>
<th>ISSUES</th>
<th>BALANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Qty</td>
<td>Price</td>
<td>Amount</td>
</tr>
<tr>
<td>2003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan. 1</td>
<td>2000</td>
<td>100</td>
<td>200,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feb. 4</td>
<td>1500</td>
<td>125</td>
<td>187,500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mar.30</td>
<td></td>
<td></td>
<td>600</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apr. 21</td>
<td>3000</td>
<td>150</td>
<td>450,000</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>150</td>
<td>837,500</td>
</tr>
</tbody>
</table>
Using the LIFO method, under perpetual inventory system, the value of closing stock (2100 Units) = ₦260,000. The total value of materials issued to different cost centres = ₦577,500.

Using the LIFO method, under periodic inventory system, the value of closing stock = 1000 x ₦150 + 1100 x ₦100 = ₦260,000. The value of materials used = ₦837,500 - ₦260,000 = ₦577,500.

LIFO method has the following advantages:

(a) It is simple to operate
(b) The Profit figures reported reflect more stability of what makes accounting information a better guide to the management.
(c) The cost charged to production is closely related to current price level.

The method, however, has the following disadvantages:

(a) It involves considerable clerical work.
(b) The valuation of stock balance may not be accepted for income tax purposes.
(c) Using stock valuation on the oldest materials in the store may present a false working capital position in the balance sheet.
(d) It can not be fairly used for the purpose of comparing job costs.

2.5.3 Weighted Average Price (WAP)

This is a method whereby the ruling price (WAP) is determined by dividing the total cost of materials in the store by the total quantity of materials in the store. This method is quite different from the others (FIFO and LIFO) in that for every receipt, there is the need to arrive at a new ruling price.

**ILLUSTRATION 2-4**

Based on the information in Illustration 2-2, prepare the Store Ledger Account of the company, using the WAP method of stock valuation and determine the value of closing stock as at 30 April, 2003.
The Weighted Average Price (WAP) is calculated each time a new purchase is made. Using the WAP method, under perpetual inventory system, the value of closing stock (2100 units) = N294,656. The total value of materials issued to different cost centres = N542,844. Using the WAP method, under periodic inventory system, the value of closing stock = 2100 x 837500/6500 = 2100 x N128.85 = N270,585. The total value of materials issued to different cost centres = N837,500 – N270,585 = N566,915.

WAP method has the following advantages:

(a) It evens out fluctuations in the price of materials.
(b) The use of the method is not limited to materials with stable prices.
(c) The need to calculate a new ruling price arises only with new receipt of stock.
(d) It presents a fair indication of stock values.

WAP method, however, has the following disadvantages:

(a) It is difficult to operate.
(b) Ruling prices usually run to a number of decimal places.
(c) Issues to production and other uses may not reflect current economic values.
2.6 JUST IN TIME (JIT) METHOD OF PURCHASING AND PRODUCTION

2.6.1 JIT Techniques

CIMA Official Terminology (1996) defines Just in Time (JIT), Just in Time Production, and Just in Time Purchasing as follows:

(a) **Just In Time (JIT)**
This is a system whose objective is to produce or to procure products or components as they are required by a customer or for office use, rather than for storage. A Just-in-Time system is a ‘pull’ system, which responds to demand, in contrast to a ‘push’ system, in which stocks act as buffers between the different elements of the system, such as purchasing, production and sales.

(b) **Just In Time Production**
This is a production system which is driven by demand for finished products whereby each component on a production line is produced only when needed for the next stage.

(c) **Just-In-Time Purchasing**
This is a purchasing system in which material purchases are contracted so that the receipt and usage of material, to the maximum extent possible, coincide.

2.6.2 JIT Philosophy

Although described as a system in the CIMA official terminology, JIT is more of philosophy or approach to management since it encompasses a commitment to continuous improvement and the search for excellence in the design and operation of the production management system.

The aim of JIT, according to February 1996 Edition, ACCA Students’ Newsletters, is to ‘streamline the flow of products through the production process and into the hands of customers’. E.J. Hay (The Just-In-Time Breakthrough) identified seven aspects of JIT as follows:
<table>
<thead>
<tr>
<th>Aspect</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>JIT Purchasing</td>
<td>The use of small, frequent deliveries against bulk contracts. This requires close integration of suppliers with the company’s manufacturing process.</td>
</tr>
<tr>
<td>Machine cells</td>
<td>The grouping of machines or workers by product or component instead of by type of work performed.</td>
</tr>
<tr>
<td>Set-up time Reduction</td>
<td>The recognition of machinery set-ups as non-value-adding’ activities which should be reduced or even eliminated.</td>
</tr>
<tr>
<td>Uniform loading</td>
<td>The operating of all parts of the productive process at a speed which matches the rate at which the final product is demanded by the customer.</td>
</tr>
<tr>
<td>Pull System (Kanban)</td>
<td>The use of a kanban, or signal, to ensure that products/components are only produced when needed by the next process. Nothing is produced in anticipation of need, to remain in stock.</td>
</tr>
<tr>
<td>Total Quality</td>
<td>The design of products, processes and vendor quality assurance programmes to ensure that the correct product is made to the appropriate quality level on the first pass through production.</td>
</tr>
<tr>
<td>Employee Involvement</td>
<td>JIT involves major cultural change throughout an organisation.</td>
</tr>
<tr>
<td></td>
<td>This can only be achieved successfully if employees at all levels are involved in the process of change and continuous improvement that is inherent in the JIT philosophy.</td>
</tr>
</tbody>
</table>

### 2.6.3 Value Added

JIT aims to eliminate all non-value-added cost. Value is added only while a product is actually processed. Whilst it is being inspected for quality, moving from one part of the factory to another, waiting for further processing and held in store, value is not being added. Non value-added activities should, therefore, be eliminated.
2.6.4 Problems Associated with JIT
JIT might not be appropriate in all circumstances because of the following reasons:

(a) It is not always easy to predict patterns of demand.
(b) JIT makes the organisation far more vulnerable to disruptions in the supply chain.
(c) JIT, originated by Toyota, was designed at a time when all of Toyota’s manufacturing was done within a 50 km radius of its headquarters. Wide geographical spread makes JIT difficult.
(d) Suppliers are likely to charge a premium price for contractually guaranteed frequent small deliveries.

2.7 ABC ANALYSIS TECHNIQUES (ABC PLAN) ON INVENTORY CONTROL
ABC plan is a systematic way of grouping materials into separate classification and determining the degree of control for each class or group. Material control for high volume items is different from that of low value item, and ABC plan is used in inventory control, when a company has a large number of individual items with each item having different value. ABC plan has been defined as “a selective method of inventory control; attempt to segregate and group materials according to total value”. It is a classification scheme for deciding what tools to use in inventory control.

ILLUSTRATION 2-5
The Soky Corporation groups its materials into separate classification for purposes of stock control. The following data is to be analysed by management.

<table>
<thead>
<tr>
<th>Stock number</th>
<th>Yearly usage (units)</th>
<th>Unit cost N</th>
</tr>
</thead>
<tbody>
<tr>
<td>126</td>
<td>7750</td>
<td>6.00</td>
</tr>
<tr>
<td>241</td>
<td>10900</td>
<td>0.50</td>
</tr>
<tr>
<td>250</td>
<td>7300</td>
<td>1.80</td>
</tr>
<tr>
<td>333</td>
<td>4500</td>
<td>60.00</td>
</tr>
<tr>
<td>401</td>
<td>3500</td>
<td>12.00</td>
</tr>
<tr>
<td>560</td>
<td>13500</td>
<td>1.00</td>
</tr>
<tr>
<td>817</td>
<td>1500</td>
<td>63.00</td>
</tr>
<tr>
<td>900</td>
<td>2000</td>
<td>14.00</td>
</tr>
</tbody>
</table>
Required:

Using the ABC method of control, arrange the materials into three classifications and prepare the chart that will be used in management’s analysis.

SUGGESTED SOLUTION 2-5

Steps:

(1) Compute the total cost of materials;

(2) List the total usage costs in descending order;

(3) Compute:
   (a) The percentage of each item’s cost to total cost
   (b) The percent of each item’s unit to total units; and

(4) Divide them into three categories.

<table>
<thead>
<tr>
<th>Stock No</th>
<th>Cost per unit</th>
<th>Yearly usage</th>
<th>% usage</th>
<th>Total usage cost</th>
<th>% total usage</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>333</td>
<td>60</td>
<td>4500</td>
<td>8.83</td>
<td>270000</td>
<td>52.62</td>
<td>A</td>
</tr>
<tr>
<td>817</td>
<td>65</td>
<td>1500</td>
<td>2.94</td>
<td>94500</td>
<td>18.42</td>
<td></td>
</tr>
<tr>
<td>126</td>
<td>6</td>
<td>7750</td>
<td>15.22</td>
<td>46500</td>
<td>9.06</td>
<td>B</td>
</tr>
<tr>
<td>401</td>
<td>12</td>
<td>3500</td>
<td>6.87</td>
<td>42000</td>
<td>8.19</td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>14</td>
<td>2000</td>
<td>3.92</td>
<td>28000</td>
<td>5.46</td>
<td></td>
</tr>
<tr>
<td>560</td>
<td>1</td>
<td>13500</td>
<td>26.50</td>
<td>13500</td>
<td>2.63</td>
<td>C</td>
</tr>
<tr>
<td>250</td>
<td>1.8</td>
<td>7300</td>
<td>14.33</td>
<td>13140</td>
<td>2.56</td>
<td></td>
</tr>
<tr>
<td>241</td>
<td>0.5</td>
<td>10900</td>
<td>21.39</td>
<td>5450</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>50950</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
<td><strong>513090</strong></td>
<td></td>
</tr>
</tbody>
</table>

11.77% of the items = 71.04% of usage cost = A
26.01% of the items = 22.71% of usage cost = B
62.22% of the items = 6.25% of usage cost = C
### Classification and Control Characteristics

<table>
<thead>
<tr>
<th>Classification</th>
<th>Control characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Needing individual more elaborate analysis. Small quantity of safety stock, frequent review, frequent orders, detailed records, most capable personnel.</td>
</tr>
<tr>
<td>B</td>
<td>Reviewed quarterly A bridge between A and C.</td>
</tr>
<tr>
<td>C</td>
<td>Essentially loose and inexpensive control Large quantity of safety stock, strict adherence, to predetermined order points with little review (one or more program) orders a year, no need for perpetual inventory, lower level personnel.</td>
</tr>
</tbody>
</table>

### 2.8 CASE STUDY

**WOMEN WORLD, PLC**

**A Case Study on Material Valuation and General Management**

Women World Plc was one of the biggest battery manufacturing companies in the 20th century, with shares allotted only to women. The company had advertised for the post of Accountant II with particular emphasis on a good knowledge of Cost Accounting. All the successful applicants who were short-listed for interview were not able to impress the company and, so, another applicant who submitted her application a little bit late was called for a special interview. Binta Jibrin was the one. She had a good degree in Accountancy from one of the highly respected Universities.

When she returned from her coffee break at 10.30 a.m. on Monday, Amina Garba, Director of Finance of the company, found a note from Amina Abubakar, Managing Director/Chief Executive, asking her to come down to her office as soon as possible for the scheduled interview. When Amina Garba arrived, she found Amina Abubakar and Jummai Othman, Company Secretary/Legal Adviser, well set to call the candidate. Bilki Sabo, Controller of Personnel and Administration, was absent as she had travelled overseas on an official duty.

Amina Garba came with the information that the company’s bank was requesting all of its major loan customers to estimate loan requirements for the remaining part of the year because of the tight money situation that was developing in the spring of the following year. Amina Abubakar had a luncheon appointment with Saude Mukhtar, the bank loan officer, on the following day – Tuesday. As Amina Abubakar wanted Amina Garba to provide her with an estimate
of financial requirements for the balance of the year, she exempted her from the interview. Amina Abubakar herself would be leaving on a business trip to London that same Monday afternoon and would return the following morning, just before the luncheon appointment.

Just as the two ladies were beginning to discuss the mechanics of the interview, Amina Abubakar’s Secretary, Binta Abduljalal, came into the office with two messages: First, Amina Abubakar had just 45 minutes in which to drive to the airport to catch her plane, and second, two very important phone calls were waiting for Jummai Othman in her office – at N20 per minute each.

Jummai Othman was allowed to go and Amina Abubakar had to get ready for the trip. But Binta Jibrin must be interviewed that day as her services, if successful, were highly needed as from Wednesday – two days away!

Amina Abubakar quickly thought of an idea. She knew that the company was looking for someone well versed in cost accounting. She just bore in mind that to be a good cost accountant, one must be very good in stock valuation. She called in Binta Jibrin, went quickly through her credentials and asked her to give a brief about herself, she forwarded these figures to her:

The following purchases of raw materials were made by WW PLC in 2003:

<table>
<thead>
<tr>
<th>Date</th>
<th>Quantity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 1</td>
<td>2,000 units</td>
<td>@ N30</td>
</tr>
<tr>
<td>Feb. 14</td>
<td>1,500 units</td>
<td>@ N35</td>
</tr>
<tr>
<td>April 28</td>
<td>3,000 units</td>
<td>@ N40</td>
</tr>
</tbody>
</table>

Issues were made to the production department from the store as follows:

<table>
<thead>
<tr>
<th>Date</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 16</td>
<td>800 units</td>
</tr>
<tr>
<td>Feb. 18</td>
<td>1,000 units</td>
</tr>
<tr>
<td>Mar. 30</td>
<td>600 units</td>
</tr>
<tr>
<td>Apr. 30</td>
<td>2,000 units</td>
</tr>
<tr>
<td>May 31</td>
<td>500 units</td>
</tr>
</tbody>
</table>

Amina Abubakar told Binta Jibrin that the figures represented purchases and issues of raw materials used to produce its product – battery. She then said:

“**Young girl, what I want you to do within 40 minutes is to prepare stores ledger accounts, using LIFO and Weighted Average Price Methods of inventory valuation and show clearly the value of closing**
stock and the total value of materials issued to the production cost centre. If you can pass this test, the post would be yours!"

Amina Abubakar then turned to Rabi Aliyu, the Chief Security Officer of the company, who had just come in on invitation, and said “I trust that you would not allow her to cheat; and that you would not add a second to her; and that you would keep the script for me; and that she would be served with a tin of Dawa Cola in the end. With this I say bye”. She took her briefcase and left for the airport.

**Required:**
(a) Take the position of Binta Jibrin and win the post for her.
(b) As a student of Management, discuss the organisational structure of Women World PLC. Was it okay? Why?
(c) How do you see the leadership ability of Amina Abubakar?
(d) What must have led to the disappearance of WW, PLC in the 21st century?
(e) Discuss the differences between LIFO and WAP methods of stock valuation.

### 2.9 SUMMARY AND CONCLUSIONS

In this chapter, efforts were made to educate the reader on materials as a cost element. Types of materials and the position they occupy in all types of organisations have been discussed. The procedures to be followed in acquiring materials, testing materials, storing materials, accounting for materials, planning and controlling materials, and valuing materials have been discussed in details, together with explanations on the documents to be used in the process.

Just in Time (JIT) as a method of purchasing and production has been introduced in respect of its terminologies, philosophy and associated problems. The reader is now in a position to give advice on the following issues:

- The five forms of materials
- The relevance of Purchase Requisition Form, Purchase Order Form, Goods Received Note, and Debit/Credit Note, which are used in Purchase procedure.
- Requirements of a good system of material planning and control.
- Documents for materials control.
- FIFO, LIFO and WAP methods of stock valuation.
- The Concept of Just-In-Time (JIT).
2.10 REVIEW QUESTIONS

2.10.1 MULTIPLE CHOICE QUESTIONS

1. A formal instruction or request to the procurement department to purchase materials is known as:
   A. Goods Received Note
   B. Materials Requisition Form
   C. Purchase Requisition Form
   D. Purchase Order
   E. Store Requisition Note

2. A Purchase Order is distributed to the following except:
   A. The Supplier
   B. Goods Receiving Department
   C. Initiating Department
   D. Accounts Department
   E. Personnel Department

3. The document prepared by the Procurement department and issued to a supplier where goods supplied are not according to specification is:
   A. Debit Note
   B. Journal Voucher
   C. Credit Note
   D. Material Return Note
   E. Local Purchase Order
   F. Stock Period

4. The time taken between the placing of an order and the receipt of a material is:
   A. Average Period
   B. Re-order Period
   C. Maximum Period
   D. Minimum Period
   E. Replacement Cost

5. Which of the following is not recognized by SAS4 as a method of valuing stocks?
   A. FIFO
   B. LIFO
   C. Specification Identification
   D. Weighted average
2.10.2 SHORT ANSWER QUESTIONS

1. The Purchasing system in which purchases are contracted so that the receipt and usage of materials to the maximum extent possible, coincide is known as:

2. What is A B C Plan?

3. What does the success or failure of a business concern depend upon to a large extent?

4. What is Stock Turnover?

5. What factors determine re-order level?

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
3.0 LEARNING OBJECTIVES

After studying this chapter, readers will be able to understand:

♦ The various methods of remuneration;
♦ General features of incentive schemes;
♦ Determination of labour cost;
♦ Accounting treatment of wages cost;
♦ Control of labour cost;
♦ Labour turnover, causes and how to minimize it.; and
♦ Job evaluation and merit rating.

3.1 INTRODUCTION

The employment of human labour in the production process gives rise to labour cost, that is, the remuneration paid. Whereas, the level of labour engagement in the production process has significantly reduced in many major manufacturing processes due to technological advancement, labour is still employed in supervising the production process. In Africa, especially, where the technological application to production has not reached such an advanced stage, labour cost forms a significant proportion of total product cost. The ability of management to manage this cost, therefore, will play a major role in the profitable management of the organisation.

3.2 METHODS OF REMUNERATION

The following methods of remuneration are common in most industries:

(a) Time-based method of remuneration;
(b) Output-based method of remuneration; and
(c) A hybrid of the above two methods.

3.2.1 TIME-BASED METHODS

Under these methods, the remuneration of the worker is a function of the amount of time that he has spent in the period in question. The time could be measured in hours, days or months. For example,

(a) **Hourly-Based Method**

Under this method, the rate of pay of the employee is specified as an amount per hour; and his remuneration for the period is calculated by multiplying that rate by the number of hours that he actually worked during that period.

\[
Wages = \text{No. of hours worked} \times \text{Rate per hour.}
\]

(b) **Day Rate**

Under this method, which is more common in many factories in Nigeria and even in some offices, pay is specified as a rate per day and remuneration is determined by multiplying that rate by the number of days actually worked during the period.

\[
Wages = \text{No. of days worked} \times \text{Rate per day.}
\]

(c) **Monthly Rate**

Here, the rate is fixed per month, or alternatively an amount is fixed per annum but payable monthly. This is more popular in remunerating office, that is, non-production, workers.

3.2.2 OVERTIME

In most time-based systems, a basic period is usually agreed as the working period. For example, a 40-hour working week may be fixed as the basic working period. Where an employee is made to work in excess of this agreed working period, then the period in excess of the agreed basic working period is paid at a higher rate than the basic rate. This payment is known as overtime.

The higher rate at which the overtime is paid will also depend on whether those hours were worked during the week or a weekend or a public holiday.
Overtime worked during ordinary days may be paid at, say, time and a quarter, that is, $1\frac{1}{4} \times$ basic rate per hour. Overtime worked during weekend or public holidays may be paid at time-and-a-half or double. The rate to be applied will always be specified.

Overtime premium forms part of indirect wages except when directly related to a particular job being costed, especially when carried out at the specific request of a customer to expedite delivery.

Overtime may sometimes be introduced as a way of increasing production output without increasing the number of employees.

**Advantages of Time-Based Method**

Some of the advantages claimed for time-based method include the following:

(a) Wages are simple to calculate and understood by workers, especially illiterate and semi-literate workers;
(b) Periodic earnings by employees are averagely stable;
(c) It helps in the attainment of quality work; and
(d) It facilitates planning for labour cost.

**Disadvantages of Time-Based Method**

Time-based methods have the following disadvantages:

(a) There is no incentive for workers to work efficiently, since all the workers in the same grade are paid at the same rate regardless of their output;
(b) Supervision may be needed to avoid idle time. Workers can be idle and yet get paid since payment is based on time rather than output; and
(c) Efficient and inefficient workers are paid the same rate within skills (especially where efficient workers are not compensated with bonus).

### 3.2.3 OUTPUT-BASED METHOD

Under these methods, wages paid are a function of the number of units of the product produced irrespective of the amount of time spent to produce them. This is common in certain industries like block making.

Output-based method may be categorized as:

(a) Straight Piece Rate; and
(b) Differential Piece Rate.
(a) **Straight Piece Rate**

Under this method, a rate per unit of production is specified. Wages are then calculated at that fixed rate multiplied by the number of units produced.

\[
\text{Wages} = \text{Number of good units produced} \times \text{Rate per piece}
\]

(b) **Differential Piece Rate**

This method encourages workers to increase output by offering an increased rate per unit at different levels of production. For example:

- 1 – 1000 units: N5 per unit
- 1001 – 1500 units: N7 per unit
- 1501 – 2000 units: N10 per unit

Thus, if an employee produces 800 units in a period, his pay would be 800 \times N5 that is, N4000. Where, however, the production is 1,200 units, his pay would be \((1000 \times N5) + (200 \times N7)\), that is, N6,400.

**Advantages of Output-Based Method**

(a) It encourages increased production, sales volume also consequently increasing;
(b) It discourages idleness of workers at work;
(c) It is simple for employees to understand and calculate; and
(d) Due to increased production volume, average fixed cost per unit can be reduced.

**Disadvantages of Output-Based Method**

(a) Employees, in a bid to achieve increased output, often overwork the machines;
(b) Quality of output can suffer due to the desire to just increase output but with less regard for quality;
(c) In order to avoid defective quality, supervision costs may have to be increased;
(d) Fixing a rate of pay per piece may become difficult and the standard output may be difficult to fix for the different workers;
(e) A flat piece rate may be unfair to new entrants (trainees) and naturally slow workers; and
(f) Where idle time occurs due to factors outside the employees’ control, such as power failure and machine breakdown, the employees become dissatisfied.
ILLUSTRATION 3-1

X, Y, and Z are engaged in the production of candies. 2000 candies are expected to be produced in a day of 8 hours and five day week. Employees in Section A are paid at the rate of N50 per hour. Section B are paid on a daily basis at the rate of N350 per day while employees in Section C are paid on the basis of the number of candies produced. The rate in section C is N4 per candy.

In the month of June 2005, it was ascertained that each worker put in 192 hours producing 1980 candies in a period of 22 days. Calculate the amount of wages earned by each employee assuming that:

X works in Section A
Y works in Section B
Z works in Section C.

(Ignore overtime and bonus).

SUGGESTED SOLUTION 3-1

### CALCULATION OF WAGES EARNED BY EMPLOYEES

<table>
<thead>
<tr>
<th>Remuneration Basis</th>
<th>Hourly rate X</th>
<th>Day rate Y</th>
<th>Piece rate Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candies Produced</td>
<td></td>
<td>1,980</td>
<td></td>
</tr>
<tr>
<td>Rate per candy (N)</td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Remuneration</td>
<td></td>
<td>N7,920</td>
<td></td>
</tr>
<tr>
<td>No of days worked</td>
<td></td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>Rate per day</td>
<td></td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>Remuneration</td>
<td></td>
<td>N7,700</td>
<td></td>
</tr>
<tr>
<td>No of hours worked</td>
<td></td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>Rate per hour</td>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Remuneration</td>
<td></td>
<td>N9,600</td>
<td></td>
</tr>
</tbody>
</table>

3.2.4 BONUS INCENTIVE SCHEMES

Due to the shortcomings of the afore-mentioned general methods of remuneration, that is, time-based and the output-based, many organisations introduce bonus schemes to compensate for the shortcomings. Bonus schemes are also introduced to compensate employees for inconveniences caused by asking them to work during unsocial hours. Some of these schemes are:
(a) **Shift Bonus**

It is common in many industries now to find shift working as a way of boosting production within few days of work or meeting the demand of customers without unduly stretching the employee. For example, a fast-food company may work two shifts, one from 7am to 2pm, and another from 2pm to 9pm, to prepare early for the morning demand and also attend to the late evening demand. An extra amount in addition to normal wages is, therefore, paid as shift bonus except where the employee is on a permanent shift.

(b) **Punctuality Bonus**

This is a bonus paid in addition to normal wages where an employee has a good record of punctuality to work.

(c) **Productivity Bonus Schemes**

These are bonus schemes based on productivity and to encourage further productivity. A standard time is fixed for the completion of a task. Where the employee is able to complete the task in less than the time specified, he becomes entitled to a bonus calculated at his usual time rate of pay multiplied by a proportion of the time he has “saved.” In the case of Hasley 50-50 system, the proportion of time paid to the worker is 50%. In the case of Hasley-Weir, this proportion is 1/3, while in the case of Rowan the proportion is given by the ratio of time taken to time allowed.

(d) **Group Incentive Schemes**

This scheme is applied to a group of workers where work is carried out in groups e.g. road construction. It could be based on ability to meet a deadline or on cost savings, for example,
ability of the group to reduce level of wastes, or an increased output, that is, where the group produced higher than a target output.

Advantages of Group Incentive Schemes

(i) It creates a strong loyalty within the group as all employees work together to ensure that the bonus is earned;
(ii) It often results in reduced absenteeism and increased output as they police one another to avoid time wastages; and
(iii) It can be applied to all categories of workers – whether direct or indirect workers.

Disadvantages of Group Incentive Schemes

(i) It may not encourage individual initiative as each member of the group is paid the same bonus; and
(ii) Efficient and inefficient employees are paid the same amount of bonus.

(e) Profit Sharing

This is a scheme whereby a specified share of the company’s profit is paid as an addition to normal wages. Frequently this is a percentage of profit and is commonly applied to Chief Executives or Profit-Centre Managers.

(f) Co – Partnership

This is a development of profit sharing schemes whereby employees are issued with shares as bonus. The shares thus entitle them to share in the company’s profit, by right as a shareholder.

It has the following advantages:

(i) It improves employee-employer relationship;
(ii) Employees benefit from the success of the company;
(iii) Employees identify themselves with the goals of the organization; and
(iv) Employees become cost conscious and ensure that company makes profits.
3.3 DETERMINATION OF LABOUR COST

It is to be noted that, the cost of labour to an organisation will include not only the basic pay but also the bonuses paid, regardless of the way the pay or bonus is calculated.

Employers are often required to make payments to an account for the benefit of the employee, for example, contribution to Social Insurance Fund (the National Social Insurance Trust Fund (NSITF) now replaced by the pensions fund under the Pensions Reform Act 2004). These also form part of total labour cost.

3.3.1 The National Social Insurance Trust Fund (NSITF)

This is a contributory pension scheme whereby a certain percentage of pay is deducted from employee earnings and a certain percentage of employee pay is also added by the employer. The employee thus becomes entitled to both percentage on withdrawal of service and attaining a specified retirement age. The employer’s contribution to this scheme also forms part of the labour cost.

3.3.2 The National Housing Fund

This is a scheme intended to enhance the chances of workers to have access to good accommodation. Under the National Housing Fund Act, employers are required to deduct from the employees pay a certain percentage (now 2\(\frac{1}{2}\)\% of Basic pay) and the employers to add 5\% of the basic pay to this for the account of the employee.

3.3.3 Gratuity Scheme

Some employers provide for the payment of a lump sum (golden handshake) to employees on retirement or withdrawal of service if they have put in a specified number of years’ service. The amount paid would depend on the length of service put in. A typical scheme provides that employee will become entitled to the gratuity after the first five years and that the calculation be one month’s basic salary for each completed year of service. The contribution by the employer also forms part of total labour cost.

In summary, from the viewpoint of employer, labour cost includes all those payments made to the employee arising because of his employment by the organisation, to work for it on a regular basis. The
accounting treatment of these payments, however, differ. This will be discussed in the next section.

3.4 ACCOUNTING TREATMENT OF LABOUR COST

For accounting purposes, wages or labour cost of production workers form part of prime cost of production while indirect wages are treated as overheads. In other words, only direct wages of production workers and those workers directly engaged in the production process, would be regarded as Direct Wages. Remuneration of other workers, even if it refers to the basic remuneration, would be regarded as indirect, that is, it is indirectly related to the production function; the only exception being where the use is related to a specific demand by a customer to expedite completion of a specific job.

For example, in a manufacturing concern, the wages of the factory operatives would be regarded as direct. In a professional accountant’s office, the remuneration of the professional staff would be regarded as direct while the remuneration of the administrative staff would be regarded as indirect.

However, within the production function, questions often arise as to the treatment of certain payments. For example:

(a) Overtime - Overtime premium should be treated as indirect wages, except as noted above where it is done at the specific request of a customer to expedite delivery;
(b) Bonus – Non-productive bonus, like shift bonus, bonus payments are generally treated as an indirect cost;
(c) Contribution to Pension Schemes or Housing Fund - This is regarded as indirect wages; and
(d) Idle Time - Where production workers are idle and the scheme of remuneration allows for wages to be paid during this idle period, the payment during the idle time will be isolated and treated as indirect wages.

In conclusion, all indirect wages are regarded as overheads. The treatment of overhead is discussed in the next chapter.

3.5 CONTROL OF LABOUR COST

Whereas all elements of cost need to be put under scrutiny to avoid costs not matched by productive efforts, labour appears the most sensitive. Any lay-off of workers in an attempt to reduce costs
frequently engender revolts, disputes and unrest from the labour unions.

In order to keep labour cost under check, therefore, a number of measures need to be put in place:

(a) Set up of Wages Department - A wages department should exist, to arrange the collection of data on employees, prepare the remuneration of employees and provide pertinent information on labour to management.

(b) Adequate Records - Necessary records to monitor the attendance of workers, time sheet, clock cards, output, idle time should be maintained.

(c) Analysis of Idle Time – Idle Time should be analysed and reported periodically to identify major causes of idle time so as to effectively address them. For example, if it is discovered that they were due more to power failure, management could decide to buy a stand-by generator.

(d) Labour Turnover - The rate at which employees leave an organisation and are replaced should be monitored, as this often has effect on productivity and labour cost – training of new staff, cost of recruitment, loss of production due to training of new staff.

Labour Turnover is calculated as follows:

\[
\text{Labour Turnover} = \frac{\text{Number of Employees replaced in the period}}{\text{Average Total Number of Employees in the period}} \times 100\%
\]

Although, it is true to say that labour turnover is inevitable in any organisation, an analysis is still necessary to identify avoidable causes of labour turnover.

Unavoidable causes are:

(i) Redundancy.
(ii) Marriage – relocating to join spouse.
(iii) Domestic reasons.
(iv) Death.

Avoidable causes would include:

(i) Poor working conditions.
(ii) Demand for increased wages or better incentive schemes.
(iii) Lack of ability of employee to perform.
(iv) Dismissal.
Suggestions for the reduction of high labour turnover rate would include:

(i) Improved selection procedures
(ii) Improved working conditions and welfare facilities, for example,
   ♦ Provision of canteen facilities
   ♦ Provision of life assurance schemes
   ♦ Provision of housing schemes
   ♦ Provision of company cars.
   This is why the rate is usually very low in oil companies.
(iii) Review of wages and general working conditions.
(iv) Introduction of/or improving training schemes.
(v) Sharing more interest in employee future through adequate career planning and individual development.

3.6 JOB EVALUATION AND MERIT RATING

Job evaluation and merit rating are techniques used to assist in the reduction of labour costs. Job evaluation seeks to objectively assess the worth of jobs and therefore rank jobs in order of importance and fix the appropriate pay scales. Merit rating is concerned with individual employees. It aims to distinguish between the performances of different individuals at the same grade level. Merit rating evaluates the performance of individual employees in a given period to determine the appropriateness of awarding promotion, bonus, etc. An example of merit rating is the annual appraisal schemes in various offices. It is more subjective than job evaluation.

3.7 SUMMARY AND CONCLUSIONS

The ability of management to manage labour cost is a major role in profitable management of the organisation. Methods of renumeration include time based, output based and hybrid of the two methods.

Bonus schemes include short bonus, punctuality and productivity bonus schemes.

The labour cost of an organisation will include basic pay and other bonuses.

Job evaluation and merit rating techniques are based to reducing labour costs.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
3.8 REVISION QUESTIONS

3.8.1 MULTIPLE CHOICE QUESTIONS

1. One of the following methods of remuneration is common in industries:
   A. Time-based method
   B. Supply-based method
   C. Demand-based method
   D. Fixed income method
   E. Profit-based method

2. Overtime premium should be treated as:
   A. Indirect Wage
   B. Prime Cost
   C. Factory Overhead
   D. Selling Overhead
   E. Direct Cost

3. The rate at which employees leave an organization and are replaced is known as:
   A. Straight Piece Rate
   B. Labour Turnover
   C. Differential Piece Rate
   D. Average Labour Rate
   E. Average Staff Strength

4. Avoidable causes of Labour Turnover include the following except
   A. Poor Working conditions
   B. Redundancy
   C. Lack of ability of employee to perform
   D. Demand for increased wages or better incentive schemes
   E. Bad recruitment policy

5. The productivity Bonus Scheme where an employee is paid according to the ratio between the time taken and time allowed is
   A. Hasley-Weir Scheme
   B. Profit Sharing
   C. Rowan
   D. Co-Partnership
   E. Hasley-Rowan Scheme

3.8.2 SHORT ANSWER QUESTIONS

1. What seeks to objectively assess the worth of jobs and ranks them in order of importance and fix the appropriate pay rate?

2. Remi works 60 hours with a team paid together as a group. The team worked for a total of 1200 hours producing 4500 units. Standard output per hour is 3.2 units. Normal pay is $5 and the group operates under the hasley-weir scheme. Calculate Remi’s bonus under the incentive scheme.
3. Outline two advantages and disadvantages of a group incentive scheme.

4. Outline two advantages and disadvantages of Time-based method remuneration.

5. Briefly explain the following terms:
   (a) Straight Piece Rate
   (b) Differential Piece Rate

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
OVERHEAD COST ACCOUNTING AND CONTROL

4.0 LEARNING OBJECTIVES

After studying this chapter readers should be able to understand the:
* Nature of expenses normally regarded as overheads;
* Various ways by which overheads can be classified;
* Apportionment of overheads including apportionment of service department costs;
* Absorption of overheads into product costs;
* Various overhead recovery rates and choice of appropriate rates; and
* Effect of activity-based costing on overhead allotment.

4.1 INTRODUCTION

The cost of a product or service is information, that is quite often required by management. This is often required for various purposes, for example, stock valuation, profit measurement or decision making. Whereas, cost is simply the aggregate of all the expenses incurred in bringing a product to its present condition and location, it will be discovered that while some expenses are directly traceable to the cost object, some are not. This has also been mentioned in the previous chapters, but the problem remains that production overheads, which are not directly traceable, should form part of the production cost of a product.

“Statement of Accounting Standard No.4 and IAS No. 2, on Stocks require that historical cost of manufactured stocks (inventories) should include a systematic allocation of those production overhead costs that relate to putting the stocks in their present location and condition.”
The process for achieving this transfer of not directly traceable expenses to the cost unit is the subject of this chapter. It is known as **Overhead Absorption**.

However, in view of the development in this area of the subject, it is necessary to mention here that there are two approaches to overhead absorption:

(a) The traditional approach - which is based on the idea of apportioning the total overhead over the production volume, irrespective of whether it is accurate to say that volume was totally responsible, and

(b) The activity – based approach – which challenges the traditional approach but takes into consideration the real factors that cause overhead to vary with production. This is because, it is believed that, what is regarded as overheads is now accounting for significant proportion of total cost and a more accurate method of apportionment is therefore, required. Using this approach would require a different way of classifying overhead cost, and the use of “Cost Roots” and ‘Cost Drivers’.

For the avoidance of doubt, the traditional approach will be discussed first, since this is still very much in use by many industries, especially in the less-technologically advanced countries.

### 4.2 IMPORTANT CONCEPTS AND TERMINOLOGIES

#### 4.2.1 Overheads

These refer to the aggregate of indirect materials, indirect labour and indirect expenses. These could be incurred under any of the functions. That is, there could be production overheads, administrative overheads, selling overheads, etc. Whereas, all these would eventually form part of the total cost of production (rates can also be set for each of these types of overheads), it is important to understand the distinction between these overheads. This is because, most organisations are more concerned with absorbing production overheads for purposes of stock valuation, and when doing so, it is only production overheads that should be covered; there should be no mix up. This is why it is necessary *abini-tio* to explain the terms. Sometimes, examination questions could test candidates understanding of this distinction by asking for the calculation of production overhead rates but including in the list of overhead expenses other classes of overheads.
4.2.2 Direct Expenses
These refer to those expenses that are directly traceable to a particular cost object, for example, direct materials, direct wages, and royalty.

4.2.3 Indirect Expenses
These refer to those expenses that are not directly traceable to a particular cost object, such as, production salaries, rent and rates, electricity, etc.

4.2.4 Overheads
By overheads, is meant, the aggregate of indirect materials, indirect labour and indirect expenses.

4.2.5 Production Overheads
Refer to the aggregate of indirect costs associated with manufacturing activities. Manufacturing activities are the sequence of operations, which begin with supplying materials to work stations and end with the primary packing of the product. Examples of these would include depreciation of plant and machinery, and production salaries.

4.2.6 Administrative Overheads
Administrative overheads encompass the costs of formulating policy, directing the organisation, and controlling the operations of an undertaking which is not related to production, selling, distribution, research or development activity or function. Examples of these include office rent, office electricity, office salaries, printing and stationery, audit and professional fees, etc.

4.2.7 Selling Overheads
These refer to those indirect costs, which are associated with marketing and selling (excluding distribution) activities. Examples of these will include salaries of sales staff, advertisement, sales commissions, etc.

4.2.8 Distribution Overheads
These represent the aggregate of indirect costs associated with the distribution of finished products. Examples of these would include haulage costs to customers, warehouse rents, etc.
4.3 FORMAT OF A COST STATEMENT

An arrangement of expenses in the determination of costs is given to illustrate the points made in paragraph 4.2.1 above.

<table>
<thead>
<tr>
<th>Simple Cost Statement</th>
<th>£</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ Direct materials</td>
<td>X</td>
</tr>
<tr>
<td>+ Direct labour</td>
<td>X</td>
</tr>
<tr>
<td>+ Direct expenses</td>
<td>X</td>
</tr>
<tr>
<td>= Prime cost</td>
<td>X</td>
</tr>
<tr>
<td>+ Production overhead</td>
<td>X</td>
</tr>
<tr>
<td>= Factory or production cost</td>
<td>X</td>
</tr>
<tr>
<td>+ Selling and distribution overhead</td>
<td>X</td>
</tr>
<tr>
<td>+ Administrative overhead</td>
<td>X</td>
</tr>
<tr>
<td>= Total Cost</td>
<td>XX</td>
</tr>
</tbody>
</table>

4.4 STEPS IN OVERHEAD ABSORPTION

In order to enhance our understanding of the subject of overhead absorption, it is recommended that the following definition of overhead absorption is kept in mind.

Overhead Absorption is the allotment of overhead cost to cost units by means of predetermined rates separately calculated for each cost centre.

(a) Cost Centre is a department of an organisation and could also be a location, or an item of equipment in relation to which costs may be ascertained. The organisation is therefore, first divided into cost centres, that is, the departments through which the product passes or work is done to facilitate production. Overhead costs are, therefore, ascertained for each of these cost centres.

(b) Since the rates are pre-determined, the overhead costs for each cost centre will be obtained through a budget process. When actuals are known, the actual overheads are also analysed according to the cost centres so that the result of the use of predetermined rates can be assessed. This study pack discusses the alternative of using rates calculated from factual information. Once the amount of overheads have been estimated or ascertained for each cost centre, the activity levels for each cost centres is also estimated or ascertained.
(c) This activity level or production volume is estimated based on what is believed largely influences overhead costs. Traditionally, the following measures are used:

(i) Machine Hours;
(ii) Labour Hours;
(iii) Amount of Material Cost;
(iv) Amount of Labour; and
(v) Amount of Prime Cost.

The following Overhead recovery rates are in use:

(a) Machine Hour Rate = \( \frac{\text{Estimated Amount of Overhead}}{\text{Estimated Machine Hours}} \)

(b) Labour Hour Rate = \( \frac{\text{Estimated Amount of Overhead}}{\text{Estimated Labour Hours}} \)

(c) Percentage of Material Cost = \( \frac{\text{Estimated Amount of Overhead} \times 100}{\text{Estimated Amount of Materials}} \)

(d) Percentage of Labour Cost = \( \frac{\text{Estimated Amount of Overhead} \times 100}{\text{Estimated Labour Cost}} \)

(e) Percentage of Prime Cost = \( \frac{\text{Estimated Amount of Overhead} \times 100}{\text{Estimated Prime Cost}} \)

4.5 OVERHEAD ANALYSIS

In estimating or ascertaining the amount of overhead for each cost centre, it will be observed that whereas some expenses are identifiable with a particular cost centre, some are common to more than one cost centre. The expenses traceable to a particular cost centre are merely allocated to that centre while the expenses common to more than one cost centre are apportioned to those cost centres using basis considered equitable, for example:

<table>
<thead>
<tr>
<th>To Share</th>
<th>The following basis may be considered equitable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent</td>
<td>Floor space</td>
</tr>
<tr>
<td>Factory Manager’s remuneration</td>
<td>No. of Employees</td>
</tr>
<tr>
<td>Maintenance of machine</td>
<td>No. of Machine hours</td>
</tr>
<tr>
<td>Insurance</td>
<td>Book value of assets insured.</td>
</tr>
</tbody>
</table>
4.5.1 Service Cost Centre

It should be noted, that some cost centres (Service Departments) exist to serve the Production Departments. It follows that the Production Departments’ total costs is not complete without the Service Department costs. Overhead Absorption Rates are not calculated for service cost centres but their costs are re-apportioned to the production departments.

An example of a service department (cost centre) is a maintenance department that run the generators and maintain the production machines in good condition. Their services are rendered to the production departments and so their costs should be borne by the production department. Therefore, to ascertain the total overheads for each Production Department, it would be necessary to go through the processes of allocation, apportionment and re-apportionment.

ILLUSTRATION 4-1

Babariga Plc manufactures three products in two production departments, a machine shop and a fitting section. It also has two service departments, a canteen and a machine maintenance section. Shown below are next year’s budgeted production data and manufacturing costs for the company:

<table>
<thead>
<tr>
<th>Product</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td>4,200 units</td>
<td>6,900 units</td>
<td>1,700 units</td>
</tr>
<tr>
<td>Prime cost:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct materials</td>
<td>N 11 per unit</td>
<td>N 14 per unit</td>
<td>N 17 per unit</td>
</tr>
<tr>
<td>Machine shop</td>
<td>N 12 per unit</td>
<td>N 4 per unit</td>
<td>N 2 per unit</td>
</tr>
<tr>
<td>Fitting section</td>
<td>N 12 per unit</td>
<td>N 3 per unit</td>
<td>N 21 per unit</td>
</tr>
<tr>
<td>Machine hours per unit</td>
<td>6 hrs per unit</td>
<td>3 hrs per unit</td>
<td>4 hrs per unit</td>
</tr>
</tbody>
</table>
It has been estimated that approximately 70% of the machine maintenance section's costs are incurred servicing the machine shop and the remainder incurred servicing the fitting section.

**You are required to:**

(i) Calculate the following budgeted overhead absorption rates,
(ii) A machine hour rate for the machine shop, and
(iii) A machine expressed as a percentage of direct wages for the fitting section.

**SUGGESTED SOLUTION 4-1**

The first step will be to carry out an analysis of Overhead to determine the amount of overhead for each cost centre, ending with the production cost centres only. Thereafter, the activity level for each cost centre will be ascertained bearing in mind the method of absorption considered appropriate for that particular cost, that is, whether machine hour, labour hour, etc.

The amount of overhead for each cost centre is then divided by the estimated activity levels to obtain the required recovery rates for each department.
Thus:

### OVERHEAD ANALYSIS

<table>
<thead>
<tr>
<th>Tutorial</th>
<th>Overhead Item</th>
<th>Basis</th>
<th>Production Departments</th>
<th>Service Departments</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Note</td>
<td></td>
<td></td>
<td>Machine Shop</td>
<td>Fitting Section</td>
<td>Canteen</td>
</tr>
<tr>
<td>1</td>
<td>Allocated Overhead</td>
<td>By Allocation</td>
<td>N27,660</td>
<td>N19,470</td>
<td>N16,600</td>
</tr>
<tr>
<td>2</td>
<td>Rent, Rates, Heat &amp; Light</td>
<td>Floor space</td>
<td>N9,000</td>
<td>N3,500</td>
<td>N2,500</td>
</tr>
<tr>
<td></td>
<td>Depreciation &amp; Insurance of Equip</td>
<td>Gross book value of equip</td>
<td>N12,500</td>
<td>N6,250</td>
<td>N2,500</td>
</tr>
<tr>
<td></td>
<td>Re-apportionment of Canteen costs</td>
<td>No. of Employees</td>
<td>N10,800</td>
<td>N8,400</td>
<td>N21,600</td>
</tr>
<tr>
<td></td>
<td>Re-apportionment of Maintenance</td>
<td>% of service enjoyed</td>
<td>N24,360</td>
<td>N10,440</td>
<td>N34,800</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>N84,320</td>
<td>N48,060</td>
<td>N132,380</td>
</tr>
</tbody>
</table>

#### Overhead Absorption Rates

1. **Machine Shop:**
   - **Machine hour rate** = \( \frac{\text{Budgeted Overhead}}{\text{Budgeted Machine hours}} \)
   - **Machine hour rate** = \( \frac{\text{N84,320}}{52,700} \text{ hrs} = \text{N1.6/ hr} \)

   **Workings**
   
   \[
   \text{Budgeted Machine hrs} = \text{hrs} \\
   X (4200 \text{ units} @ 6\text{hrs/unit}) = 25,200 \\
   Y (6900 \text{ units} @ 3\text{hrs/unit}) = 20,700 \\
   Z (1,700 \text{ units} @ 4\text{hrs/unit}) = 6,800 \\
   \text{Total} = 52,700
   \]

2. **Fitting Section:**
   - **% of Direct Wages:** \( \frac{\text{Budgeted Overhead}}{\text{Budgeted Wages}} \times 100 \)
   - **% of Direct Wages** = \( \frac{\text{N38,060}}{106,800} \times 100 = 35.64\% \)

   **Working**
   
   \[
   \text{Budgeted Wages} = \text{N} \\
   X (4200 \text{ units} @ \text{N12/unit}) = 50,400 \\
   Y (6900 \text{ units} @ \text{N3/unit}) = 20700 \\
   Z (1700 \text{ units} @ \text{N21/unit}) = 35,700 \\
   \text{Total} = 106,800
   \]
### 4.6 INTER-SERVICE COST CENTRES SERVICES

The above example has simply assumed that the canteen department served the fitting department but the fitting department did not provide services to the canteen.

In some other cases, service cost centres may render service to one another. The implication of this is that some amount remain to be shared in a service cost centre when the cost of the other service cost centre is apportioned. In other words, it gives rise to repeated re-apportionment. In this type of situation, the solution is to continue the re-apportionment until the figures become insignificant. This is known as the Continuous Allotment (or Repeated Distribution) method.

Other quicker methods are, however, available, there are:

(a) The elimination method; and
(b) The simultaneous equation (or algebraic) method.

### ILLUSTRATION 4-2

A company has three production and two service departments. The overhead analysis sheets provide the following totals analysed into production and services:

<table>
<thead>
<tr>
<th>Production Dept.</th>
<th>-x</th>
<th>₦48,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-y</td>
<td>₦42,000</td>
</tr>
<tr>
<td></td>
<td>-z</td>
<td>₦30,000</td>
</tr>
<tr>
<td>Service Dept.</td>
<td>-p</td>
<td>₦14,040</td>
</tr>
<tr>
<td></td>
<td>-q</td>
<td>₦18,000</td>
</tr>
</tbody>
</table>

The service departments’ costs are apportioned as follows:

<table>
<thead>
<tr>
<th>Production Depts.</th>
<th>Service Depts</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>p</td>
</tr>
<tr>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>y</td>
<td>q</td>
</tr>
<tr>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>z</td>
<td></td>
</tr>
<tr>
<td>%</td>
<td>%</td>
</tr>
</tbody>
</table>

| Service Dept. p   | 20 | 40 | 30 | 10 |
| Service Dept. q   | 40 | 20 | 20 | 20 |

Using simultaneous equation method, calculate the total overheads charged to the production departments.

### SUGGESTED SOLUTION 4-2

Let P represent total overhead of department p
And Q total overhead of department q
P received 20% of Q’s services
Thus P = 14,040 + 0.2 Q
Likewise Q = 18,000 + 0.1P

Using substitution method of simultaneous equation
P = 14,040 + 0.2 (18,000 + 0.1P)
P = 14,040 + 3,600 + 0.02P
P - 0.02P = 17,640
0.98P = 17640
P = 18,000

Q = 18,000 + (0.1 x 18,000)
Q = 19,800

Overhead charged to Production

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>48,000</td>
<td>42,000</td>
<td>30,000</td>
</tr>
</tbody>
</table>

Allocated Overhead
Share of P’s service
(N18,000 X % served) 3,600 7,200 5,400
Share of Q’s service 7,920 3,960 3,960
(N19,800 X % served) 59,520 53,160 39,360

Using Continuous Re-Apportionment Method

<table>
<thead>
<tr>
<th>Department</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>P</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocated Overhead</td>
<td>48,000</td>
<td>42,000</td>
<td>30,000</td>
<td>14,040</td>
<td>18,000</td>
</tr>
<tr>
<td>P’s Overhead</td>
<td>2,808</td>
<td>5,616</td>
<td>4,212</td>
<td>(14,040)</td>
<td>1,404</td>
</tr>
<tr>
<td>Q’s Overhead</td>
<td>7,762</td>
<td>3,881</td>
<td>3,881</td>
<td>3,880</td>
<td>(19,404)</td>
</tr>
<tr>
<td>P’s Overhead</td>
<td>776</td>
<td>1,552</td>
<td>1,164</td>
<td>(3,880)</td>
<td>388</td>
</tr>
<tr>
<td>Q’s Overhead</td>
<td>155.2</td>
<td>77.6</td>
<td>77.6</td>
<td>77.6</td>
<td>(388)</td>
</tr>
<tr>
<td>P’s Overhead</td>
<td>15.52</td>
<td>31.04</td>
<td>23.28</td>
<td>(77.6)</td>
<td>7.76</td>
</tr>
<tr>
<td>Q’s Overhead</td>
<td>3.104</td>
<td>1.552</td>
<td>1.552</td>
<td>1.552</td>
<td>(7.76)</td>
</tr>
<tr>
<td>Total</td>
<td>59,520</td>
<td>53,160</td>
<td>39,360</td>
<td>1,552</td>
<td>-</td>
</tr>
</tbody>
</table>

4.7 USE OF ACTUAL RATES

It should be noted from the illustration 4-2 that estimated amounts of overhead and estimated activity levels were used in calculating the recovery rates, hence they are called pre-determined absorption rates. The problem that the use of estimates, that is, budgeted amounts will create is that actuals may differ from the estimates and this will give rise to over or under absorption of the overheads.

It may, therefore, be suggested that the absorption rates be computed from the actual information than waiting for the end of the period to know the actual amounts of overheads for each cost centre (though, there
would still be apportionment of common costs) and the actual activity levels. The objections to the use of rates calculated from the factual information are:

(a) It would result in a delay in providing information on job cost, whereas this is needed quickly if it is to be of any value in management decision making.

(b) The above timing problem can be resolved by calculating the overhead rates at more frequent intervals, say, on a monthly basis, but a large amount of overhead is fixed whereas activity will vary from month to month. This will give rise to large fluctuations in overhead rates.

4.8 USE OF SINGLE (BLANKET) RATE

It would be observed from the chosen definition of overhead absorption that separate rates are calculated for each cost centre. In a layman’s language, this is saying that each department has a rate at which it charges the job or product passing through it for the overheads it has incurred in the course of attending to all the jobs.

These rates have to be different because the factors that give rise to overheads in one department may differ from the factors that cause overheads in another. A department may be machine intensive and so incur most of its overhead by way of machine related expenses while another may be labour-intensive, like a packaging department.

However, arguments may be put up for the use of a single rate throughout the factory regardless of the nature of work in the different department, that is, using a single, plant-wide, or blanket rate. This can be allowed in circumstances where:

(a) Only one product is produced;
(b) All the products use the services of all the departments approximately equally; and
(c) Overheads form an insignificant part of total cost.

4.9 ACTIVITY BASED COSTING (ABC) AND OVERHEAD ABSORPTION

It has been stated that the major drawback of the traditional approach to overhead absorption is that overhead is absorbed into product cost on the basis of production volume (measured generally in machine hours, labour hours, etc) regardless of the fact that most of the overhead expenses may not have been the result of that production volume. This, therefore, results
in inaccurate allotment of overhead cost with a consequent effect on product pricing and profitability or profit measurement.

The modern approach called “Activity Based Costing” challenges this traditional approach by arguing that in these days of technological advancement, different factors do influence the overhead expense. These are called cost drivers. It attempts to find a causal relationship between overhead and the cost driver. A cost driver is, therefore, any factor which causes a change in the cost of an activity. An activity may have multiple cost drivers associated with it. For example:

<table>
<thead>
<tr>
<th>For an activity like</th>
<th>The following cost drivers are possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Order Processing</td>
<td>No of customers</td>
</tr>
<tr>
<td></td>
<td>No of sales order</td>
</tr>
<tr>
<td></td>
<td>Size of orders</td>
</tr>
<tr>
<td>Production Control</td>
<td>No of production batches</td>
</tr>
<tr>
<td></td>
<td>No of production set-up</td>
</tr>
<tr>
<td></td>
<td>No of production orders</td>
</tr>
</tbody>
</table>

**Classification of Overheads**

ABC suggests that cost should be ‘pooled’, that is, collected together on the basis of the activities that ‘drives’ the cost regardless of the department where the cost was incurred. In other words, ABC assigns cost to activities cost centres rather than departments. It will, therefore, not be strange to discover that there may be more cost pools than departments.

**ILLUSTRATION 4-3**

Two products X and Y are made using similar equipment and methods. The data for last period are:

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units produced</td>
<td>6,000</td>
<td>8,000</td>
</tr>
<tr>
<td>Labour hours per unit</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Machine hours per unit</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Set-ups in period</td>
<td>15</td>
<td>45</td>
</tr>
<tr>
<td>Orders handled in the period</td>
<td>12</td>
<td>60</td>
</tr>
<tr>
<td>Overheads for period</td>
<td>Ø</td>
<td>89,500</td>
</tr>
<tr>
<td>Relating to production set-ups</td>
<td>89,500</td>
<td></td>
</tr>
<tr>
<td>Relating to order handling</td>
<td>15,000</td>
<td></td>
</tr>
<tr>
<td>Relating to machine activity</td>
<td>27,500</td>
<td></td>
</tr>
</tbody>
</table>
Calculate the overheads to be absorbed per unit of each product based on:
(a) Conventional absorption costing using a labour absorption rate
(b) An ABC approach using suitable cost drivers.

**SUGGESTED SOLUTION 4-3**

Under the traditional method, the formula for labour hour rate is:

\[
\text{Labour hour rate} = \frac{\text{Estimated amount of overhead}}{\text{Estimated labour hours}}
\]

\[
= \frac{132,000}{22,000} = \text{₦}6 \text{ per hour}
\]

Amount of overhead absorbed

\[
\text{Product X} = 6,000 \times (\text{₦}6 \times 1) = \text{₦}36,000
\]

\[
\text{Product Y} = 8,000 \times (\text{₦}6 \times 2) = \text{₦}96,000
\]

\[
\text{Total} = 132,000
\]

Under Cost pools by the drivers are

\[
\text{Set-up} = 89,500
\]

\[
\text{Order handling} = 15,000
\]

\[
\text{Machine activity} = 27,500
\]

Therefore, Cost Driver Rates

\[
\text{Production set-ups} = \frac{\text{₦89,500}}{60} = \text{₦1491.67 per set up}
\]

\[
\text{X = 15 set ups}
\]

\[
\text{Y = 45 set ups}
\]

\[
\text{Order Handling} = \frac{\text{₦15,000}}{72} = \text{₦208.33 per order handled}
\]

\[
\text{Machine Costs} = \frac{\text{₦27,500}}{40,000} = \text{₦0.6875 per machine hour}
\]

\[
\text{X = 6000 x 4 = 24000 hours}
\]

\[
\text{Y = 8000 x 2 = 16000 hours}
\]
**Overheads Absorbed using ABC**

<table>
<thead>
<tr>
<th></th>
<th>Product X</th>
<th>Product Y</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set ups (15x1491.67)</strong></td>
<td>22,375</td>
<td>67,125</td>
<td>89,500</td>
</tr>
<tr>
<td><strong>Orders (12x208.33)</strong></td>
<td>2,500</td>
<td>12,500</td>
<td>15,000</td>
</tr>
<tr>
<td><strong>Machine cost</strong></td>
<td>16,500</td>
<td>11,000</td>
<td>27,500</td>
</tr>
<tr>
<td></td>
<td>41,375</td>
<td>90,625</td>
<td>132,000</td>
</tr>
</tbody>
</table>

**Note:**

The difference between the two approaches is in the allotment of the overheads to the cost units – the products.

<table>
<thead>
<tr>
<th>Product X</th>
<th>Product Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Traditional approach</td>
<td>36,000</td>
</tr>
<tr>
<td>ABC approach</td>
<td>41,375</td>
</tr>
</tbody>
</table>

This could, therefore, have effect on the pricing of the individual products.

**Shortcomings of ABC**

Whereas arguments may be put up to justify the superiority of ABC over the traditional approach, it also has the following limitations:

(a) The assumption of a linear cause/effect relationship between a cost driver and the overhead expense may not be realistic in all cases.
(b) It does not totally remove the problem of common costs apportionment where some overheads cost support several activities.
(c) It is sometimes difficult to trace cost to product where more than one product benefit from a single activity, for example, a single set of activity that supports many products.

**4.10 SUMMARY AND CONCLUSION**

This chapter highlights the nature and classification of overheads. Methods of overhead allocation and apportionment using the continuous allotment or repeated distribution method are discussed. The chapter emphasizes the important concepts, terminologies steps in overhead absorption and concludes with the concepts of Activity Based Costing (ABC).
Activity based costing (ABC) is a modern approach for overhead absorption that says cost should be collected together in the basis of activities that drives the cost regardless of the department where the cost was incurred.

However, ABC assumed linear relationship between cost and overheads; which may not be true in all cases.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

4.11 REVISION QUESTIONS

4.11.1 MULTIPLE CHOICE QUESTIONS

Use the following information to answer questions 1 and 2. Actual production hours planned was 48000 hrs. Standard was 144 minutes per product and fixed overhead was recovered at N3 per unit.

1. If actual production was 2200 units in 4,600 hours, what is the over/under absorption of overhead?
   (a) N600 under absorption
   (b) N600 over absorption
   (c) N300 under absorption
   (d) N300 over absorption
   (e) N400 under absorption

2. If the units produced were sold at N20 each, per unit prime cost was N12 while direct labour was N5 each, using absorption costing what is the reported profit?
   (a) N11,600
   (b) N10,400
   (c) N11,300
   (d) N10,700
   (e) N11,400

3. A cost accountant was handed the following data for further processing

<table>
<thead>
<tr>
<th>Month</th>
<th>Units</th>
<th>Total overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,500</td>
<td>N16,000</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>N21,000</td>
</tr>
</tbody>
</table>

Expected costs for the month are as follow
Direct material N12,000
Direct labour N9,000

Budgeted output is 3000 units. How much is production cost per unit, assuming absorption costing is employed?
   (a) N7.27
   (b) N12.00
   (c) N12.25
   (d) N12.78
   (e) N12.15
4. Budgeted overhead was ₦5000 and expected output was 1250 units. At the end of the period, 1100 units were produced. Actual overhead scheme was ₦5250. How much was overhead absorbed?
   (a) ₦250 under absorption
   (b) ₦250 over absorption
   (c) ₦850 under absorption
   (d) ₦850 over absorption
   (e) ₦500 under absorption

5. The department, location or item of equipment through which a product passes or work is done to facilitate production is known as:
   a. Cost driver
   b. Cost department
   c. Cost centre
   d. Production department
   e. Materials department

4.11.2 SHORT ANSWER QUESTIONS
1. What is Activity Based Costing?
2. Under what circumstances will a single (blanket) rate be used in the absorption of cost?
3. What are the approaches to overhead absorption?
4. Briefly describe the differences between these terms:
   (i) Cost allocation
   (ii) Cost apportionment
5. Briefly explain:
   (i) The meaning of cost centres
   (ii) Overhead absorption

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
LEARNING OBJECTIVES

After studying this chapter, readers will be able to understand:

- The various ways of classifying costs;
- The cost behavioural patterns;
- Cost estimation or segregation processes; and
- The importance of cost analysis in managerial decision making process.

INTRODUCTION

In Chapter 1, we defined cost as the value of economic resources used in the production of goods, services, income or profit. The question to ask is ‘which cost’? That question is welcome in the sense that cost accounting readers must be aware of cost absorption classification.

This chapter attempts to conduct analysis on cost in respect of various ways of classifying it; its behavioural patterns; and its estimation process. The classification and behavioural patterns of cost are necessary for an understanding of the type of cost being referred to while talking about a particular economic resource used in production and the circumstance in which the cost is incurred.

COST CLASSIFICATION

There are many ways of classifying costs for better understanding of the term. The following are the most usual ways:

(a) Classification by Nature

The cost of a cost unit can be broken down into:
- Direct material cost;
- Direct labour cost; and
- Overhead cost.
(i) Direct Material cost is the material cost that can be directly and economically traced as an integral part of the final product or service. Examples include cost of leather used in producing a handbag, cost of cement used in building a house, or cost of flour used in making a loaf of bread. The cost accountant should be able to determine the direct material content of a cost unit and the cost of the material component is to be directly traced to the cost unit.

(ii) Direct Labour Cost is the cost of labour (skilled or unskilled) that can be directly identified with the final product. For example, if a chair requires 3 hours of a carpentry work, then these 3 hours can be directly identified with the cost of the chair. The hourly rate might be N40 and, total direct labour cost per chair is N120.

(iii) Overhead cost is the cost that cannot be directly identified with the cost unit in exact amount, but generally incurred for the production of all goods and services. It is, therefore, about the cost of indirect materials, indirect labour and other manufacturing, selling and administrative costs which can only be apportioned to the final product for absorption. Indirect materials are essentially required for the completion of production, but their use in such a small quantity brings about difficulties in tracing them in a cost unit or even cost centre. They consist of rent, rates and other charges (water bill, telephone bill, electricity bill, etc.), general management salaries, etc. Overhead may be either fixed or variable.

(b) Direct and Indirect Cost

Direct costs are those that can be directly allotted or traced to a particular cost unit or cost centre. Direct material and direct labour costs fall under this class. They vary with volume of output.

Indirect costs, on the other hand, are those that can not be allotted or traced to a particular cost unit or cost centre. They are only to be subjected to apportionment for an accurate determination of total cost per unit or per cost centre. Examples are rent, insurance, depreciation, bad debts, rates and other charges, salaries, etc. Indirect costs may come in the form of
cash expenses or non-cash expenses. In the examples given here, depreciation and bad debt are non-cash expenses whereas others are cash expenses.

(c) **Fixed and Variable Cost**

Fixed costs are those that would remain unchanged no matter the level of activity, for example, production level, sales level, telephone calls, etc. Variable costs (prime costs), on the other hand, are those that vary proportionately with changes in levels of activity. The higher the level of activity, the higher the total variable costs, while total fixed costs will remain constant, in the short-run, despite changes in the levels of activity.

(d) **Controllable and Uncontrollable costs**

This classification is a fall-out of the concept of ‘Responsibility Accounting’. Each responsibility centre manager should be mindful of costs under his control and be aware of costs that are controllable by him. Sales department manager see costs approved to be incurred in his department as controllable, whereas those approved to be incurred in production department, for example, are to be seen as uncontrollable. Direct costs (as they vary with the volume of production) can be termed controllable costs, whereas indirect cost (which must be incurred whether there is production or not) are to be termed uncontrollable costs of a particular responsibility centre.

(e) **Avoidable and Unavoidable Cost**

Avoidable costs are those that can be avoided or eliminated if an activity is stopped or discontinued, whereas unavoidable costs are those that can not be eliminated even if you stop the activity. Direct costs and variable costs are clearly avoidable costs whereas fixed costs are unavoidable costs. This classification is not aimed at the simple explanation given here. The aim is to show how fixed costs can be treated as avoidable costs. When an activity is stopped, management may decide to rent out some of the facilities being used, dispose of some fixed assets and release some rented buildings and equipment back to the owners (landlords). The management would, by so doing, avoid some rent and depreciation expenses, which are fixed costs. The facilities rented out will generate some income for the business and the amount generated can be used to offset other fixed production cost or cost of purchases from another manufacturer.
It is only the fixed costs that would be incurred despite the decision taken by management as a result of stoppage of an activity that are to be considered as unavoidable costs. These are the sunk costs.

(f) Full and Marginal Costs

Full cost is the total cost, mixed costs, semi-variable or semi-fixed costs. It is the cost that reflects ‘absorption’ of some general overhead costs by the cost unit or cost centre. Marginal cost, on the other hand, represent the additional cost of an extra cost unit produced. It is the incremental cost of producing one extra unit. Whereas, full cost is made up of variable cost and fixed cost, marginal cost is about variable cost only, since fixed cost is to be considered as a period cost which must be incurred whether there is production or not, and, is irrelevant in total cost per unit determination.

(g) Joint and Separate Costs

Joint cost is the cost of processing a single raw material to produce two or more products. Separate cost, on the other hand, is the cost of processing the identified products further for them to become consumable or marketable. Joint cost is to be shared or apportioned to the joint products using one basis of apportionment or the other. Separate costs is to be added to the share of joint cost of a joint product to arrive at the total production cost of the product. The single raw material, when processed to the point of split-off, may produce joint products that can be categorised into main products and by-products. The net realizable values of the by-products are to be used in offsetting the joint cost before apportionment to the main products.

(h) Incremental and Sunk Costs

Incremental costs comprise the additional costs of making a change, such as increasing the volume of output, changing production methods, changing product mix, or redesigning a product or service. They are the additional costs incurred resulting from a decision to increase production, and they are variable costs in the context of an increase in output. Marginal cost is a special version of incremental or variable cost – it is the additional cost incurred from increasing output of a cost unit. Sunk costs, on the other hand, comprise all those costs which are
not affected by a decision to increase or decrease production level. They are past costs which have been incurred and cannot be changed. They are never relevant for a decision in respect of additional cost units to be produced.

The cost classifications above can be relevant for decision making purposes as well as for cost control and product cost measurement. They are also relevant for decision as to the profitable extent of further production or processing and for pricing decision by the firm. The distinction between fixed and variable costs, in particular, is crucial for \textit{budgeting purposes} and for \textit{cost control}, and for many short-run tactical decisions such as elimination of appreciably unprofitable products, allocation of scarce resources between competing needs, make or buy decisions, acceptance or rejection of special orders from customers, etc.

5.3 \textbf{COST BEHAVIOURAL PATTERNS}

An important part of cost and management accounting is the provision of information for managerial planning and control. In order to provide the required information, the cost accountant is not only concerned with recording what actual costs were, he/she has to be concerned also with issues like:

(a) What costs are expected to be in the future (for budgeting and planning purposes); and

(b) What should costs have been (for comparison with actual cost and taking appropriate control action).

In order to estimate what costs will be or what cost should have been, for proper control of activities, the accountant must demonstrate clear understanding of cost behaviour. Cost behaviour refers to the way in which costs respond to decrease or increase in the activity level of an organisation.

5.3.1 \textbf{Level of Activity}

This refers to the amount of work done or the number of events that have occurred. Depending on circumstances, level of activity may refer to the volume of production in a period, number of items sold, value of items sold, number of invoices issued or received, number of units of electricity consumed, labour turnover, etc. Level of activity is simply in relation to
quantity of product or service and is normally presented as the *independent variable* on a graph.

### 5.3.2 Basic Principle

The basic principle of cost behaviour is that as the level of activity rises, cost will usually rise. Naturally, it will cost more to produce 400 units of output than it will cost to produce 200 units; it will cost more to make 30 telephone calls than it will cost to make 20 calls, etc. The problem for the accountant to solve, however, is to determine for each item of cost:

(a) In what way do costs rise; and  
(b) By how much, as the level of activity increases?

### 5.3.3 Patterns of Cost Behaviour

Total cost or mixed cost is what shows the behaviour illustrated for basic principle above. Mixed cost can be basically categorized into two: fixed and variable costs. Fixed cost, variable cost and mixed cost behave differently in response to changes in levels of activity. They follow the following patterns:

(a) **Fixed Cost**: This is the cost which tends to be unaffected by increase or decrease in the volume of output. Fixed cost is a *period cost* in that it relates to a span of time and not to the level of activity, in the short run. Total fixed cost (TFC), therefore, would remain unchanged, no matter the level of activity. Fixed cost per unit, however, would be decreasing as activity level increases. The graphs below illustrate the points:

![Graphs illustrating fixed cost and fixed cost per unit](image-url)
An important behavioural pattern of fixed costs is that total fixed cost remains constant but fixed cost per unit decreases with increase in volume of output and vice versa. This shows that fixed cost per unit and volume are inversely related.

Examples of fixed cost are:

(i) Salary of Managing Director;
(ii) The rent of a factory building; and
(iii) Straight-line depreciation of a machine, etc.

(b) **Variable Cost:** This is the cost which varies directly with the volume of output. Variable cost per unit, however, is the same for each unit produced, that is, it remains unchanged no matter the level of activity, in the short-run. Below are graphs that illustrate this concept.

When level of activity is zero, there will be no variable cost. As the level of activity rises from 1 upward, total variable cost (given by variable cost/unit x the number of units) would rise. As activity level decreases, total variable cost (TVC) will also decrease. The relationship between TVC and level of activity is direct.

Variable cost per unit, however, would remain unchanged, in the short-run, no matter the level of activity.

Examples of variable cost are:

(i) Cost of raw materials (where there is no quantity discount);
(ii) Cost of direct labour;
(iii) Sales commission; and
(iv) Bonus payments after achieving a certain performance level.

(c) **Mixed Cost:** This is also referred to as total cost, full cost, semi-variable or semi-fixed cost. Examples of this cost are:

(i) Electricity bills
(ii) Telephone bills, etc. Graphically, it can be illustrated as follows:

```
+---------------------+
|                    |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
|                     |
+---------------------+

Mixed cost behave in the same way as variable cost, that is, showing direct relationship with level of output. The major difference between the two is that whereas TVC starts from zero level, mixed cost starts from TFC level. Mixed costs are simply defined as costs in which there is a standing basic charge and a variable charge per unit of consumption.

(d) **Step Costs:** Many items of cost might be seen as fixed cost in nature within certain levels of activity only. For example, depreciation charge on machine may be fixed if production remains within the capacity of one machine. If more units are to be produced to satisfy increasing demand, another machine has to be purchased and the depreciation on the second machine would make total depreciation charge to go up a step. Step costs are fixed costs that are subject to gradual changes in response to abnormally high productivity. Graphically, they form the pattern of a ladder:
Other examples of step cost are:

(i) Rent, where accommodation requirements increases as output levels get higher; and

(ii) Basic salaries, where more employees are employed on account of demand for increase in output.

5.4 COST ESTIMATION

It is often assumed that within the normal range of output, costs are variable, fixed or semi-fixed. Step costs are fixed within a certain range. For this reason, cost accountants usually treat all costs as fixed or variable, and semi-fixed costs are segregated into their fixed and variable elements.

There are several ways in which fixed costs elements and variable cost elements within semi-fixed cost may be estimated. Each way/method is only an estimate, and would, therefore, give different results from other methods. Some of the estimation methods are:

(a) High-low method;
(b) Scatter-graph (which is about putting cost on Y-axis and Volume of output on X-axis and then drawing a line of best fit through the points of intersection); and
(c) Statistical analysis (simple regression or multiple regression analysis).

5.4.1 The High-Low Method

This method of segregating costs into their fixed and variable elements is simple and crude as it relies on two extreme points (the highest and the lowest levels of activities). It may not be
COST ANALYSIS

very representative of the data. Since this and other methods are emphasising on estimation to be used in guiding decision making pertaining to cost elements, the method’s simplicity should be capitalised upon. It is to be used mathematically, as follows:

(a) Total Cost (TC) of High output = FC + VC (of highest level of activity);
(b) Less: TC of Low output = FC + VC (of lowest level of activity);
(c) Equals difference in variable cost, because the fixed cost is the same at each level of activity;
(d) The variable cost per unit = Difference In VC (as FCs have cancelled themselves) / Difference In output; and
(e) Using the High output, Fixed Cost = TC – (VC/Unit x No. of Units at this level).

**PROFORMA OF HIGH-LOW METHOD**

<table>
<thead>
<tr>
<th>Output</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>XXX</td>
</tr>
<tr>
<td>Low</td>
<td>XXX</td>
</tr>
<tr>
<td>Difference</td>
<td>XX</td>
</tr>
<tr>
<td>VC/Unit</td>
<td>xx = xx</td>
</tr>
</tbody>
</table>

**ILLUSTRATION 5-1**

The operating costs of Afro Master Limited, a firm of electronics manufacturers, for the last six (6) months are as follows:

<table>
<thead>
<tr>
<th>Month</th>
<th>Cost (₦'m)</th>
<th>Production Volume ('000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
<td>15.0</td>
</tr>
<tr>
<td>2</td>
<td>260</td>
<td>18.5</td>
</tr>
<tr>
<td>3</td>
<td>220</td>
<td>14.5</td>
</tr>
<tr>
<td>4</td>
<td>255</td>
<td>15.1</td>
</tr>
<tr>
<td>5</td>
<td>258</td>
<td>15.3</td>
</tr>
<tr>
<td>6</td>
<td>230</td>
<td>14.8</td>
</tr>
</tbody>
</table>

**Required:**

(a) Using the high-low method of cost estimation; determine the total fixed cost, variable cost per unit and the cost function.
(b) What should be the cost in month 7 when output is expected to be 13,000 units of electronics?

**SUGGESTED SOLUTION 5-1**

**AFRO MASTER LIMITED**

(a) From the data above, the highest level of activity (which is accompanied by the highest total cost) is in month 2 and the lowest level of activity is in month 3 and, therefore, they are the two required extreme points.

<table>
<thead>
<tr>
<th>Output</th>
<th>TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>N260,000,000</td>
</tr>
<tr>
<td>Low</td>
<td>N220,000,000</td>
</tr>
<tr>
<td>Difference</td>
<td>N40,000,000</td>
</tr>
</tbody>
</table>

VC/Unit = \( \frac{N40,000,000}{4,000} = N10,000 \)

Using the highest level:

Fixed Cost = N260,000,000 \( - (N10,000 \times 18,500) \)

= N260,000,000 \( - 185,000,000 \)

= N75,000,000

Cost Function = N10,000X + N75,000,000

TVC + TFC

(b) To determine what the cost should be in month 7, we use the cost function developed above and substitute \( x \):

TC in month 7 = N10,000 \( \times 13,500 \) + N75,000,000

= N135,000,000 + N75,000,000

= N210,000,000
ILLUSTRATION 5-2

Consider the following costs of Tiga Company Limited over the relevant range of 5,000 to 20,000 units produced:

<table>
<thead>
<tr>
<th>Units Produced</th>
<th>5,000</th>
<th>10,000</th>
<th>15,000</th>
<th>20,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable cost</td>
<td>N200,000</td>
<td>N?</td>
<td>N?</td>
<td>N?</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>N1,800,000</td>
<td>N?</td>
<td>N?</td>
<td>N?</td>
</tr>
</tbody>
</table>

Cost per unit:

<table>
<thead>
<tr>
<th></th>
<th>5,000</th>
<th>10,000</th>
<th>15,000</th>
<th>20,000</th>
</tr>
</thead>
</table>

Required:

(a) Compute the missing amounts.
(b) What is the total cost of producing 18,000 units?

SUGGESTED SOLUTION 5-2

(a) Tutorial Note: Knowledge of cost behavioural patterns is to be applied to advantage here.
   (i) Total Fixed cost would remain the same for all levels.
   (ii) Variable cost per unit would remain unchanged as in the first level, that is, \( \frac{N200,000}{5,000} = N40 \)
   (iii) Fixed cost per unit would be decreasing as activity level increases; as you divide TFC by the increasing levels of activities.
   (iv) Total variable costs for the last 3 levels are obtained by multiplying the VC/unit by the units produced.
   (v) Total costs and total cost per unit can now be determined.
### Units Produced

<table>
<thead>
<tr>
<th>Units Produced</th>
<th>5,000</th>
<th>10,000</th>
<th>15,000</th>
<th>20,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable cost</td>
<td>₦200,000</td>
<td>₦400,000</td>
<td>₦600,000</td>
<td>₦800,000</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>₦1,800,000</td>
<td>₦1,800,000</td>
<td>₦1,800,000</td>
<td>₦1,800,000</td>
</tr>
<tr>
<td>Total cost</td>
<td>₦2,000,000</td>
<td>₦2,200,000</td>
<td>₦2,400,000</td>
<td>₦2,600,000</td>
</tr>
</tbody>
</table>

**Cost per unit:**

<table>
<thead>
<tr>
<th></th>
<th>5,000</th>
<th>10,000</th>
<th>15,000</th>
<th>20,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable cost</td>
<td>₦40</td>
<td>₦40</td>
<td>₦40</td>
<td>₦40</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>₦360</td>
<td>₦180</td>
<td>₦120</td>
<td>₦90</td>
</tr>
<tr>
<td>Total cost</td>
<td>₦400</td>
<td>₦220</td>
<td>₦160</td>
<td>₦130</td>
</tr>
</tbody>
</table>

(b) Cost function = ₦40x + ₦1,800,000

TC of producing 18,000 units:

= ₦40 x 18,000 + ₦1,800,000

= 720,000 + 1,800,000

= ₦2,520,000

5.5 **SUMMARY AND CONCLUSIONS**

In this chapter, effort has been made to classify cost in eight different ways giving sufficient justification for so doing. It is for management of an organisation to select the classification that best fits the organisation's peculiarities and the decision to be made in respect of the cost of a product, project, department or division within the organisation. The behavioural patterns of fixed costs, variable costs, mixed costs and step costs have been explained and graphically illustrated. The high-low method of segregating mixed costs into their fixed and variable components has been discussed. This is just one of the many methods of cost estimation that could be used with a view to guiding management on cost function and its use in determining the total cost of various levels of activities of the organisation.

If this chapter is well understood, reader should be seen to be well prepared to give satisfactory explanation on:

(a) Cost classifications;
(b) Important cost behavioural patterns;
(c) High-low method of cost estimation; and
(d) Cost analysis.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
5.6 REVISION QUESTIONS

5.6.1 MULTIPLE CHOICE QUESTIONS

1. The costs that can be directly allotted or traced to a particular cost unit or cost centre are:
   A  Production costs
   B  Direct costs
   C  Indirect costs
   D  Indirect materials
   E  Indirect Labour

2. Which of the following is not an example of variable cost?
   A  Cost of raw material
   B  Cost of direct labour
   C  Bonus payment for achieving a certain performance level
   D  Cost of fuel for generating plant
   E  Indirect Overhead

3. Cost items which might be seen as fixed in nature with certain levels of activity only but are subject to gradual changes in response to abnormally high productivity are known as
   A  Mixed costs
   B  Fixed costs
   C  Step costs
   D  Semi-variable costs
   E  Cost-drive

4. Which of the following is not a cost estimation method?
   A.  Accounting techniques
   B.  Scatter graph
   C.  Regression analysis
   D.  High-low method
   E.  Algebraic method

5. In High-Low method of estimating costs, the two extreme points are:
   A.  Highest level and lowest level of activities
   B.  Highest level and lowest level of costs
   C.  Highest level of activity and lowest level of costs
   D.  High level of cost and lowest level of activity
   E.  Medium level of cost and medium level of activity
5.6.2 SHORT ANSWER QUESTIONS

1. Differentiate between incremental costs and sunk costs

2. Why should a Cost Accountant be involved with cost estimation?

3. Why is behavioural pattern of cost important?

4. In studying cost behavioural patterns, dependent and independent variables must be identified.
   Give examples of independent variables and discuss how variable cost is dependent on volume of production

5. Indirect costs apportioned to a responsibility centre may be termed ---
   -------------------costs of that responsibility centre

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
6.0 LEARNING OBJECTIVES
After studying this chapter, readers will be able to understand the:

- Distinction between costing methods and costing techniques;
- Various branches of specific order costing;
- Various branches of unit (or average) costing;
- Method of job costing;
- Method of batch costing;
- Method of contract costing; and
- Method of service or operating costing.

6.1 INTRODUCTION
From time to time, questions on what an item of goods or service has cost do arise. One good example is a question that is often raised when people comment on the necessity of the frequent labour strikes over petroleum prices. The question is often raised to challenge government’s claim that it was subsiding petroleum cost. A subsidy is an amount borne by someone, especially government, on behalf of another as excess of cost over the price paid by the other.

The question of cost is often necessary to answer a number of other fundamental questions, such as:

(a) At what price should a product be sold?
(b) At what price should a service be rendered?
(c) What is the amount of subsidy that government is providing on a service to the public?
(d) How much exactly has a product cost?
(e) How profitable is a product relative to other products?
6.1.1 Costing definition

Costing is concerned primarily with ascertaining in financial terms the cost of producing a product or rendering a service. Such information is usually useful to management to effectively carry out their functions of planning, controlling and decision making.

6.1.2 Principles of Cost Ascertainment

Whereas cost is ascertained by simply aggregating all the expenses incurred in getting a product to a required state or rendering a service, but the system or the procedure for cost ascertainment, that is, bringing the expenses together. In other words, methodology of accumulating the cost depends on the type of industry.

6.2 COSTING METHODS

Costing methods, therefore, refer to the systems of cost finding and ascertainment. They are devised to suit the methods by which goods are manufactured or services are provided, for example:

(a) Where work is undertaken to customer's special requirements, the method of costing that will be in operation is job costing (specific order costing).
(b) Where standardized goods result from a sequence of repetitive and more or less continuous operation or process, then process costing is used.

There are basically two methods of accumulating costs, namely:

(a) Specific Order Costing

Where expenses are linked to the cost object, for example, Job, Contract or a Batch. Hence, Job Costing, Contract Costing, Batch Costing fall under specific order costing method.

(b) Unit Costing (Average Costing)

This is where expenses cannot be linked directly with the cost object and are, therefore, charged to the department or process. The total costs are then averaged over the total units produced. Examples of these are batch costing (where the
cost of a unit within the batch is desired), Service or Operating Costing, and Process or Operation Costing.

6.3 COSTING TECHNIQUES

It may be helpful at this point to quickly explain costing techniques, as many readers are often confused when asked to distinguish between costing methods and Cost techniques.

Whereas costing methods refer to the systems of cost finding and ascertainment, as explained above, costing techniques refer to the manner of presenting the cost information. There are also two major costing techniques, namely: Absorption costing and Marginal costing. Absorption costing is the practice of charging all manufacturing costs, both fixed and variable to cost units, while Marginal costing is the practice of charging only variable manufacturing costs to cost units.

These can be presented diagrammatically as follows:

COSTING METHODS

Specific Order Costing
(or Job Costing)

Unit Costing
(or Average Costing)

Job Costing

Contract Costing

Batch Costing

Batch Costing

Service or Operating Costing

Process or Operation Costing

COSTING TECHNIQUES

Absorption Costing

Marginal Costing

Historical Absorption Costing

Standard Absorption Costing

Historical Marginal Costing

Standard Marginal Costing
6.4 JOB COSTING

6.4.1 What is Job Costing?

Job Costing is that form of costing which applies where jobs are carried out to customer’s specific requirement, for example, printing, construction, furniture, general engineering. The job becomes the cost unit.

In job costing, production is not for stock but in accordance with the special requirements of a customer. Some of the noticeable features of industries where Job Costing will be applicable are:

(a) The specific request of a customer is received.
(b) A Technical Personnel meets the customer and agrees with him the precise details of his requirement.
(c) An estimate of cost is prepared by the Estimating Department or a designated officer, which forms the basis of a quotation that is then submitted to the customer.
(d) If the quotation is acceptable to the customer, he signifies his acceptance, and most commonly asked to make a deposit payment.
(e) A distinct Job number is then assigned and a Job Card is opened. The job number will be quoted on all Material Requisition Notes, Job, etc. for proper cost accumulation.
(f) The job may be for a short period of time, that is, within one accounting period or longer. Where the Job is for a long period straddling more than one accounting period, them some accounting issues will arise, making contract accounting rules to be applied.

6.4.2 JOB CARD

A Job Card is opened for each job that is undertaken by the company. The format of a Job Card varies from one organisation to the other but essentially, it should contain certain information as contained in the following typical Job Card:
**COSTING AND QUANTITATIVE TECHNIQUES**

**JOB CARD**

<table>
<thead>
<tr>
<th>Date 2005</th>
<th>Ref</th>
<th>Material</th>
<th>Labour</th>
<th>Overhead</th>
<th>Other charges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Estimate</td>
<td>Estimate</td>
<td>Estimate</td>
<td>Estimate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N1,250</td>
<td>N100</td>
<td>N176</td>
<td>N25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cost</td>
<td>Cum Dir</td>
<td>Cost</td>
<td>Cum DLH rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>lab hour</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov</td>
<td>MR. 1714</td>
<td>182</td>
<td>1,200</td>
<td>170</td>
<td>110</td>
</tr>
<tr>
<td>6</td>
<td>Consultant’s for Test fee</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>MR. 1937</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>MRN 213</td>
<td>19</td>
<td>1,401</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Labour</td>
<td>(26)</td>
<td>1,375</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analysis</td>
<td></td>
<td>50</td>
<td>28</td>
<td>138</td>
<td>55</td>
</tr>
</tbody>
</table>

**SUMMARY**

- Material: N1,375
- Labour: N138
- Overhead: N242
- Other charges: N23

**COMMENTS**

- Invoice price: N1,778
- (Invoice No. 7147 dated 12/12/08): N2,400
- Profit: N622
It should be appreciated that the Job Cost Card is supplementary to the cost book-keeping system, that is, it exists independently of the cost ledger system, for up to date controls of costs. This is why in most jobbing concerns, such as a Motor Garage, the Job Card is kept in the shop floor while costs (expenses) are recorded in the Cost Ledger via the normal source documents like Material Requisition Note, Wages Analysis Journals and Expenses Analysis Books.

6.5 BATCH COSTING

Batch Costing is a special type of Job Costing, or a modification of Job Costing.

Batch Costing arises where a customer orders a quantity of identical products at the same time. The job consists of a number of units of similar products covered by a single job number; for example, a batch of 1000 books printed by a printer. The batch cost must be divided by the total quantity to give a cost per item.

Peculiar features of Batch Costing System include:

(a) **Set-Up Costs**: This is the cost of setting up production facilities, for example, the Engineering time, Factory consumable to set up moulds, etc.

(b) **Unit Costs**: Average Cost per unit will tend to decrease as the batch size increases due to some costs (such as set-up costs mentioned above) which will remain the same irrespective of the batch size.

(c) **Batches of Dissimilar Products**: Sometimes batches of dissimilar products are brought together because a common operation can be performed on them so as to save set-up time on the machine. The common cost of this Joint Operation becomes a problem to apportion. However, it should be realized that common costs should be apportioned on equitable basis.

**ILLUSTRATION 6-1**

1. Ayinba Press Limited was asked to quote for supplying 5,000 and 25,000 booklets. The company normally expects a profit of 10% on sales. Costs were reckoned to be:

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper and other materials, (per 1,000 copies)</td>
<td>₦30.00</td>
</tr>
<tr>
<td>Wages (per 1,000 copies)</td>
<td>₦20.00</td>
</tr>
</tbody>
</table>
COSTING AND QUANTITATIVE TECHNIQUES

Draft a cost computation, showing minimum selling prices that might be quoted per 1,000 copies for each of the supplies. Though this question is asking for the selling price to quote, we will also use it to demonstrate how cost per batch and how cost per unit are obtained in addition to solving the simple arithmetical exercise of calculating selling price.

**SUGGESTED SOLUTION 6-1**

**Calculation of Batch Cost**

As earlier on mentioned, this is just the accumulation of all the expenses incurred in producing the batch of similar products, in this case, the booklets.

<table>
<thead>
<tr>
<th></th>
<th>1000 copies</th>
<th>5000 copies</th>
<th>25000 copies</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set up cost</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layout cost</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td><strong>Material</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Paper</td>
<td>30</td>
<td>150</td>
<td>750</td>
</tr>
<tr>
<td><strong>Labour</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wages</td>
<td>20</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td><strong>Overhead</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed overhead</td>
<td>200</td>
<td>200</td>
<td>500</td>
</tr>
<tr>
<td>Variable overhead</td>
<td>2.4</td>
<td>12</td>
<td>60</td>
</tr>
<tr>
<td><strong>Total cost</strong></td>
<td>752.40</td>
<td>962</td>
<td>2310</td>
</tr>
<tr>
<td>Profit margin</td>
<td>83.60</td>
<td>107</td>
<td>223</td>
</tr>
<tr>
<td><strong>Selling price</strong></td>
<td>836</td>
<td>1069</td>
<td>2533</td>
</tr>
</tbody>
</table>

It should be noted that Total cost was obtained by simply aggregating the expenses, set-up cost, material, labour and overhead. If cost per unit is desired, it will be obtained by simply averaging the total cost per batch over the units, that made up the batch size. At 5000 units, total cost was N962; cost per unit,
therefore, will be N962/5000, i.e. 19.24 kobo. At 25000 copies, total cost was N2310; cost per copy therefore is 8.04 kobo. This proves a common saying, that cost decreases as the batch size increases, meaning that cost per unit decreases as the batch size increases. Selling Price per copy will also be calculated in the same way.

6.6 SERVICE COSTING

This is the application of costing principles to the costing of service operations as opposed to the manufacture of goods. For example, hotel, transport and airline businesses, schools, etc. It may also, however, be applied to a service cost centre in a manufacturing concern. It may be decided to treat the haulage department of a manufacturing and trading concern as a service cost centre after determining whether it is worthwhile to run the delivery of goods to customer internally or to sub-contract it to a haulage company. To take such a decision, information is required on how much it costs to run it in-house.

The principles of cost ascertainment remain the same in that all expenses incurred in the process of rendering the service are appropriately classified and aggregated and then averaged over the total number of units of that service to obtain cost per unit.

The manner in which costs are classified would vary from one industry to the other. The following formats for a transport and canteen businesses are illustrated.

Format of a Cost Statement for a Transport Business

<table>
<thead>
<tr>
<th>Fixed or Standing Charges</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Insurance</td>
<td>x</td>
</tr>
<tr>
<td>Vehicle Licences</td>
<td>x</td>
</tr>
<tr>
<td>Drivers Remuneration</td>
<td>x</td>
</tr>
<tr>
<td>General Administration</td>
<td>x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Running Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel</td>
</tr>
<tr>
<td>Servicing</td>
</tr>
<tr>
<td>Lubricants</td>
</tr>
<tr>
<td>Repairs and Maintenances</td>
</tr>
<tr>
<td><strong>Total Cost</strong></td>
</tr>
</tbody>
</table>
Fixed or standing charges represent those expenses which are incurred irrespective of the distance covered.

### Canteen Cost

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Wages and Salaries</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supervisor</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Cooks</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Counter helpers</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Provision</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meat</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fish and poultry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vegetable and fruit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eggs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coffee</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Miscellaneous</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maintenance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crockery and Glassware</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Insurance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Table linen</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deduct income from:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Company subsidy</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Cost Unit in Service Industry

The major problem with service costing is the determination of the cost unit to use in measuring the output of the service. For example, for a transport business, service could be measured in terms of kilometers travelled or the weight of goods carried, or a combination of both.
Other cost-units in the service industry are as follows:

<table>
<thead>
<tr>
<th>Service</th>
<th>Cost unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hotel</td>
<td>Occupied room-day</td>
</tr>
<tr>
<td>Restaurant</td>
<td>meal served</td>
</tr>
<tr>
<td>College</td>
<td>Full time equivalent student</td>
</tr>
<tr>
<td>Electricity</td>
<td>Kilowatt-hour</td>
</tr>
<tr>
<td>Hospital</td>
<td>Patient-day</td>
</tr>
<tr>
<td>Transport</td>
<td>Miles travelled, Tonne-mile</td>
</tr>
</tbody>
</table>

**Costing of Professional Services are:**

In most professional offices such as those of Architects, Accountants and Tax Practitioners, services are rendered and classified as belonging to the service industry, it may be inappropriate to apply service costing principles. This is because the services rendered by professional offices usually differ from one client to another as opposed to the service organisation that render standardized services. In such a case, job costing method is used in the costing of such professional services.

### 6.7 CONTRACT COSTING AND ACCOUNTING

Contract Costing is similar in all respects to job costing. It is a form of job costing. Perhaps the following features would explain how contract costing differ from job costing.

#### 6.7.1 Peculiar features of Contract Costing System are:

(a) The jobs are usually of a long term duration, often more than one accounting period.

(b) The contracts are often carried out in the contractee’s premises, that is, site-based.

(c) There may be sub-contractors where there are some special jobs to be done such as plumbing, and electrical installation.

(d) There is often an architect engaged by the contractee who supervises the work and issues a valuation certificate upon which payment would be made to the contractor. This is otherwise known as work certified or architects valuation.
6.7.2 Accounting for Contracts

It should be appreciated by the reader that the problem with contract costing is not the ascertainment of cost. Cost is ascertained by simply aggregating all the expenses relating to each contract. A contract account is opened for each contract wherein the expenses will be recorded as they are incurred.

The major problem is the calculation of profit if any, that can be taken on a contract, even though it is yet to be completed, but has reached an advanced stage.

This major problem (accounting for profit on uncompleted contracts) will, therefore, be given proper attention. In this section, contracts can be classified into three (3) different categories according to their stage of completion.

(a) Contracts yet at the early stage.
(b) Contracts that have reached an advanced stage but for which cost to complete cannot be reasonably estimated.
(c) Contract that are nearing completion, that is, have reached an advanced stage and for which cost to complete can be reasonably estimated.

Procedure

For any of these types of contract, the accountant is advised to first open the contract account and debit thereto all relevant expenses and apportionments. This will give the cost to date of the contract, and then move to the next stage of considering whether profit can be taken on the contract at the given stage of the contract. It is at this stage that the three classifications given above becomes important.

A Notional profit is usually first obtained before a calculation of profit is taken. The procedure for calculating Notional Profit and profit to be taken is discussed according to the stage of the contract.

(a) **Contract at the Early Stage**

Where the contract has only just been started the expenses debited to the contract account will simply be carried forward as cost of the contract or the value of the Work-In-Progress just in the same manner as Job Costing.
(b) **Fairly Advanced Stage**

At this stage of a contract’s life, profit may be recognized on the efforts on the contract to date, although clear estimate of the cost to complete may not be available. The profit taken is usually limited by the excess of the architect’s value of work certified over the contractors cost to date (notional profit) adjusted for the cash actually received. This approach accords with the prudence concept.

Where a contract is at this stage, the profit recognized is usually computed as follows:

**Step 1** Calculate the Notional Profit as follows:

\[
\begin{align*}
\text{Value of work certified} & \quad XX \\
\text{Cost of work certified} & \quad (X) \\
\text{Notional Profit} & \quad XX
\end{align*}
\]

It should be noted that cost of work certified might not be the same with the cost to date as costs may be incurred on work which for one reason or the other the architect is yet to certify.

**Step 2** Calculate the Profit taken as follows:

\[
\frac{2}{3} \times \frac{\text{Notional Profit}}{\text{Value of work certified}} \times \frac{\text{Cash received}}{\text{Value of work certified}}
\]

Readers should also note that proportion of notional profit will be as determined by the contract paper or in case of students, by the examiner.

**ILLUSTRATION 6-2**

OLANIYAN Nigeria Limited is a medium sized construction company based in Okoja, Kogi State of Nigeria. At present, the company has two contracts in progress. Details extracted at 31 December were:

<table>
<thead>
<tr>
<th>Contract name (project)</th>
<th>Plumbing</th>
<th>Surfacing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Commencement Date</strong></td>
<td>1 January</td>
<td>1 July</td>
</tr>
<tr>
<td><strong>Contract price</strong></td>
<td>550,000</td>
<td>700,000</td>
</tr>
<tr>
<td><strong>Expenditure:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>25,360</td>
<td>38,560</td>
</tr>
<tr>
<td>Wages</td>
<td>96,886</td>
<td>74,436</td>
</tr>
<tr>
<td>Site expenses</td>
<td>13,000</td>
<td>17,240</td>
</tr>
</tbody>
</table>
Head office charges of N45,000 are charged in proportion to their prime costs. The plant was installed at the commencement of the contracts and depreciation is calculated at 20% per annum. Both contracts have been estimated to give an overall profit on completion.

You are required to prepare Contract accounts for the company.

**SUGGESTED SOLUTION 6.2**

**OLANIYAN NIGERIA LIMITED**

**CONTRACT ACCOUNT 31 DECEMBER**

<table>
<thead>
<tr>
<th></th>
<th>Plumbing</th>
<th>Surfacing</th>
<th></th>
<th>Plumbing</th>
<th>Surfacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>25,360</td>
<td>38,560</td>
<td>Materials c/d</td>
<td>4,200</td>
<td>12,800</td>
</tr>
<tr>
<td>Wages</td>
<td>96,886</td>
<td>74,436</td>
<td>Cost not yet</td>
<td>7,000</td>
<td>4,400</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>certified</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Site expenses</td>
<td>13,000</td>
<td>17,240</td>
<td>Cost of work</td>
<td>215,704</td>
<td>152,296</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>certified</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depreciation of</td>
<td>60,000</td>
<td>13,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>plant</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Accrued wages</td>
<td>8,434</td>
<td>4,484</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Head office</td>
<td>23,224</td>
<td>21,776</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>charges</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>226,904</td>
<td>169,496</td>
<td></td>
<td>226,904</td>
<td>169,496</td>
</tr>
<tr>
<td>Cost of work</td>
<td>215,704</td>
<td>152,296</td>
<td>Value of work</td>
<td>220,000</td>
<td>170,000</td>
</tr>
<tr>
<td>certified</td>
<td></td>
<td></td>
<td>certified</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit taken (W1)</td>
<td>2,434</td>
<td>8,852</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit c/f</td>
<td>1,862</td>
<td>8,852</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>220,000</td>
<td>170,000</td>
<td></td>
<td>220,000</td>
<td>170,000</td>
</tr>
<tr>
<td>Cost to date</td>
<td>222,704</td>
<td>156,696</td>
<td>Work- in-</td>
<td>225,138</td>
<td>165,548</td>
</tr>
<tr>
<td>Profit taken</td>
<td>2,434</td>
<td>8,852</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Profit taken on Contracts

<table>
<thead>
<tr>
<th></th>
<th>Plumbing</th>
<th>Surfacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of work certified</td>
<td>220,000</td>
<td>170,000</td>
</tr>
<tr>
<td>Cost of work certified</td>
<td>(215,704)</td>
<td>(152,296)</td>
</tr>
<tr>
<td>Notional profit</td>
<td>4,296</td>
<td>17,704</td>
</tr>
</tbody>
</table>

\[
\frac{2}{3} \times 4296 \times \frac{187,000}{220,000} = \text{₦}2,434
\]

\[
\frac{2}{3} \times 17,704 \times \frac{127,500}{170,000} = \text{₦}8,852
\]

### Depreciation of Plant

<table>
<thead>
<tr>
<th></th>
<th>Plumbing</th>
<th>Surfacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant issued to site</td>
<td>300,000</td>
<td>130,000</td>
</tr>
<tr>
<td>Depn @ 20% for 1 yr</td>
<td>(60,000)</td>
<td>(\text{---})</td>
</tr>
<tr>
<td>Depn @ 20% for month</td>
<td>(\text{---})</td>
<td>(13,000)</td>
</tr>
<tr>
<td>Plant b/f</td>
<td>(240,000)</td>
<td>(117,000)</td>
</tr>
</tbody>
</table>

### Head Office Charges

<table>
<thead>
<tr>
<th></th>
<th>Plumbing</th>
<th>Surfacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total head office charges</td>
<td>45,000</td>
<td>(\text{---})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Plumbing</th>
<th>Surfacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime costs</td>
<td></td>
<td>(\text{---})</td>
</tr>
<tr>
<td>Plumbing</td>
<td>143,680</td>
<td>(\text{---})</td>
</tr>
<tr>
<td>Surfacing</td>
<td>134,720</td>
<td>(\text{---})</td>
</tr>
<tr>
<td></td>
<td>278,400</td>
<td>(\text{---})</td>
</tr>
</tbody>
</table>

\[
\text{Plumbing} \times \frac{45,000}{278,400} = 23,224
\]

\[
\text{Surfacing} \times \frac{45,000}{278,400} = 21,776
\]

\[
45,000 = \text{---}
\]
Nearing Completion State

At this point, reliable estimates could be made of the cost to complete thus the total contract cost is in sight. Profits are taken on contracts at this stage using a fairly different basis compared to those discussed above. Here the profit taken is with reference to the estimated total profit of the contract over its life while adjusting for whatever profit that have been recognized previously.

The calculation of profit taken follows these steps:

**Step 1: Calculate the Estimated Total Profit (ETP)**

| &nbsp; | \( 
\begin{array}{c|c|c|c} 
\hline
\text{Total contract price} & xx \\
\hline
\text{Less Total contract cost:} & \text{xx} \\
\hline
\text{Cost to date} & x \\
\hline
\text{Cost to complete} & x \\
\hline
\text{Estimated total cost} & (x) \\
\hline
\text{Estimated total profit} & xx \\
\hline
\end{array} \\
\end{array} \) |

**Step 2: Calculate Profit Realized to Date**

This is the lower of

(i) \[
\text{Value of work certified} \times \text{ETP} \\
\text{Total contract price (TCP)}
\]

(ii) \[
\text{Cost to date} \times \text{ETP} \\
\text{Estimated total cost}
\]

**Step 3: Profit taken in Current Period**

| &nbsp; | \( 
\begin{array}{c|c|c|c} 
\hline
\text{Profit realized to date} & x \\
\hline
\text{Profit recognized in prior periods} & (x) \\
\hline
\text{Profit taken in current period} & xx \\
\hline
\end{array} \\
\end{array} \) |

**Where Losses are Expected**

Where it is expected that the contract as a whole will result in a loss for the organisation, the loss must be written off to the maximum possible extent immediately. If the contract is unprofitable, the acceptance of the contract was unprofitable. The foreseeable loss should be written
off in the period it is recognized in line with the prudence concept. This concept reduces the value of the asset that may be carried in respect of the contract asset.

Valuation of Work in Progress for Balance Sheet

A final step is to provide the value at which the contract should be carried in the balance sheet. This should not pose a big problem as what is required is to deduct payments received from the contractee from the value of the asset created so far. This is the amount to which the contractor remains entitled.

The calculation is usually done thus:

<table>
<thead>
<tr>
<th>Work-in-progress for balance sheet</th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost to date</td>
<td>X</td>
</tr>
<tr>
<td>Profit taken on contract</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Less cash received from contractee</td>
<td>(x)</td>
</tr>
<tr>
<td>WIP per balance sheet</td>
<td>X</td>
</tr>
</tbody>
</table>

ILLUSTRATION 6-3

The following information relates to contract No. ZC121 which was started on January 10, 2009 and expected to last for 12 months. The Cost Accountant estimated the total cost of the contract to be ₦118,750. By 31 December 2009, the end of the contractor’s accounting year, the costs incurred to date were as follows:

<table>
<thead>
<tr>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour</td>
</tr>
<tr>
<td>Materials issued to site</td>
</tr>
<tr>
<td>Overheads</td>
</tr>
<tr>
<td>Plant transferred to site (1/3/09)</td>
</tr>
<tr>
<td>Written down value of (31/12/09)</td>
</tr>
</tbody>
</table>

The full contract price was agreed at ₦112,500. At 31 December, 2009, the value of work certified was ₦87,500 and progress payment received to date ₦8,750.

Required:

(a) Contract Account;
(b) Contractee Account; and
(c) Calculate the value of work-in-progress for balance sheet purpose.

**SUGGESTED SOLUTION 6-3**

(a) **Contract Account for Contract No. ZC121**

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour</td>
<td>45,000</td>
<td>Materials c/d</td>
<td>2,500</td>
</tr>
<tr>
<td>Materials issued to site</td>
<td>30,000</td>
<td>Plant c/d</td>
<td>35,000</td>
</tr>
<tr>
<td>Overheads</td>
<td>7,500</td>
<td>Cost to date</td>
<td>95,000</td>
</tr>
<tr>
<td>Plant transferred to site</td>
<td>50,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>132,500</td>
<td></td>
<td>132,500</td>
</tr>
<tr>
<td>Cost to date</td>
<td>95,000</td>
<td>Contract loss</td>
<td>6,250</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Work-in-progress</td>
<td>88,750</td>
</tr>
<tr>
<td></td>
<td>95,000</td>
<td></td>
<td>95,000</td>
</tr>
</tbody>
</table>

(W1) **Calculation of Total Profit (Loss)**

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate price</td>
<td>112,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less estimate cost of Contract</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost to date</td>
<td>95,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost to complete (118,750 – 95,000)</td>
<td>23,750</td>
<td>(118,750)</td>
<td></td>
</tr>
<tr>
<td>Estimated Total Loss</td>
<td></td>
<td></td>
<td>6,250</td>
</tr>
</tbody>
</table>

The expected loss is provided for immediately in line with the prudence concept.

(b) **Contractee Account**

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of work certified</td>
<td>87,500</td>
<td>Progress payment received</td>
<td>78,750</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Balance b/d</td>
<td>8,750</td>
</tr>
<tr>
<td></td>
<td>87,500</td>
<td></td>
<td>87,500</td>
</tr>
</tbody>
</table>

(c) **Value of work-in-progress for Balance sheet purpose**

<table>
<thead>
<tr>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value per contract account</td>
<td>88,750</td>
</tr>
<tr>
<td>Less cash received</td>
<td>(78,750)</td>
</tr>
<tr>
<td>Work-in-progress for balance sheet</td>
<td>10,000</td>
</tr>
</tbody>
</table>
6.8 SUMMARY AND CONCLUSIONS

Costing is concerned primarily with ascertaining in financial terms, the cost of producing a product or rendering a service. The information is useful to management for planning, controlling and decision making.

Costing methods refer to the system of cost ascertainment.

Job costing applies where jobs are carried out to customer’s specific requirement.

Batch costing arises where a customer orders a quality of identical products at the same time.

Service costing is the application of costing principles to the costing of service operations as opposed to the manufacture of goods.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

6.9 REVISION QUESTIONS

6.9.1 MULTIPLE CHOICE QUESTIONS

1. The systems of cost findings and cost ascertainment are
   A. Cost ascertainment
   B. Costing techniques
   C. Costing methods
   D. Cost allocation
   E. Cost apportionment

2. The manner of presenting cost information is
   A. Cost classification
   B. Costing technique
   C. Cost absorption
   D. Costing methods
   E. Cost ascertainment

3. The costs incurred on contracts that are still at an early stage and not nearly completed are, at the end of the period:
   A. Debited to Profit and loss account
   B. Set off against the revenue for the period
   C. Treated as work-in-progress
   D. Considered as raw material
   E. Treated as stock of finished goods

4. The method of applying the same basic costing methods, principles and techniques to several undertakings that are in the same industry, trade association or group is known as
A Standard costing  
B Process costing  
C Uniform costing  
D Absorption costing  
E Contract costing

5. The most suitable method to use where products manufactured differ in materials and labour requirements and are made according to customers’ specification is  
A Job costing  
B Uniform costing  
C Contract costing  
D Batch costing  
E Unit costing

6.9.2 SHORT ANSWER QUESTIONS

1. Where a customer orders a quantity of identical products at the same time, the costing method to be used is  

2. The major costing techniques are  

3. The two main methods of accumulating costs are  

4. What are the peculiar features of batch costing?

5. How will an accountant take credit for profit on a contract that is nearing completion?

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
7.0 LEARNING OBJECTIVES

After studying this chapter, readers will be able to understand:

- The meaning and characteristics of process costing;
- The meaning, concept of equivalent units and accounting of spoilages;
- The preparation of cost of production report;
- The meaning of joint costs, joint-products and methods of apportioning joint costs to joint-products; and
- The accounting treatment of by-products and total costs of producing joint products.

7.1 INTRODUCTION

Process costing is a method used in a situation where production follows a series of sequential processes. The method is used to ascertain the cost of a product or service at each stage of production, manufacture or process. It is generally applied in particular industries where continuous mass production is possible. In view of the continuous nature of the process and the uniformity of the output, it is not possible or necessary to identify a particular unit of output with a time of manufacture. The cost of any particular unit must be taken as the average cost of manufacture over a period. This can be complicated because of the need to apportion costs between completed output and unfinished production at the end of the period. Wastage must also be accounted for. In process costing, it is the average cost incurred that concerns management.

Process costing is used in a variety of industries, including food processing, paper milling, chemical and drug manufacturing, oil
refining, soap making, textiles, box-making, paint and ink
manufacturing, brewery, flour milling, bottling and canning, biscuits
products, meat products, sugar making, etc. It is probably the most
widely used cost accounting system in the world.

7.2 CHARACTERISTICS OF PROCESS COSTING

Although, details will vary from one business concern to another, there
are common features in most process costing systems that should be
taken note of. These are:

(a) Clearly defined process cost centres will normally be set up for
each operational stage, which can be identified. Expenditure for
each cost centre is collected and, at the end of the accounting
period, the cost of the completed units are then transferred into
a stock account or to a further process cost centre. Accurate
records are, therefore, required of units produced and part-
produced units and the total cost incurred by the cost centres.
(b) The cost unit chosen should be relevant to the organisation.
(c) The cost of the output of one process is the raw material input
cost of the following process. The cost incurred in a process cost
centre could include, therefore, costs transferred from a previous
process plus the raw materials, labour and overhead costs
relevant to the cost centre.
(d) Wastage due to scrap, chemical reaction or evaporation is
unavoidable. The operation or manufacturing should, however,
be in such a way that wastage can be reduced to the barest
minimum.
(e) Either the main product or by-product of the production process
may require further processing before reaching a marketable
state.

ILLUSTRATION 7-1

<table>
<thead>
<tr>
<th>Material labour overhead</th>
<th>Dept A</th>
<th>Dept B</th>
<th>Dept C</th>
<th>Finished goods stock</th>
</tr>
</thead>
</table>

Figure 1: It shows that no further raw material is introduced into
the process and that the finished product emerges from
Dept. C and is passed to the store for sale or use.
Figure 2: It shows that the output of Dept A (transferred in cost) is a raw material input to Dept B and that further material must be added, thereby incurring additional cost on material.

Note: Conversion cost represents the cost of labour and overhead put together.

7.3 PRODUCT FLOW

As a product passes from one cost centre to another, per unit cost and total cost should be determined. As shown in figure 2, the total cost incurred at the lower level of processing is to be seen as the transferred in cost of the higher level to which cost of additional material and conversion cost must be added before arriving at its total costs. That total cost may be a transferred in cost, if the production process is not complete, or the final total cost of production, if finished products have been arrived at. Product flows have to be accompanied by their total costs at each level of processing.

ILLUSTRATION 7-2

A product passes through three distinct processes (A, B, and C) to completion. During the period 15th May, 2009, 1000 litres were produced. The following information is obtained:

<table>
<thead>
<tr>
<th></th>
<th>Process A</th>
<th>Process B</th>
<th>Process C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials cost</td>
<td>40,000</td>
<td>15,000</td>
<td>5,000</td>
</tr>
<tr>
<td>Labour cost</td>
<td>20,000</td>
<td>25,000</td>
<td>15,000</td>
</tr>
<tr>
<td>Direct overhead expenses</td>
<td>5,000</td>
<td>3,000</td>
<td>3,000</td>
</tr>
</tbody>
</table>

Indirect overhead expenses for the period were ₦30,000 apportioned to the processes on the basis of wages. There was no work-in-process at the beginning or end of the period.

Required:

Calculate the cost of output to be transferred to finished goods stock and the cost per litre.
### Suggested Solution 7-2

#### Process A

<table>
<thead>
<tr>
<th>Details</th>
<th>Cost/litre</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>40</td>
<td>40,000</td>
</tr>
<tr>
<td>Labour</td>
<td>20</td>
<td>20,000</td>
</tr>
<tr>
<td>Dir. Exps.</td>
<td>5</td>
<td>5,000</td>
</tr>
<tr>
<td>Indir. Exps.</td>
<td>10</td>
<td>10,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>75</strong></td>
<td><strong>75,000</strong></td>
</tr>
</tbody>
</table>

#### Process B

<table>
<thead>
<tr>
<th>Details</th>
<th>Cost/litre</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process A</td>
<td>75</td>
<td>75,000</td>
</tr>
<tr>
<td>Material</td>
<td>15</td>
<td>15,000</td>
</tr>
<tr>
<td>Labour</td>
<td>25</td>
<td>25,000</td>
</tr>
<tr>
<td>Dir. Exps.</td>
<td>3</td>
<td>3,000</td>
</tr>
<tr>
<td>Indir. Exps.</td>
<td>12.5</td>
<td>12,500</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>130.50</strong></td>
<td><strong>130,500</strong></td>
</tr>
</tbody>
</table>

#### Process C

<table>
<thead>
<tr>
<th>Details</th>
<th>Cost/litre</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process B</td>
<td>130.50</td>
<td>130,500</td>
</tr>
<tr>
<td>Materials</td>
<td>5.00</td>
<td>5,000</td>
</tr>
<tr>
<td>Labour</td>
<td>15.00</td>
<td>15,000</td>
</tr>
<tr>
<td>Dir. Exps.</td>
<td>3.00</td>
<td>3,000</td>
</tr>
<tr>
<td>Indir. Exps.</td>
<td>7.50</td>
<td>7,500</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>161.00</strong></td>
<td><strong>161,000</strong></td>
</tr>
</tbody>
</table>

**Note:**

(a) Indirect expenses were apportioned as follows:

- **Process A**
  \[ \text{Total} = 20,000 \times \frac{30,000}{60,000} = \text{₦10,000} \]

- **Process B**
  \[ \text{Total} = 25,000 \times \frac{30,000}{60,000} = \text{₦12,500} \]

- **Process C**
  \[ \text{Total} = 15,000 \times \frac{30,000}{60,000} = \text{₦7,500} \]
(b) The cost per litre of the product is ₦161 and, so, the selling price must be higher than that amount if the business is to make any profit.

(c) Indirect expenses include all expenses that cannot be directly traced to the productive process and, so, they include general administrative, selling and distributive cost.

7.4 EQUIVALENT UNITS

At the end of a given period, in the course of the production process, it is virtually certain that some items will only be partly completed (work-in-process). Some of the cost of the period, therefore, are attributable to these partly completed units as well as to those that are fully completed. In order to spread the costs equitably over part-finished and fully completed units, the concept of ‘equivalent units’ is used.

For the calculation of costs, the number of equivalent units is the number of equivalent fully completed units which the partly completed units represent. For example, in a given period production was 3,000 completed units, and 1,600 partly completed were deemed to be 60% complete.

Total equivalent production = completed units plus equivalent units produced in work-in-progress (WIP).

= 3,000 + (60% of 1,600)

= 3,000 + 960

= 3,960 units

The total costs for the period would be spread over the total equivalent production as follows:

Cost per unit = \[
\frac{\text{Total Costs}}{\text{Total equivalent production (units)}}
\]

In calculating equivalent units, it is more desirable to consider the percentage completion of each of the cost elements: material, labour and overhead. Here, each cost element must be treated separately and then the costs per unit of each element are added to give the cost of a complete unit.
ILLUSTRATION 7-3

The production and cost data of Elsemco Shoemakers for the month of January, 2005 were as follows:

<table>
<thead>
<tr>
<th>Elements</th>
<th>Cost</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>422,400</td>
<td></td>
</tr>
<tr>
<td>Labour</td>
<td>395,600</td>
<td></td>
</tr>
<tr>
<td>Overhead</td>
<td>225,000</td>
<td></td>
</tr>
<tr>
<td>Total cost</td>
<td>1,043,000</td>
<td></td>
</tr>
</tbody>
</table>

Production was 8,000 fully completed units and 2,000 partly completed. The percentage completion of the 2,000 units work-in-process was:

- Material: 80%
- Labour: 60%
- Overhead: 50%

Required:

Find the value of completed production and the value of work-in-process (WIP).

**SUGGESTED SOLUTION 7-3**

<table>
<thead>
<tr>
<th>Cost Elements</th>
<th>Equiv. units in WIP</th>
<th>Fully compl. units</th>
<th>Total prod. units</th>
<th>Total cost</th>
<th>Cost/unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>80% of 2,000 = 1,600</td>
<td>8,000</td>
<td>9,600</td>
<td>422,400</td>
<td>44</td>
</tr>
<tr>
<td>Labour</td>
<td>60% of 2,000 = 1,200</td>
<td>8,000</td>
<td>9,200</td>
<td>395,600</td>
<td>43</td>
</tr>
<tr>
<td>Overhead</td>
<td>50% of 2,000 = 1,000</td>
<td>8,000</td>
<td>9,000</td>
<td>225,000</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1,043,000</td>
<td>112</td>
</tr>
</tbody>
</table>

Value of completed units = N112 × 8,000 = N896,000

Value of WIP = TC – Value of completed units

= 1,043,000 - 896,000 = N147,000

To check the value of WIP, the cost per each cost element is to be multiplied by the number of equivalent units of production in WIP related to each cost element.
### COSTING AND QUANTITATIVE TECHNIQUES

<table>
<thead>
<tr>
<th>Element of units in WIP</th>
<th>No. of equiv.</th>
<th>Cost/unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>1,600</td>
<td>44</td>
<td>70,400</td>
</tr>
<tr>
<td>Labour</td>
<td>1,200</td>
<td>43</td>
<td>51,600</td>
</tr>
</tbody>
</table>
| Overhead                | 1,000        | 25        | 5,000 | **₦147,000**

#### Process Account

<table>
<thead>
<tr>
<th>Element</th>
<th>Units</th>
<th>Total Cost</th>
<th>Element</th>
<th>Units</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>10,000</td>
<td>422,400</td>
<td>Goods transferred</td>
<td>8,000</td>
<td>896,000</td>
</tr>
<tr>
<td>Labour</td>
<td>395,600</td>
<td>225,000</td>
<td>WIP c/d</td>
<td>2,000</td>
<td>147,000</td>
</tr>
<tr>
<td>Overhead</td>
<td>10,000</td>
<td>1,043,000</td>
<td>WIP b/d</td>
<td>2,000</td>
<td>147,000</td>
</tr>
</tbody>
</table>

#### 7.5 TRANSFERRED IN

It is important to remind the reader that the output of one process level forms the input material to the next process level. The full cost of the completed units transferred forms the input material cost of the subsequent process and, by its nature, must be 100% complete. Material introduced is an extra material required by the process and should always be shown separately.

If there are partly completed units at the end of one period, there will be opening WIP at the beginning of the next period. The values of the cost elements of the brought forward WIP are normally known and they are to be added to the costs incurred during the period.

#### 7.6 ACCOUNTING TREATMENT OF SPOILAGES

In many industries, the amount of the process output will be less than the amount of the materials input. Such shortages are known as process losses or spoilages, which may arise due to a variety of factors such as evaporation, scrap, shrinkage, unavoidable handling, breakages, etc.

If the losses are in accordance with normal practice they are known as normal process losses. But where losses are above expectation, they are
known as abnormal losses, and as such they should be charged to an appropriate account pending investigation.

Normal process spoilages are unavoidable losses arising from the nature of the production process and, so, it is logical and equitable that the cost of such losses is included as part of the cost of good production. This is because in the production of good units normal spoilage occur. Since the spoilage arises under efficient operating conditions, it can be estimated with some degree of accuracy.

Abnormal process spoilages are those above the level deemed normal in the production process. Abnormal spoilage cannot be predicted and may be due to special circumstances such as plant breakdown, inefficient working, or unexpected defects in materials. Abnormal spoilage is the difference between actual spoilage in the period and the normal (estimated) spoilage. Abnormal gain is where the actual spoilage is less than the normal spoilage.

The cost of abnormal spoilage is to be charged to the profit and loss account unlike the cost of normal spoilage which is to be part of the good products’ total cost. Process account is to be credited as abnormal loss account is debited. The abnormal loss account is then to be closed to the profit and loss account.

Abnormal gain realized is to be credited to the abnormal gain account as process account is debited. The abnormal gain account is to be closed to the credit of profit and loss account.

**ILLUSTRATION 7-4**

A process has a normal spoilage of 5% which has a resale value of \( N\)150 per kg. Find the cost per kg of good production, if material cost is \( N\)27,000 and conversion cost is \( N\)13,000 of producing 100 kg.

**SUGGESTED SOLUTION 7-4**

| Units produced (100 kg) | Cost  \\
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>27,000</td>
</tr>
<tr>
<td>Conversion</td>
<td>13,000</td>
</tr>
<tr>
<td>100</td>
<td>40,000</td>
</tr>
<tr>
<td>Less Normal spoilage (5%)</td>
<td>5</td>
</tr>
<tr>
<td>Good Production</td>
<td>95</td>
</tr>
</tbody>
</table>
Cost per kg of good production \( = \frac{₦39,250}{95} = ₦413.16 \)

**ILLUSTRATION 7-5**

Using the information given in Illustration 7-4, find the abnormal spoilage and its value if good production was 91 kg and cost per kg of good production is the same (that is ₦413.16 per kg).

**SUGGESTED SOLUTION 7-5**

Abnormal spoilage \( = 9 \text{ kg} - 5 \text{ kg} = 4 \text{ kg} \)

<table>
<thead>
<tr>
<th>Process Account</th>
<th>Kg</th>
<th>Value</th>
<th>Kg</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>100</td>
<td>27,000</td>
<td>Good production</td>
<td>91</td>
</tr>
<tr>
<td>Conversion</td>
<td>13,000</td>
<td>750</td>
<td>Normal spoilage</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1,652</td>
<td>Abnormal spoilage</td>
<td>4</td>
</tr>
</tbody>
</table>

**Note:**

Abnormal spoilage cost was determined as follows:
\[
\text{Total cost - (cost of good prod. + Cost of normal spoilage)}
\]
\[
40,000 - (91 \times ₦413.16 + 5 \times ₦150)
\]
\[
40,000 - (37,598 + 750)
\]
\[
40,000 - 38,348 = ₦1,652
\]

<table>
<thead>
<tr>
<th>Abnormal Spoilage Account</th>
<th>₦</th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process A/c</td>
<td>1652</td>
<td></td>
</tr>
<tr>
<td>Profit &amp; Loss a/c</td>
<td>1652</td>
<td></td>
</tr>
</tbody>
</table>

**7.7 VALUATION PROCESS FOR COST STATEMENT**

A number of stages are passed through in the valuation process for cost statement. First, the physical flow of the units of production must be calculated having regards to the total number of units to be accounted for, regardless of the degree of completion.

Secondly, the equivalent units involved in the physical flow are to be calculated. In this respect, it is often necessary to divide the flow into its material cost element and conversion cost element as the degree of completion may vary between them.
Thirdly, having already established the physical units to be accounted for by means of the first two stages, the total equivalent units and the current equivalent units involved are to be calculated. These are to be accounted for in respect of the cost elements (transferred in cost, material cost and conversion cost).

Fourthly, the unit costs are to be calculated, paying attention to the stock valuation method assumed (FIFO, WAP, LIFO, etc.).

Fifthly and finally, the total cost of the transferred out products and work-in-process are to be calculated, ensuring that all costs are accounted for.

7.8 COST OF PRODUCTION REPORT

This report is to show the number of units of output to be accounted for, the total equivalent units of completed output, the cost statement showing the impact of all the cost elements and the cost of completed units as well as that of the work-in-progress at the end of the reporting period.

In the illustration that follows, two methods of stock valuation, FIFO and WAP, would be used and two processes of production are assumed.

ILLUSTRATION 7-6

Within the production department of Savannah Sugar Company Limited, there are two processes which produce the finished product. Raw materials are introduced initially at the commencement of Process 1 and further raw materials are added at the end of process 2. Conversion costs accrue uniformly throughout both processes. The flow of the product is continuous, the completed output of process 1 passes immediately into process 2 and the completed output of process 2 passes immediately into the finished goods warehouse.

The following information is available for the month of June:

**Process 1**

<table>
<thead>
<tr>
<th>Description</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening WIP</td>
<td>35,000 units</td>
</tr>
<tr>
<td>Materials</td>
<td>₦210,000</td>
</tr>
<tr>
<td>Conversion (2/5 complete)</td>
<td>₦52,500</td>
</tr>
<tr>
<td>Completion of units in June</td>
<td>168,000 units</td>
</tr>
<tr>
<td>Units commenced in June</td>
<td>140,000 units</td>
</tr>
<tr>
<td>Closing WIP (½ complete as to conversion)</td>
<td>7,000 units</td>
</tr>
<tr>
<td>Material introduced in June</td>
<td>₦770,000</td>
</tr>
<tr>
<td>Conversion cost added in June</td>
<td>₦630,000</td>
</tr>
</tbody>
</table>
Process 2
Opening WIP 42,000 Units
Material from process 1 ₦343,000
Conversion (2/3 complete) ₦392,000
Completion of units in June 154,000 Units
Units commenced in June
Closing WIP (2/8 complete as to conversion) 56,000 Units
Material introduced in June ₦462,000
Conversion costs added in June ₦2,205,000

Required:

Give the cost of production report of Theresa Alice Sugar Company Limited for the month of June, using each of the WAP and FIFO methods, and showing clearly the cost of finished production and WIP at end of the period.

SUGGESTED SOLUTION 7-6

Tutorial Note: The units to be accounted for, total equivalent units and current equivalent units are to be determined before going to the cost statement, using each of the two stock valuation methods. The heading of the report should be well expressed.

Cost of Production Report of Theresa Alice Sugar Company Limited for the month of June, using Weighted Average Price (WAP) Method

Process 1

Physical flow of units of material:
WIP (beginning) 35,000
Material introduced 140,000
Total units to be accounted for 175,000

Units Accounted for
Units completed & transferred out 168,000
WIP (ending) 7,000
Total units accounted for 175,000

Equivalent Units
Material Conversion
168,000 168,000
7,000 (100%) 3,500 (1/2 of

Total equivalent units (TEU) 175,000 171,500
Less WIP (beginning) 35,000 14,000
Current Equivalent units (CEU) 140,000 157,500
Note

(a) Conversion WIP ending = \(1/2 \times 7,000\) = 3,500 units
(b) Conversion WIP beginning = \(2/5 \times 35,000\) = 14,000 units

Cost Statement

<table>
<thead>
<tr>
<th>Cost Element</th>
<th>Cost of WIP (beginning)</th>
<th>Current Cost</th>
<th>TC</th>
<th>TEU</th>
<th>Cost/unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Material</td>
<td>210,000</td>
<td>770,000</td>
<td>980,000</td>
<td>175,000</td>
<td>5.60</td>
</tr>
<tr>
<td>Conversion</td>
<td>52,500</td>
<td>630,000</td>
<td>682,500</td>
<td>171,500</td>
<td>3.98</td>
</tr>
<tr>
<td></td>
<td>262,500</td>
<td>1,400,000</td>
<td>1,662,500</td>
<td></td>
<td>9.58</td>
</tr>
</tbody>
</table>

Cost of units completed and transferred out = 168,000 x N9.58
= N1,609,440

Cost of WIP (Ending)

| Material     | 7,000 x 1 x N5.6 =      | N39,200      |
| Conversion   | 7,000 x 1/2 x N3.98 =   | N13,930      |
|              |                         | N53,130      |

Another way (which is easier) of determining the cost of WIP ending is to find the difference between total cost and cost of the completed units.

Cost of WIP (end) = TC – Cost of completed units
= 1,662,500 - 1,609,440
= N53,060

Note:
The difference of N70 is due to the approximation made to two decimal places.

Cost of Production Report Using First-In-First-Out (FIFO) Method

Process 1

<table>
<thead>
<tr>
<th>Current costs</th>
<th>C.E. Units</th>
<th>Units cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>770,000</td>
<td>N5.50</td>
</tr>
<tr>
<td>Conversion</td>
<td>630,000</td>
<td>N4.00</td>
</tr>
</tbody>
</table>

Cost of Closing WIP

| Material     | 7,000 x 1 x N5.5 =      | N38,500      |
| Conversion   | 7,000 x 1/2 x N4.0 =    | N14,000      |
|              |                         | N52,500      |
Units completed & transferred out = 168,000 units
Cost of the completed unit = TC – cost of closing WIP
= 1,662,500 – 52,500
= ₦1,610,000

Process 2, Using WAP Method

Physical flow of units of material

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>WIP (beginning)</td>
<td>42,000</td>
</tr>
<tr>
<td>Units transferred in</td>
<td>168,000</td>
</tr>
<tr>
<td>Units to be accounted for</td>
<td>210,000</td>
</tr>
</tbody>
</table>

Equivalent Units

<table>
<thead>
<tr>
<th>Units Accounted for:</th>
<th>Transferred in</th>
<th>Material</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units completed in the period</td>
<td>154,000</td>
<td>154,000</td>
<td>154,000</td>
</tr>
<tr>
<td>WIP (ending):</td>
<td>56,000</td>
<td>56,000</td>
<td>0</td>
</tr>
<tr>
<td>Total units accounted for</td>
<td>210,000</td>
<td></td>
<td>210,000</td>
</tr>
<tr>
<td>Total equivalent units</td>
<td>210,000</td>
<td>154,000</td>
<td>175,000</td>
</tr>
<tr>
<td>Less WIP (beginning)</td>
<td>42,000</td>
<td>0</td>
<td>28,000</td>
</tr>
<tr>
<td>Current Equivalent units (CEU)</td>
<td>168,000</td>
<td>154,000</td>
<td>147,000</td>
</tr>
</tbody>
</table>

Cost Statement

<table>
<thead>
<tr>
<th>Cost Element</th>
<th>Cost of WIP (beginning)</th>
<th>Current Cost</th>
<th>TC</th>
<th>TEU</th>
<th>Cost/unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred in</td>
<td>343,000</td>
<td>1,609,440</td>
<td>1,952,440</td>
<td>210,000</td>
<td>9.2973</td>
</tr>
<tr>
<td>Material</td>
<td>0</td>
<td>462,000</td>
<td>462,000</td>
<td>154,000</td>
<td>3.0000</td>
</tr>
<tr>
<td>Conversion</td>
<td>392,000</td>
<td>2,205,000</td>
<td>2,597,000</td>
<td>175,000</td>
<td>14.8400</td>
</tr>
<tr>
<td></td>
<td>735,000</td>
<td>4,276,440</td>
<td>5,011,440</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Cost of complete units = 154,000 x ₦27.1373 = ₦4,179,144.20

Cost of WIP (Ending)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred in</td>
<td>56,000 \times 1 \times ₦9.2973 = ₦520,648.80</td>
</tr>
<tr>
<td>Material</td>
<td>56,000 \times 0 \times ₦3 = ₦0</td>
</tr>
<tr>
<td>Conversion</td>
<td>56,000 \times \frac{3}{8} \times ₦14.84 = ₦311,640.00</td>
</tr>
<tr>
<td></td>
<td>₦832,288.80</td>
</tr>
</tbody>
</table>
Another Way

Cost of Ending WIP = TC – Cost of completed units
= 5,011,440 – 4,179,144.20 = 832,295.80

Note that the difference of N7 is due to the approximation made to four decimal places.

Process 2: Using FIFO Method

<table>
<thead>
<tr>
<th>Current Costs</th>
<th>Current Equiv.</th>
<th>Unit Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Units</td>
<td>₹</td>
</tr>
<tr>
<td>Transferred in</td>
<td>1,609,440</td>
<td>168,000</td>
</tr>
<tr>
<td>Material</td>
<td>462,000</td>
<td>154,000</td>
</tr>
<tr>
<td>Conversion</td>
<td>2,205,000</td>
<td>147,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TC/C.E.U</td>
</tr>
</tbody>
</table>

Cost of Closing WIP

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>₹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred in</td>
<td>56,000 x 1 x 9.58</td>
<td>536,480</td>
</tr>
<tr>
<td>Material</td>
<td>56,000 x 0 x N3</td>
<td>0</td>
</tr>
<tr>
<td>Conversion</td>
<td>56,000 x 3/8 x 15</td>
<td>315,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>851,480</td>
</tr>
</tbody>
</table>

Units completed and transferred out = 154,000 units
Cost of completed units = TC – Cost of ending WIP
= 5,011,440 – 851,480
= 4,159,960

ILLUSTRATION 7-7

Mabel Akparandu Oil Milling Company Limited processes oil in two departments: A and B, before the final product is made ready for consumption. The company uses process costing for its operations. You are given the following information by the cost accountant of the company for the month ended 31 December, 2008.

Department A

Work-in-Process (beginning) 170,000 units
Units started during the year 430,000
Normal spoilage 1.5% of total
Unit completed 490,000
Work-in-Process (ending) 98,000

Stages of completion of Work-in-Process

WIP beginning: Material 100%
COSTING AND QUANTITATIVE TECHNIQUES

Direct labour 15%
Overhead 15%

WIP ending: Materials 100%
Direct labour 25%
Overhead 25%

Stages of Completion of Spoilage

Material 100%
Direct labour 30%
Overhead 30%

Costs

WIP beginning = ₦684,250
Cost incurred during the month = ₦3,459,700
(Material: ₦1,612,500; Direct Labour: ₦738,900; Overhead: ₦1,108,350)

Department B

Work-in-Process (beginning) 115,000 units
Transferred in 490,000
Gain on efficiency = 5% of transferred in 24,500
Normal spoilage 3,000
Units completed 595,000
Work-in-Process (ending) 31,000

Stages of completion of WIP

WIP beginning: Transferred in 100%
  Material 100%
  Direct labour 45%
  Overhead 45%
WIP ending: Transferred in 100%
  Materials 100%
  Direct labour 75%
  Overhead 75%

Stages of Completion of Spoilage

Material 100%
Transferred in 100%
Direct labour 70%
Overhead 70%
Costs

WIP beginning = ₦1,259,250
Cost incurred during the month = ₦7,415,125
(material: ₦900,375; Transferred in: ₦3,670,000; direct labour: ₦1,137,900; Overhead: ₦1,706,850)

Required:

Using the FIFO method, draw up a cost of production report showing the cost of completed units, cost of normal and abnormal spoilages and the cost of closing inventory (WIP) of each of the two departments. Show all relevant computations in detail.
Spoilages are detected at the end of operations.

SUGGESTED SOLUTION 7-7

Cost of Production Report of Mabel Akparandu Oil Milling Company Ltd for the month ended 31/12/2005, using the FIFO method of stock Valuation

Department A’s Report
Physical flow of Units:
WIP beginning 170,000
Units started 430,000
Units to be accounted for 600,000

Equivalent Units

<table>
<thead>
<tr>
<th>Units Accounted for</th>
<th>Material</th>
<th>Direct lab.</th>
<th>Overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units completed</td>
<td>490,000</td>
<td>490,000</td>
<td>490,000</td>
</tr>
<tr>
<td>Normal spoilage (1.5% of 600,000)</td>
<td>9,000</td>
<td>9,000</td>
<td>2,700</td>
</tr>
<tr>
<td>WIP (end):</td>
<td>98,000</td>
<td>98,000</td>
<td>24,500</td>
</tr>
<tr>
<td>Abnormal spoilage</td>
<td>3,000</td>
<td>3,000</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>600,000</td>
<td>600,000</td>
<td>518,100</td>
</tr>
</tbody>
</table>

Less: Old equiv. units of
WIP (beginning) 170,000 25,500 25,500
Current Equivalent units (CEU) 430,000 492,600 492,600

Note that:
Abnormal spoilage is the difference between Actual spoilage and normal spoilage = 12,000 – 9,000 = 3,000 units
Alternatively, abnormal spoilage
= 600,000 – (490,000 + 9,000 + 98,000)
\[
\begin{align*}
\text{Current Cost} & \quad \text{Current Equiv.Units} & \quad \text{Cost/Unit} \\
\text{Direct material} & = 1,612,500 & = 430,000 & = 3.75 \\
\text{Direct labour} & = 738,900 & = 492,600 & = 1.50 \\
\text{Overhead} & = 1,108,350 & = 492,600 & = 2.25 \\
\hline
\text{Total cost} & = 3,459,750 \\
\end{align*}
\]

**Cost of WIP (end)**

\[
\begin{align*}
\text{Direct material} & = 98,000 \times 1.00 \times 3.75 = 367,500 \\
\text{Direct labour} & = 98,000 \times 0.25 \times 1.5 = 36,750 \\
\text{Overhead} & = 98,000 \times 0.25 \times 2.25 = 55,125 \\
\hline
\text{Total cost} & = 459,375 \\
\end{align*}
\]

**Cost of Spoilages**

\[
\begin{align*}
\text{Direct material} & = 12,000 \times 1.00 \times 3.75 = 45,000 \\
\text{Direct labour} & = 12,000 \times 0.3 \times 1.5 = 5,400 \\
\text{Overhead} & = 12,000 \times 0.3 \times 2.25 = 8,100 \\
\hline
\text{Total cost} & = 58,500 \\
\end{align*}
\]

**Apportionment**

\[
\begin{align*}
\text{Normal spoilage} & = \frac{9,000 \times 58,500}{12,000} = 43,875 \\
\text{Abnormal spoilage} & = \frac{3,000 \times 58,500}{12,000} = 14,625 \\
\hline
\text{Total cost} & = 4,144,000 - (459,375 + 14,625) \\
& = 3,670,000 \\
\end{align*}
\]
Note:

The abnormal spoilage cost is to be charged to the profit and loss account, while the normal spoilage cost is to be part of the good production cost.

Department B’s Report

Physical flow of units
- WIP beginning: 115,000
- Transferred in: 490,000
- Gain (5% of T. in): 24,500
- Units to be accounted for: 629,500

Equivalent Units

<table>
<thead>
<tr>
<th>Units</th>
<th>Units Accounted for</th>
<th>Units Transferred in</th>
<th>Direct Mat.</th>
<th>Direct Lab.</th>
<th>Overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>completed</td>
<td>595,000</td>
<td>595,000</td>
<td>595,000</td>
<td>595,000</td>
<td>595,000</td>
</tr>
<tr>
<td>Normal spoilage</td>
<td>3,000</td>
<td>3,000</td>
<td>3,000</td>
<td>2,100</td>
<td>2,100</td>
</tr>
<tr>
<td>WIP end</td>
<td>31,000</td>
<td>31,000</td>
<td>31,000</td>
<td>23,250</td>
<td>23,250</td>
</tr>
<tr>
<td>Abnormal spoilage</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>350</td>
<td>350</td>
</tr>
<tr>
<td></td>
<td>629,500</td>
<td>629,500</td>
<td>629,500</td>
<td>620,700</td>
<td>620,700</td>
</tr>
</tbody>
</table>

Less: Old equiv. unit
- (WIP beginning): 115,000
- Current Equivalent units: 514,500

Cost Statement

<table>
<thead>
<tr>
<th>Current Costs</th>
<th>Current Equiv. units</th>
<th>Cost/Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred in</td>
<td>3,670,000</td>
<td>7.13</td>
</tr>
<tr>
<td>Direct material</td>
<td>900,375</td>
<td>1.75</td>
</tr>
<tr>
<td>Direct labour</td>
<td>1,137,900</td>
<td>2.00</td>
</tr>
<tr>
<td>Overhead</td>
<td>1,706,850</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>7,415,125</td>
<td></td>
</tr>
<tr>
<td>Add: cost of WIP</td>
<td>1,259,250</td>
<td></td>
</tr>
<tr>
<td>(beginning)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total cost</td>
<td>8,674,375</td>
<td></td>
</tr>
</tbody>
</table>
Costing and Quantitative Techniques

Cost of WIP (end)  

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred in</td>
<td>31,000 x 1 x 7.13 = 221,030</td>
</tr>
<tr>
<td>Direct material</td>
<td>31,000 x 1 x 1.75 = 54,250</td>
</tr>
<tr>
<td>Direct labour</td>
<td>31,000 x .75 x 2.00 = 46,500</td>
</tr>
<tr>
<td>Overhead</td>
<td>31,000 x .75 x 3.00 = 69,750</td>
</tr>
<tr>
<td>Normal loss (apportionment)</td>
<td>1,839</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>393,369</strong></td>
</tr>
</tbody>
</table>

Costs of spoilage (losses) are determined as follows

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total spoilage</td>
<td>3,500</td>
</tr>
<tr>
<td>Transferred in</td>
<td>3,500 x 1 x 7.13 = 24,955</td>
</tr>
<tr>
<td>Direct material</td>
<td>3,500 x 1 x 1.75 = 6,125</td>
</tr>
<tr>
<td>Direct labour</td>
<td>3,500 x 0.7 x 2.00 = 4,900</td>
</tr>
<tr>
<td>Overhead</td>
<td>3,500 x 0.7 x 3.00 = 7,350</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>43,330</strong></td>
</tr>
</tbody>
</table>

Allocation of Spoilages cost

Normal spoilage = 3,000 x N43,330 = N37,140

Abnormal spoilage = 500 x N43,330 = N6,190

The abnormal loss cost is to be debited to abnormal loss account and credited to process account (or department B’s account). The abnormal loss account is to be closed to the profit and loss account.

As for the normal loss, the whole amount can be made to form part of the cost of units transferred to store or be apportioned between the units transferred to the store for sale and the WIP end. It is the second approach that is adopted in this example and, so, the figure charged to WIP end (N1,839) was arrived at as follows:

Apportionment to units transferred in

= 595,000 x N37,140 = N35,301

= 626,000

Apportionment to WIP end

= 31,000 x N37,140 = N1,839

= 626,000

= 37,140
7.9 JOINT AND BY-PRODUCTS COSTING

The process costing principle discussed in this chapter is about determining the cost of processing some inputs that yield the same type of product. At the end of the processing activities, only one type of product would result from the processed raw material.

However, it is not always that we have only one type of product from a processing operation. It is possible for a single raw material to yield two or more products simultaneously when processed. Such products are known as joint products. For example, when crude oil (a single raw material) is processed or refined, petrol, kerosine, gas, etc, could be obtained from it.

The cost of processing a production input (raw material) that would amount to joint products is known as joint cost. The joint cost is to be restricted to the split-off point (point after which each joint product would be incurring separate processing cost). Joint cost is not to be traced to any particular product but rather to all the joint products as a group. There are many ways of apportioning joint cost to joint products for financial accounting purposes. These would be discussed in this chapter.

In practice, it is normal to identify one product out of the joint products as the main or principal product and the rest to be treated as joint products or as by-products. In the example above, it is clear that petrol is the main product to be identified as crude oil is processed. Pairs of shoes could be main products as leather is processed, while bags, wallets, etc, could be joint or by-products.

One way of differentiating between by-product and joint product is to consider their cost of production or sales value. A product that cost between 10% to 15% of the main product cost should be treated as a by-product. Any product that costs between 15% to 40% of the main product cost is a joint product. Any product that costs above 40% of the identified main product cost should also be treated as a main product. As a result of changes in price, therefore, a by-product can become a joint-product or even a main product and vice versa.
7.10 BY-PRODUCT AND ITS ACCOUNTING TREATMENT

A by-product is a secondary product arising as a result of a processing activity aimed at producing a certain main product. The market value of a by-product less the processing cost after the split off point is usually negligible, compared to the total market value of all the joint products or the market value of the main product.

The usual treatment of by-product is to deduct its Net Realizable Value (NRV) from the total joint cost (JC) and then divide the net joint cost among the joint or main products. The NRV of the by-product is the difference between its market value and its separate processing cost.

ILLUSTRATION 7-8

Wambai Shoemakers has a process that yields two main products: A and B and a by-product C at a total cost of Naira 3,000,000. There are 1000 units of C requiring no further processing and each can be sold at Naira 60 with negligible market cost. The two main products take equal share of joint cost.

Required

What should be the share of Product A from the Joint Cost?

SUGGESTED SOLUTION 7-8

The total market value of Product C = 1000 x Naira 60 = Naira 60,000. This is its NRV, since its market cost is negligible.

Net Joint Cost = 3,000,000 – 60,000 = Naira 2,940,000

Share of Product A = \( \frac{2,940,000}{2} \) = Naira 1,470,000

Note

It can be concluded that in deducting the NRV of by-product C from the Joint Cost, we are in effect assigning to the by-product a joint cost which is equal to its NRV.

7.11 ACCOUNTING TREATMENT OF JOINT COST

There are three usual bases of sharing joint cost to the joint (or main) products. These are the Physical Units Basis, Sales Value (at the point of separation) and Net Realization Basis.
7.11.1 PHYSICAL UNIT BASIS

Under this method, the joint cost is shared among the joint products on the basis of the quantities of physical units, provided all the products are measured by a common unit of measurement, such as kilograms or litres. The problem with this method is that consideration is not given to price and, so, it does not consider the value of the products. Usually, the value of products is the most important factor to be considered.

ILLUSTRATION 7-9

Anadariya Company Ltd., Tiga has a processing system that produces three products: Kuli, Sudi and Tuni with 5,000 kg, 3,000 kg and 2,000 kg, respectively, in a year. The total cost incurred up to the split off point in the year 2000 was N1,000,000. Use the physical units basis to share the joint cost among the three products. Calculate also their unit cost.

SUGGESTED SOLUTION 7-9

(a) The Ratio
\[ K = \frac{5,000}{10,000} \times 100 = 50\% \]
\[ S = \frac{3,000}{10,000} \times 100 = 30\% \]
\[ T = \frac{2,000}{10,000} \times 100 = 20\% \]

Share of joint cost
\[ K = 50\% \text{ of } N1,000,000 = N500,000 \]
\[ S = 30\% \text{ of } N1,000,000 = N300,000 \]
\[ T = 20\% \text{ of } N1,000,000 = N200,000 \]

(b) Unit Cost based on the share of joint cost
\[ K = \frac{N500,000}{5,000} = N100/\text{unit} \]
\[ S = \frac{N300,000}{3,000} = N100/\text{unit} \]
\[ T = \frac{N200,000}{2,000} = N100/\text{unit} \]
7.11.2 Sales Value (At the Point of Separation)

Under this method, the joint cost is shared among the joint products on the basis of their sales value before further processing. At the split-off point, market value can be estimated per unit of each of the joint products. The ratios of the sales value of the joint products are to be used as basis of apportioning the joint cost.

The problems with this method are two-fold: One, a product may have zero value at the point of separation but significant value with little processing cost after the split-off point. Secondly, a product may have high selling price at the split-off point and hence high sales value but may involve large selling and distribution cost (advert, carriage, etc) so that its value is much less than its selling cost.

**Illustration 7-10**

Assuming that Anadariya Company Ltd has estimated the following selling prices for its three products at the point of separation:

- \( K = \text{₦400/unit} \)
- \( S = \text{₦440/unit} \)
- \( T = \text{₦340/unit} \)

Use the Sales Value method to apportion the joint cost and determine the per unit cost of each of the three products.

**Suggested Solution 7-10**

(a) | Product | Unit | SP/Unit | Sales Value | Ratio | Share of JC |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>5,000</td>
<td>400</td>
<td>2,000,000</td>
<td>50%</td>
<td>500,000</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>3,000</td>
<td>440</td>
<td>1,320,000</td>
<td>33%</td>
<td>330,000</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>2,000</td>
<td>340</td>
<td>680,000</td>
<td>17%</td>
<td>170,000</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{Total Share of JC} = 1,000,000 \]

(b) Unit cost based on the share of joint cost:

- \( K = \frac{\text{₦500,000}}{5000} = \text{₦100/Unit} \)
- \( S = \frac{\text{₦330,000}}{3000} = \text{₦110/Unit} \)
- \( T = \frac{\text{₦170,000}}{2000} = \text{₦85/Unit} \)
7.11.3 NET REALIZABLE VALUE BASIS

This method is used only when further processing is necessary or when high marketing and distribution costs are involved. Net realization is sales value less the incremental cost of further processing after the split-off point and any other cost necessary for the selling or distribution of the product.

ILLUSTRATION 7-11

Assuming that the sales values in illustration are market prices after further processing and that separate processing and marketing costs are as follows:

\[ K = \text{₦200,000} \]
\[ S = \text{₦300,000} \]
\[ T = \text{₦160,000} \]

Determine the share of the joint cost to the three (3) products. Show also the per unit cost of each of the three products.

SUGGESTED SOLUTION 7-11

<table>
<thead>
<tr>
<th>Product</th>
<th>Units</th>
<th>SP/Unit</th>
<th>Sales</th>
<th>SPC</th>
<th>NRV</th>
<th>Share of JC</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>5,000</td>
<td>400</td>
<td>2,000,000</td>
<td>200,000</td>
<td>1,800,000</td>
<td>538,922</td>
</tr>
<tr>
<td>S</td>
<td>3,000</td>
<td>440</td>
<td>1,320,000</td>
<td>300,000</td>
<td>1,020,000</td>
<td>305,389</td>
</tr>
<tr>
<td>T</td>
<td>2,000</td>
<td>340</td>
<td>680,000</td>
<td>160,000</td>
<td>520,000</td>
<td>155,689</td>
</tr>
<tr>
<td>Total</td>
<td>10,000</td>
<td>4,000,000</td>
<td>3,340,000</td>
<td></td>
<td></td>
<td>1,000,000</td>
</tr>
</tbody>
</table>

Note:

(a) Net Realizable Value (NRV) = Sales Value Less separate processing costs (SPC).

(b) The total of the NRV of all the joint products is obtained and the joint cost is shared in proportion to the NRV of each product.

(c) This method is the ‘best’ as it considers the quantity (units) produced of all the joint products, their sales values and their further processing costs.

2. Unit cost based on the share of joint cost:

K = \( \frac{₦538,922}{5,000} = ₦108/\text{unit} \)
7.12 TOTAL COST PER UNIT DETERMINATION USING NRV METHOD

Total cost of a joint product is given by its share of joint cost plus its further processing and marketing cost. To arrive at its total cost per unit, the total cost is divided by the units produced.

Using illustration 7-11, total cost per unit could be determined for each of the three products as follows:

\[
K = \frac{N538,922 + 200,000}{5,000} = \frac{N738,922}{5,000} = N147.78
\]

\[
S = \frac{N305,389 + 300,000}{3,000} = \frac{N605,389}{3,000} = N201.80
\]

\[
T = \frac{N155,689 + 160,000}{2,000} = \frac{N315,689}{2,000} = N157.84
\]

If there are closing inventory of Product K (900 units), S (500 units) and T (400 units), the value of closing stock for reflection in the balance sheet could be determined as follows:

\[
K = 900 \times N147.78 = N133,002
\]

\[
S = 500 \times N201.80 = N100,900
\]

\[
T = 400 \times N157.84 = N63,136
\]

\[
N297,038
\]

\[\text{Note:} \]

It should be understood that profit is always the difference between total revenue (sales value) and total cost. That economics principle is very much applicable in joint-product costing!

7.13 SUMMARY AND CONCLUSIONS

This chapter has introduced the readers to the meaning of process costing, its application areas, and how it can be put to use for proper accountability. The characteristics of process costing, how products flow in the course of processing, the equivalent units of production to be transferred to the next stage of production, accounting for spoilages/losses and the valuation process for cost of production report have all been treated. Finally, cost of production and report write-ups have been adequately illustrated, using highly standardized exercises.
Process costing, which is arguably the most widely used costing in the world, has been given adequate coverage it deserves.

The chapter has also put the readers through joint products costing, where three different methods of apportioning joint cost to joint-products were discussed. By-product, and its accounting treatment, has also been discussed.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

7.15 REVISION QUESTIONS

7.14.1 MULTIPLE CHOICE QUESTIONS

1. Which of the following methods calculates equivalent production based upon the activity of current period only?
   A. FIFO
   B. Specific Identification
   C. Weighted Average
   D. LIFO
   E. Replacement Cost

2. Transferred in costs are accounted for in the same manner as
   A. Materials added at the beginning of the process
   B. Materials added at the end of the process
   C. Conversion costs
   D. Labour costs
   E. Other indirect expenses

3. In process costing, where losses have a positive scrap value, abnormal gains account is
   A. Debited with the normal production cost of the abnormal gains units
   B. Debited with the normal production cost of the abnormal gain units and credited with scrap value of the abnormal gain units
   C. Credited with normal production cost of the abnormal gains units and debited with scrap value of abnormal gain units
   D. Credited with the normal production cost of the abnormal gains units and credited with scrap value of abnormal gains units
   E. Totally ignored in the cost computation

4. Where abnormal loss arises in process costing, the process account generally is
   A. Debited with the scrap value of abnormal loss units
   B. Debited with the full production cost of abnormal loss units
   C. Credited with the scrap value of abnormal loss units
   D. Debited with the full production cost of abnormal loss units
   E. Computation ignored in the cost computation.
5. Joint Product costs may not be accounted for through one of the following bases
   A. Physical units
   B. Sales value at point of separation
   C. Observation cost at split-off point
   D. Net realisable value
   E. Value after further processing

7.14.2 SHORT ANSWERS QUESTIONS

Use the information given below to answer questions 1-5. Briggs Ltd manufactures a product that passes through two processes, 1 and 2. All manufacturing costs are added uniformly in Process 1. Information for Process 1 in May is as follows:

Work in Process May 1 units (30% complete) \( \text{N}30,000 \)
- Direct materials \( \text{N}4,000 \)
- Direct labour \( \text{N}3,000 \)
- Overhead \( \text{N}4,752 \)

During May, 200,000 units were completed and transferred to Process 2.

The following costs were incurred by process 1 during May:
- Direct materials \( \text{N}32,210 \)
- Direct labour \( \text{N}55,400 \)
- Overhead \( \text{N}31,300 \)

16,000 units that were 70% complete remained in the Process at May 31 using FIFO method.

1. Equivalent units of production for May would be
2. Total cost to account for would be \( \text{N} \)
3. Total cost per Equivalent unit of production would be \( \text{N} \)
4. The cost of goods transferred to Process 2 would be \( \text{N} \)
5. The costs of May’s closing work in process would be

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
8.0 LEARNING OBJECTIVES

After studying this chapter, readers will be able to understand the:

♦ Meaning and significance of budget, budgeting process, types of budgets based on activity, time and quantitative perspectives;
♦ Budgetary control process, including comparison between budgeted and actual results and significance of budget committee, budget manual, budget officer, etc;
♦ The use of budgetary improvement techniques such as Incremental Budgeting; Zero-base Budgeting, Planning, Programming, Budgeting System and Rolling Budgeting; and
♦ Preparation of functional and master budgets of a business organisation.

8.1 INTRODUCTION

Budget is a very popular term to many people both in business and governmental organisations. It has been widely popularized by government through its annual budget pronouncements. All the three components of the economy (individuals, firms and governments) need budgets as part of their everyday lives. Finance, time, material, men and other resources need to be budgeted for in view of the fact that they are scarce, in every sector of the economy, when compared against needs.

Budgeting to an individual might not be a formal plan, but all his/her activities should be based on some degree of budgeting and reflect some level of planning if control is to be exercised. Business and other organisational activities of firms must be well planned so that the firms may operate effectively and efficiently, utilizing all resources
economically to achieve the desired results. Government at various levels must budget for the revenue to be generated and the expenditure to be incurred, using its scarce resources to cater for the general well-being of the masses. Budget indiscipline on the part of any of the three components of the economy would surely lead to a lot of problems.

8.2 WHAT IS A BUDGET?

The term budget has been defined in several ways, each emphasising the issues of planning and control of future activities. It could be viewed as a process of setting performance standard for future activities so as to exercise control (on cost, revenue, income and other financial or non-financial activities). It could also be viewed as a means of obtaining accountability and control over the use of money or over all activities.

The Institute of Cost and Management Accountants (ICMA) defined budget as a plan quantified in monetary terms, prepared and approved prior to a defined period of time, usually showing planned incomes to be generated and/or expenditures to be incurred during that period, and the capital to be employed to attain a given objective. Budget could also be defined as an exercise in communication by which the expectations of management about levels of performance of subordinates are communicated to the subordinates.

These and many other definitions of budget are emphasising the need for good planning and control of financial and non-financial activities by all the three components of an economy. The definitions also show that since resources are scarce, there is the need for proper and adequate planning as to how to utilize them to achieve maximum results.

8.3 BUDGETING FRAMEWORK (PROCESS)

Budgets are to be prepared within a given framework. The process of budgeting entails passing through some steps, which are outlined on page 137:
Step 1  Identify goal(s) of budgetee

Step 2  Collect and analyse data about alternatives

Step 3  Choose decision rules

Step 4  Rank alternative courses of action

Step 5  Make a decision and state expected outcome

Step 6  Monitor actual outcome to ensure actions

Step 7  Report actual results for comparison with budgeted results

The illustration on budgeting (process) describes the framework for effective managerial planning, decision making and control to be used in both private and public sector organisations. The framework is explained as follows:

8.3.1 Goal(s) Identification

The basic goal(s) to be achieved must be identified. This is necessary for accurate estimation of the money, time and other resources to be used in the short-run and in the long-run for the achievement of the set objectives.

8.3.2 Collection and Analysis of Data

In budgeting, alternative courses of action, for the achievement of goals/objectives, have to be mobilized. This would create room for analysis and interpretation of the data to be collected on the alternative courses of action so that cost effectiveness could be achieved. For example, it may be better to develop some human resources using on-the-job training instead of off-the-job training as this may save cost greatly and allow room for effective monitoring and supervision. The data collected when analysed and interpreted would guide a lot of decisions.
8.3.3 Choose Decision Rules

As you analyse and interpret alternative courses of action, using the data collected, decision has to be reached. Yardstick for decision-making must be adopted and this should be in line with the objectives to be achieved. For capital budgeting, decision rules like Net Present Value (NPV), Pay Back Period (PBP), Internal Rate of Returns (IRR), Accounting Rate of Return (ARR), etc, are applied in taking decisions in respect of projects to be embarked upon.

8.3.4 Ranking of Alternative Courses of Action

The competing alternative courses of action should be ranked in their order of decreasing benefit or just in order of importance to the organisation. Budgetary decision should be based on the ranking to be made in view of the fact that the scarce resources cannot take care of all the competing activities or projects that would be proposed for the achievement of organisational objectives.

8.3.5 Make a decision and state expected outcome

On the basis of the ranking made, decision has to be taken and the subordinates, who are to implement the decision (budget), have to be informed as to what is expected of them. Budgetary decision is to be made by the top management after following the due process of approval. The approved budgets are to be communicated to the departments or divisions for implementation.

8.3.6 Monitor actions to ensure results

There is the need for effective monitoring and supervision of the implementors of approved budgets to ensure that everything is going according to plan. The cost of this monitoring and supervision is to be estimated and included in the budgeted expenditure. Since budget (standard) is prepared at the beginning of an accounting period for the achievement of various results, there is always the need for control, which is just about making sure that things are going according to plan (the budget). It is then that the desired results could be achieved by the end of the accounting period.
8.3.7 Measurement and reporting of actual results

At the end of an accounting period (which may be a year, a quarter, a month, a week or a day), actual results on set objectives, should be measured (determined) and reported to the appropriate authorities for consideration.

The budgeting framework described clearly shows that budgeting is an integral part of both planning process and control process in any serious organisation.

8.4 BUDGET LIMITING FACTORS

Budget decision has to take account of the interdependency existing in the organisation. For example, in a manufacturing organisation, sales department depends on the production department. Production department depends on both purchases department and personnel department. These two departments depend on the finance department. Finance department depends on sales department. All the departments are dependent on one another, in one way or another. This interdependency has to be taken into consideration while preparing budgets or setting standards for the departments to attain some level of performance. It would be unfair, for example, to expect a production department to produce up to a certain level of output if enough raw materials could not be purchased for it to use; if adequate skilled and unskilled labour force is not available, or if machines are not in good shape.

Another important limitation of budget is the market. If market is not available for a certain number of an organisation’s product or service, that number should not be produced until the market is created. Other environmental factors like the political, economic, social, legal and technological circumstances of the budget period are to be taken into consideration while preparing budgets.

8.5 GENERAL PURPOSES OF BUDGET

Budget serves a number of purposes in a business organisation. The following are the summarized purposes of budget:

8.5.1 Accountability and Control

This is the traditional purpose which budget serves. Accountability is about giving reasons for or explanations on what you do. Budget serves as the basis of making budgetees account for what they have done by the end of the budget
period. Accountability leads to the establishment of a pattern of control over receipts and expenditure that permits a determination of whether funds have been used for the purpose for which they are meant or not. Accountability is achieved through specific allocation of funds and responsibilities and ensuring that all laws, rules, and procedures established are obeyed and followed to the letter. Budget is an instrument of checking the excesses of officials through various control measures.

8.5.2 Management

The management purpose which budget serves involve efforts toward carrying out approved plans and policies efficiently and effectively; and since management is about formulating and implementing decisions, budget is, therefore, its essential tool to discharge these basic duties. Budget, as a management tool, represents an operational document which specifies, directly or indirectly, the cost, time and nature of the expected results of specific budgetary outlays. Budget, as a tool, is very much applicable in all fields of human endeavour.

8.5.3 Planning

Forecasts and estimates as to what resources should be mobilized and utilized are made in the budget. Planning is, therefore, central in budgetary process. Planning involves the determination of objectives, evaluation of alternative courses of action and the authorization for selecting programme. The planning purpose of budget is very central to the realization of budget objectives.

8.5.4 Economic Policies

At government levels, in particular, budget enunciates government's economic policies. It indicates the direction of the economy and expresses intentions regarding the utilization of the societal resources. It also shows how government strives to promote macro-economic balance in the economy. Companies use budget as a means of expressing policy objectives and directions with human resources development occupying prominent position.

The four general purposes of budget suggest that budgeting should be geared toward enforcing prudence, accountability and
integrity. Indeed, a sound and cohesive budgeting is necessary for ensuring control in all fields of human endeavours.

8.6 **BUDGETARY CONTROL PROCESS**

Budgetary control is the establishment of a quantitative and financial statement, showing the effect of following a given policy objective during a specified period and then comparing the actual results with the previous estimates. Variances (deviations) are investigated and corrective actions taken. These may include the modification of plans and the changing of operational strategies. The following processes are necessary for effective budgetary control:

8.6.1 **Establishment of Objectives**

There should be well established overall and divisional/departmental objectives. It is on the basis of these objectives, which could be short, medium or long-term, that budgets are prepared. Without these objectives, the preparation of budgets will lack direction.

8.6.2 **Budget Centres**

Budgets are to be prepared by budget centres based on the set objectives to be achieved. Budget centres are the divisions, departments, sections or units within an organisation. The centres are to use the budget manual or guidelines given to them by the budget committee to prepare their budgets.

8.6.3 **Budget Co-ordination**

The estimates made by various budget centres on cost of various projects and programmes and the revenue to be generated are to be compiled by a responsible official (a budget officer) and be presented before a budget committee for consideration. The budget committee is to be made up of budget holders (departmental/divisional heads) and chaired by a high-ranking officer (possibly an Executive Director or General Manager). The committee is to resolve differences and submit a final comprehensive budget for approval. Budget preparation should be a “bottom-up” process.

8.6.4 **Budgets Approval**

After the committee’s deliberations, adjustments and modifications on the compiled budgets, approval is to be sought
from the highest ruling body/person, such as board of directors of the organisation. The body/person may need to make some adjustments on the submission of the committee, before giving the final approval. For companies, budgets are summarized into a master budget which consists of a cashflow statement, a budgeted income statement and a budgeted balance sheet. After the master budget has been approved, the budgets are then passed to the appropriate responsibility centres for implementation.

8.6.5 Measurement of Actual Performance

At the beginning of the budget period, budget centres would be directed to implement the approved budgets. The implementation is to be subjected to effective monitoring and supervision to make sure that things are going according to plan. At the end of the budget period, actual result/performance is to be measured for comparison against the initial plan. Since budgeted results are based on a number of assumptions, which might be nullified by events, actual results would hardly agree with the budgeted results and, therefore, variances are bound to arise. These variances should be investigated, as to their reasons or causes, before reaching a conclusion on whether they are favourable or adverse. The comparison of actual results with budgeted results is called Budget Review and its report should be sent to the appropriate budgetees so that it could have the maximum motivational impact.

8.6.6 Feedback Actions

After taking a decision on budget variance, appropriate actions (in the form of reward or punishment of different dimensions) are to be taken on the budget centre or budget holder concerned. The actions would serve as feedback for future planning and control.

The budgetary control process discussed above shows that budgets are used to ensure effectiveness in planning of activities, controlling of activities and motivation of budget centres or holders. Motivation comes in because, as goals are achieved, rewards would be given, while sanctions would follow inability to achieve set goals.
8.7 BUDGET COMMITTEE AND BUDGET OFFICER

The co-ordination role of a budget committee, to be set up by top management, has been highlighted in the budgetary control process described above. The members of a budget committee, however, would vary depending on the size of the organisation and the nature of its business. The committee should, in principle, consists of representatives of the following three main groups within an organisational set up:

(a) Executive Management: In order to provide decision making authority and to re-inforce corporate objectives.

(b) The Accounting Section / Department: In order to integrate the budgets, highlight interdependencies and constraints, and provide specialist financial assistance to the committee.

(c) Senior Budget Holders: In order to establish meaningful and acceptable targets on a participatory basis. Other individuals and interest groups, to be treated as budget holders, should be involved where appropriate.

8.7.1 Duties of Budget Committee

The major duties of budget committee are to:

(a) act as an advisory committee to the chief executive;

(b) direct executives on budgetary matters; largely on how to approve the budgets or amend them;

(c) receive and examine the subsidiary budgets and approve them or call for amendments;

(d) interview budget holders concerning their budgets; and

(e) consider request for budget preparation during the course of the year.

8.7.2 Budget Officer

In addition to the budget committee, a budget officer (who should be an accountant) is also to be appointed to discharge the following duties in order to ensure that the co-ordination and review of the budget committee is very effective:
(a) undertake work in connection with the budgeting cycle and to generally maintain the discipline of the system;
(b) collate the budgets and produce draft master budget;
(c) detect errors and inconsistencies in budget submission and to liaise with budgetees;
(d) convene meetings of the budget committee; and
(e) prepare and issue a Budget Manual which will describe the objectives and procedures involved in the budgeting process.

The manual, which serves as a very useful reference source for budgetees, may include a time table which specifies the order in which the budgets should be prepared and the date when they should be presented to the budget committee.

8.8 TYPES OF BUDGET

Budgets may take different forms, depending on the situation. For proper discussion of types of budget, some perspectives have to be borne in mind. As time period could be used to differentiate budgets, we can take *Time perspective* to classify budgets into two: a *short-term budget* and a *long term budget*, and probably a medium-term budget. The activities involved could dictate the classification as we can differentiate budgets into financial budgets and non-financial (operational) budgets. This categorization could be seen as *Activity perspective*. Another way of differentiating types of budget, which is more popular in the public sector, could be termed *Quantitative perspective* as budgets are differentiated on the basis of the monetary value involved. Thus, budgets could be classified into surplus budget, deficit budget and balanced budget.

8.8.1 Time Perspective of Types of Budget

*Short-term budget:* relates to current conditions and it usually covers a period of one year. Even annual budgets are in turn broken down into quarterly, monthly, fortnightly or weekly budgets for control purposes, as management may wish to take corrective actions before a situation gets out of hand. The period selected, for short-term budgets preparation, will depend upon such factors as:

(a) the stability of demand for the firm’s products;
(b) the life cycle of the firm’s product;
(c) the characteristics of the industry within which the firm operates; and
(d) the general economic climate.

A widely used variant of the short-term annual budget is the *rolling* or continuous budget. The budget is formulated initially for a period of one year or more and is broken down into smaller periods. As each month, quarter or week passes, two actions take place. First, a budget for the corresponding period of the following year is prepared, ensuring that a short-term budget is always in existence for the immediate future of twelve (12) months; and second, the budget is to be revised in the light of the results of the period which has elapsed, thus ensuring that the current budget is revised constantly and kept up-to-date.

**Long-term budget** relates to the development of the organisation or its business over many years. It is usually drawn up in any general terms which cover the nature of the business, its position in the industry, the expected level of inflation and its impact on the business. A period of between three to ten years may be appropriate for a long term budget. Matters such as capital assets purchase and long term finance between debt and equity are considered in long term budgeting decisions.

**Medium-term budget** could be formulated to relate to any financial or non-financial budgets that may cover the period of between one to five years, with a period of thirteen (13) months as the floor and a period of fifty-nine months as the ceiling. The categorization is not popular, as long-term budget is readily serving its purpose.

### 8.8.2 Activity Perspective of Types of Budget

**Operating Budgets:** These are budgets that reflect day-to-day activities or operations of an organisation. This category deals with items of manufacturing, trading and profit and loss accounts like material purchases, labour cost, production and overhead, sales, purchases, ending inventory, opening inventory, etc, budgets. It also deals with revenue or incomes budgets and expenses or expenditure budgets. Operating budget is synonymous with recurrent expenditure budget of the government financial year.

**Financial Budget:** This relates to financing of assets and generally indicate cash inflow and outflow. Capital budgeting is
part of financial budget. This category is the budget to be prepared on the funds to be generating through different sources for the financing of various projects. The budget would indicate ownership of assets and insurance of liabilities and, so it gives the information which would enable a budgeted balance sheet to be prepared. Capital budgeting, which is part of financial budget, is synonymous with capital expenditure budget of the government financial year.

**Master Budget:** This is the summary of all the operating and financial budgets and it consists of a budgeted profit and loss account, a budgeted balance sheet and a cashflow statement. The master budget, is, therefore, an overall budget.

### 8.8.3 Quantitative Perspective of Types of Budget

**Surplus Budget:** is arrived at when the total estimated revenue is greater than the total estimated expenditure. While some ministries of government are revenue centres, generating more revenue than the cost they incur, most government ministries are cost centres - incurring more cost than the revenue they generate. The total of the estimated revenues of the relevant ministries and the total of the estimated expenditures of all the ministries are aggregated and compared when the budgets are approved.

**Deficit Budget:** is the reverse of surplus budget, that is, it is arrived at where the total anticipated expenditure is greater than the total anticipated revenue.

**A Balanced Budget:** is arrived at where the total estimated expenditure is equal to the total estimated revenue or when the difference between the two is insignificant.

### 8.9 BUDGETARY CONTROL AND RESPONSIBILITY ACCOUNTING

Budgetary control can only be achieved when budget holders (managers) are held accountable for any variance (favourable or adverse) which may occur. The budget, as agreed, acts as a policy plan for the coming financial period, and whilst the accountant may be responsible for preparing the master budget, issuing operating statements for budget implementation to budgetees and comparing actual results with the budgeted results, the responsibility for keeping within the framework of the budget rests with the individual budget holders (managers). This is known as responsibility accounting.
Budgetary control, with budgetary planning and discipline, is part of the overall system of responsibility accounting within an organisation.

Responsibility accounting is a system of accounting in which costs and revenues are analysed in accordance with areas of personal responsibilities so that the performance of the budget holders can be monitored in financial or non-financial terms. If budget holders are to be held accountable, then there is always the need for a proper organisational structure from which every official will know his position in the organisation and his responsibilities and to whom he is accountable.

A budgetary control system may be well designed, stimulating carefully prepared budgets for comparison with actual results. It is the use made of the budget information and the result of the comparative analysis that counts. This is where the issue of "responsibility" comes in. Without the effort and determination to control costs and to generate revenue to a budgeted level, the entire budgeting exercise will be futile.

8.10 BUDGETARY IMPROVEMENT TECHNIQUES

Budgets preparation, in an organisation, is to start from a certain time and is to be a continuous exercise. As the organisation continues to prepare budgets, there is the need for improvement on the previous budgets. In order to bring about improvement in the budget exercise, there are techniques that should be put to use to achieve the desired organisational objectives.

Although, the techniques for budgetary improvement are more popular and more applicable in the government sector than in any other sector of the economy, the government and the other two components of the economy are consciously or unconsciously using one of the techniques or the other in their continuous budget preparation.

The most popular of these budgetary improvement techniques are:

(a) Incremental Budgeting (IB);
(b) Zero Based Budgeting (ZBB);
(c) Continuous (Rolling) Budgeting (CB); and
(d) Planning Programming Budgeting System (PPBS).
8.10.1 Incremental Budgeting (IB)

This technique does not believe in any comprehensive budgeting that requires budget holders to look beyond one year in their budget preparation. They are not required to critically justify all their proposed expenditures as they are based on adding a percentage to the previous year’s budgets to arrive at current year’s budget.

Proponents of incremental budgeting would argue that budgeting is incremental, not comprehensive and that budget is almost never actively reviewed as a whole every year, that budgeting is to be based on last year’s budget with special attention given to a narrow range of increases and decreases, and that budget preparer’s attention is to be focused on small number of items over which the budgetary battle is fought.

The basic assumption of this technique is that the previous period’s level of expenditure or activity is appropriate and, so, there is no need for comprehensive review of activities, no need for much defense of proposal, no need for looking beyond the specific budget period, no need for coming up with various alternatives to a proposed project or activity and no need for taking long term decision(s). All that is needed in budgeting, is a decision on the activities to be dropped, added, expanded or reduced, based on the available resources.

This technique is a clear reflection of the budgetary behaviour of the vast majority of governments, firms and individuals in most developing countries.

Advantages of Incremental Budgeting

Incremental budgeting (IB) has the following advantages:

(a) **Moderation of conflicts:** Conflicts between managers of the interdependent departments or units of an organisation could easily be moderated or avoided since decision as to what to allocate to what unit, department or project would be based on the last period’s budget.

(b) **Reduction of cost:** Search costs as to the overall review of projects, feasibility studies, comparison of alternative courses of action would be avoided or reduced to the barest minimum, using the IB technique.
(c) **Saving of Time:** The technique brings about reduction of the amount of time that budget holders must invest in budgeting, if it is to be comprehensive.

**Disadvantages of Incremental Budgeting**

The technique has a number of disadvantages which could be summarised as follows:

(a) **Future Cost Implication ignored:** The focus of the technique is one year or shorter period only, without looking at the future cost implications of the decision to be taken in the short period. The philosophy is that of attaining a level of activity in the short period, even if that would bring about setback in the future.

(b) **Transferring Inefficiency to the Future:** By beginning the budgeting process for each period with the funding level of the previous period, the technique assumes that the activity performed in the previous period were efficient and cost effective. As a result, any inefficiencies of the past are enshrined as standards for the future.

(c) **Provision of Poor Information:** The technique does not emphasise on the provision, by budget holders, of adequate information on the services (projects) to be provided, the reason(s) for providing the services, the beneficiaries of the services or the resources needed to maintain specific levels of services.

(d) **No Evaluation of Alternatives:** The technique does not provide any meaningful way of evaluating alternative courses of action to executing projects or programmes, alternative methods of providing a service or trade off between different services or levels of services. In short, the technique does not encourage the development of alternative courses of action to various activities or projects.

(e) **Lack of Budget Expertise:** Budget committees, in an organisation using IB technique, do not need to be made up of experts - the composition is usually that of heads of budget centres and very few accounting staff. It is for this reason that departments, units or ministries inflate their
blindly prepared budget in anticipation of arbitrary cuts by the committee.

(f) **Flexible Budgeting Ignored:** Incremental Budgeting, with its emphasis on line-items budgeting, does not permit the use of technique of flexible budgeting, where one can relate costs with level of activity performed or achieved.

It is in response to the above criticisms of incremental budgeting that many other budgetary improvement techniques emerged in the budgeting literature.

### 8.10.2 Zero Base Budgeting (ZBB)

This budgetary improvement technique assumes that current operations will start from zero level. It, therefore, calls for total review of all proposed activities and expenditures of an organisation, irrespective of whether they are new or continuing ones. The idea is to justify every Naira of an organisation’s expenditure. This rational model of decision making assumes that the decision-makers in government and industry know where they are going and that what is needed is a decision on how to get there through the choice of the most efficient route. That route can only be found after a comprehensive and critical assessment of all the proposals that would embody an organisation’s budgets.

ZBB involves allocation of the resources of an organisation on the basis of the cost-benefit-analysis to be conducted on each of its activities. In essence, positive and practical attempts are to be made to eliminate inefficiencies and wastes from all operations. The technique’s basic philosophy is that budget holders are to look at their activities or priorities afresh and that they have to justify their entire budget requests anew.

**Major Steps of ZBB**

The major steps of ZBB are three:

(a) Breaking down each activity or project into decision packages. A decision package is a document containing information about:

(i) Cost-benefit-analysis on a proposal;
(ii) A comparison of the result of the analysis with those of other alternatives; and

(iii) The consequences of not approving the proposal.

(b) Comprehensive evaluation of various alternative courses of action to a proposed activity or project and ranking of the proposals in order of decreasing benefit or importance to the organisation.

(c) Allocation of resources to the competing projects or activities in accordance with the final ranking established.

**Advantages of ZBB**

The following advantages could be derived from the application of ZBB technique:

(a) **Availability of Alternatives:** Budget holders must discover and present various alternative courses of action to achieving a given objective. This means that bases of comparison must be provided by anybody proposing projects or activities to be undertaken. The alternatives would be extensively evaluated before decision is taken.

(b) **Future cost Implications of Decisions are considered:** The technique considers all the future cost implications of current decisions as part of the cost-benefit-analysis to be conducted on each proposed project or activity.

(c) **Elimination of Low Priority Programmes:** These programmes or activities can be easily cut-off or eliminated with more confidence, since decision as to where to allocate scarce resource would be based on the ranking made after a comprehensive cost-benefit-analysis.

(d) **Creativity and Initiative Enhanced:** ZBB model makes budget preparers to be creative and imaginative, since they have to think afresh when drafting their budgets and they have to forward alternative ways of achieving a desired objective. The presentation of alternatives would be done with greater clarity so as to give way for objective analysis and review before decision is taken.
Disadvantages of ZBB

All the advantages of IB technique could be the disadvantages of ZBB. Specifically, the following are the disadvantages of ZBB:

(a) **Conflict of Interest:** The technique is bound to bring about conflict of interest among various budget holders, since it encourages defense of budget proposals so as to justify every naira of proposed expenditure. The competition for favourable allocation of the scarce resources may result into a conflict that may prove difficult to moderate.

(b) **High Cost:** The cost of preparing the budget, which must be initiated afresh, and from the bottom upwards, would be high.

(c) **Time Wasting:** A lot of time must be spent preparing budgets using ZBB technique, since everything would start as original and be subjected to justification.

(d) **Difficulty of Understanding:** Inadequate understanding of the aims, strengths and weaknesses of ZBB among most budget holders, would affect their ability to gather relevant data needed for its implementation.

(e) **High Level of Paper work:** ZBB model entails high degree of analysis and security which would be energy and other resources consuming.

All the shortcomings (disadvantages of ZBB) can be overcome through proper training of budget holders and foresighted administration of the entire ZBB technique. The technique requires the commitment of a lot of resources - money, time, and energy to succeed; but, it is more comprehensive, accurate and objective than the incremental model.

8.10.3 **Continuous (Rolling) Budgeting (CB)**

Rolling budgeting can be defined as the continuous updating of short-term budgets so that previously prepared budgets for future periods could reflect current conditions. This is, therefore, an attempt to prepare short-term budgets which are very realistic. A rolling budget is formulated initially for a period, say one year or one decade. This would be broken down into monthly, quarterly or yearly budgets for execution. As each month, quarter or year passes, two actions take place:
(a) A budget for the corresponding period of the following period is prepared, ensuring that a budget is always in existence for the immediate future of 12 months or ten years.

(b) The budget of the following month or year would be revised in the light of the results of the period which has elapsed, so ensuring that the current budget is revised constantly and kept up to date. This budgetary improvement technique has the same philosophy as ZBB model. The only difference between the two is that, while ZBB is about starting afresh always, CB is about rolling the assessed result of the previous period to adjust the budget prepared for the present period. The budgets to be initiated under the rolling budgeting thinking are to be based on the principles of ZBB. Therefore, all the advantages and disadvantages of ZBB technique are applicable to CB technique.

8.10.4 Planning, Programming, Budgeting System (PPBS)

This technique is basically related to government budgeting. It started in the USA in 1968 and its philosophy introduced in Nigeria in 1980. The aims and objectives of PPBS are in line with those of ZBB. They, however, differ in two respects which would be explained later. First, let us discuss the objectives and problems of PPBS.

Aims and Objectives of PPBS

They are as follows:

(a) Budgeting on a multi-year basis, for example, 5-year National Development Plan or Three-year Rolling Plan.

(b) Identification of real and actual objectives of expenditures (projects) before money is spent on them. It calls, therefore, for adequate defence of proposals.

(c) Segregation of projects into core and non-core before setting targets of performance on them.

(d) All other aims and objectives of ZBB technique.
Problems of PPBS

More than two decades after the PPBS exercise has been introduced in Nigeria, not much has been achieved in terms of budgetary improvement. This be unconnected with the following problems:

(a) **Inability to identify objectives**: The objectives of projects which the technique is aimed at identifying could not be specified by the relevant authorities, let alone devise the expective means of achieving them.

(b) **Instability of Leadership**: The multi-year objective of the technique cannot be realized due to the persistent unscheduled changes of leadership, which bring about frequent changes in the budget direction of the different governments. Each leadership will come with its policies which would necessitate preparation of new budgets. The years 1984, 1985, 1993, 1998 and 1999 had two different budgets each at the Nigerian Federal Government level and they were prepared just to suit the need of the incumbent, thereby faulting the long term plan initiated.

(c) **Lack of Sense of Direction**: The aim of segregating projects into core and non-core and the setting of target result could not be accomplished because of the perceived lack of sense of direction in the civil service. This is another result of constant change of government. For PPBS to be fully implemented, the above problems have to be tackled by the interested budgetee.

Where PPBS and ZBB Differ

The two techniques differ in two respects. These are:

(a) The focus of ZBB is on the current budget alone, while PPBS has a longer focus. PPBS is concerned with budgets of the future years, especially as they affect the current decision.

(b) ZBB requires that all alternative projects have to be adequately evaluated and ranked in order of priority and that acceptance or rejectance of projects should be based on the ranking and the availability of funds. PPBS does
not rank projects or activities as it has no formal methodology for such exercise.

8.11 PREPARATION OF FUNCTIONAL AND MASTER BUDGETS

For obvious reasons, we shall concentrate on budgets preparation of firms - one of the three components of the economy. Their functional and master budgets preparation would be illustrated and this should serve as guide to budgets preparation at the governmental and individual levels - the other two components of the economy.

Functional Budgets

These are the subsidiary budgets that would combine to make up the master budgets. Sales budgets, purchase budget, production budget, operating expenses budget, etc, are to be prepared before preparing the master budgets.

(i) Sales Budget Preparation

Sales budget is the foundation of all budgets. Production or Purchase budgets are to be prepared on the basis of the sales budgeted. Where a company is selling only one product, the budgeted sales revenue would be given by:

Units to be sold X selling price (SP) per unit = \( \text{N} \)x

But where two or more products are in the sales mix of a business, its sales budget should be as follows:

<table>
<thead>
<tr>
<th>Product</th>
<th>Qty. to be sold</th>
<th>SP/unit</th>
<th>Sales value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>B</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>C</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

(b) Purchase and Production Budget Preparation

In preparing purchase or production budget, the knowledge of trading account preparation is to be employed. Trading account arrangement is to be taken upside down so that purchase value (for a distributive business) or production cost (for a manufacturing business) could be determined as follows:
BUDGETING AND BUDGETARY CONTROL

Debit side of Trading Account       Purchase or Production Budget
Opening stock xx                    Unit or stock sold xx
Add: Purchase xx                    Add: closing stock xx
COG avail. for sales xx            Stock available for sales xx
Less Closing stock xx              Less: Opening stock xx
Cost of Units sold xx              Purchase or Production xx

Note:
The purchase budget could be on finished goods purchases or raw materials purchases.

(c) Operating Expenses Budget

These budgets are for all business running expenses expected to be incurred in the budget period. The budget may be prepared on monthly, quarterly or annual basis. All cash and non-cash expenses as well as accrued expenses are to be reflected. If disbursements budget is to be prepared, it is only the cash expenses and payments expected to be made within the budget period that are to be reflected.

(d) Total Cost per Unit Budget

Here, the estimated direct material cost, direct labour cost and overhead cost per unit must be determined. It is the total of the per unit cost of these cost elements that would give the budgeted total cost per unit of a product.

(e) Beginning and Ending Inventory Budgets

These budgets are to be prepared on all the types of materials that the business is dealing with - raw materials, finished goods, work-in-progress, etc. The estimated opening or closing units of each type of materials are to be multiplied by budgeted total cost per unit of each material type. On raw materials, for example, Budgeted Value of Closing Stock would be arrived at as follows:

<table>
<thead>
<tr>
<th>Type</th>
<th>Quantity</th>
<th>Cost/Unit</th>
<th>Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Y</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Z</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

Total estimated value of closing stock of raw materials xx
(f) **Direct Labour Budget**

A manufacturing business may have to prepare direct labour cost budget in the process of determining its budgeted total cost of production. This is more so when there are different types of direct labour, paid at different rates. The budget is prepared as follows:

<table>
<thead>
<tr>
<th>Types</th>
<th>DL hours needed</th>
<th>Rate/hour</th>
<th>DL Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>B</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

OR

<table>
<thead>
<tr>
<th>Product</th>
<th>Units to be produced</th>
<th>DLHs/unit</th>
<th>Total DL Hrs</th>
<th>Rate/Hr DL Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>B</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>C</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>D</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

8.11.1 **Master Budgets**

These are the budgeted cashflow statement (cash budget), budgeted income statement (or profit and loss account) and budgeted balance sheet. They are made up of one combination of functional budgets or the other.

(a) **Cash Budget**

This budget takes account of all projected receipts and disbursements during a future period and shows the estimated closing cash balance at the end of a budget period. It can be prepared on weekly, monthly, quarterly or annual basis. It is very helpful in cash management and in making estimate as to how much to be borrowed from outside sources in order to meet various demands. The budget can be prepared as follows:
Specimen Cash Budget

<table>
<thead>
<tr>
<th>A. Receipts</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Income</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Other incomes</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Loan</td>
<td>xx</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total Estimated Cash Receipt</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Disbursements</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw materials purchases</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Operating expenses</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Machinery purchases</td>
<td>xx</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Loan Repayment &amp; Interest</td>
<td>-</td>
<td>-</td>
<td>xx</td>
<td>-</td>
</tr>
<tr>
<td>Total Estimated Cash payments</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Net Cash Flow (A - B)</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add Opening balance</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Closing Balance</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

The Net cashflow can be positive, negative or zero.

Sometimes, a minimum cash balance might be targeted, and the cash budget might be for the determination of estimated loan requirements, and, if so, it should be prepared as follows:

<table>
<thead>
<tr>
<th>A. Receipts:</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening balance</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Sales Revenue</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Other incomes</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Total Receipt</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Disbursements:</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw materials purchases</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Operating expenses</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Machinery purchases</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Other expenses</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Total</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Surplus / Deficit:</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loan Requirements</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
<tr>
<td>Closing cash balance</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
<td>xx</td>
</tr>
</tbody>
</table>
The closing cash balance must not be less than the minimum required and it is to be the opening balance for the next budget period. The monthly loan requirements are to be aggregated to arrive at the total amount to be sought as loan from a bank or any other financial institution.

(b) **Budgeted Income Statement**

This budget is to follow the format for trading and profit and loss accounts preparation for either internal use or external use. The budgeted trading account is to disclose the budgeted gross profit, while the budgeted profit and loss account is to disclose the budgeted net profit before tax. Budgeted profit and loss appropriation account for a partnership or a company can also be prepared, if given the relevant data for that purpose.

(c) **Budgeted Balance Sheet**

This, also, is to be prepared in the format of a balance sheet, using the available data. Budgeted balance sheet is to be prepared using the format approved for internal use or the format approved for external use, depending on the availability of relevant data.

**ILLUSTRATION 8-1**

Zango Stores plans the following inventory levels (at cost) on one of the items sold in the stores. The inventory at the end of May should be N1,700,000; June N1,500,000; July N1,900,000 and August N1,600,000. Sales on the item are expected to be: June N3,500,000; July N2,500,000 and August N3,300,000. Cost of goods sold is 60% of sales value. The business estimated purchases in April is expected to be N1,900,000 and in May N1,600,000. A given month’s purchases are to be paid as follows: 10% during the month; 80% the next month and the final 10% two months after.

**Required:**

Prepare purchases budget for the months of June, July and August. Prepare also disbursements budget on Purchases for the three-month period.
SUGGESTED SOLUTION 8-1

Purchases Budget

<table>
<thead>
<tr>
<th></th>
<th>June</th>
<th>July</th>
<th>Aug</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of goods (units) sold</td>
<td>2,100,000</td>
<td>1,500,000</td>
<td>1,980,000</td>
</tr>
<tr>
<td>Add: cost of closing units</td>
<td>1,500,000</td>
<td>1,900,000</td>
<td>1,600,000</td>
</tr>
<tr>
<td>Cost of units available for sales</td>
<td>3,600,000</td>
<td>3,400,000</td>
<td>3,580,000</td>
</tr>
<tr>
<td>Less: Cost of opening units</td>
<td>1,700,000</td>
<td>1,500,000</td>
<td>1,900,000</td>
</tr>
<tr>
<td>Purchases</td>
<td>1,900,000</td>
<td>1,900,000</td>
<td>1,680,000</td>
</tr>
</tbody>
</table>

Disbursement on Purchases Budget

<table>
<thead>
<tr>
<th></th>
<th>June</th>
<th>July</th>
<th>Aug</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% During the Month</td>
<td>190,000</td>
<td>190,000</td>
<td>168,000</td>
</tr>
<tr>
<td>80% (May - July purchases)</td>
<td>1,280,000</td>
<td>1,520,000</td>
<td>1,520,000</td>
</tr>
<tr>
<td>10% (April - June purchases)</td>
<td>90,000</td>
<td>160,000</td>
<td>190,000</td>
</tr>
<tr>
<td></td>
<td>1,660,000</td>
<td>1,870,000</td>
<td>1,878,000</td>
</tr>
</tbody>
</table>

ILLUSTRATION 8-2

Alaka Showboy Stores has the following budgeted sales, which are uniform throughout the period: May ₦3,000,000; June ₦2,500,000; July ₦2,200,000 and August ₦2,800,000. The Store’s employees earn fixed salaries of ₦120,000 monthly and commissions of 10% of the current month’s sales. Disbursements are made semi-monthly - that is half to be paid a month after salaries and commissions are earned. Other expenses are rent, ₦30,000, paid on the first of each month for that month’s occupancy; miscellaneous expenses, 6% of sales, paid as incurred; insurance, ₦3,000 per month, related to a one-year policy that was paid for on 2nd January; and depreciation, ₦19,000 per month.

Required:

(i) Prepare an operating expenses budget for the months of June, July and August.

(ii) Prepare a disbursement budget on the operating expenses for the period.
SUGGESTED SOLUTION 8-2

(i) Operating Expenses Budget

<table>
<thead>
<tr>
<th></th>
<th>June</th>
<th>July</th>
<th>Aug</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>₦</td>
<td>₦</td>
<td>₦</td>
</tr>
<tr>
<td>Salaries and commission</td>
<td>370,000</td>
<td>340,000</td>
<td>400,000</td>
</tr>
<tr>
<td>Rent</td>
<td>30,000</td>
<td>30,000</td>
<td>30,000</td>
</tr>
<tr>
<td>Miscellaneous expenses</td>
<td>150,000</td>
<td>132,000</td>
<td>168,000</td>
</tr>
<tr>
<td>Insurance</td>
<td>3,000</td>
<td>3,000</td>
<td>3,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>19,000</td>
<td>19,000</td>
<td>19,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>572,000</td>
<td>524,000</td>
<td>620,000</td>
</tr>
</tbody>
</table>

Note: Salaries and commission for May

\[= 120,000 + 300,000\]
\[= ₦420,000\]

(ii) Disbursement Budget

<table>
<thead>
<tr>
<th></th>
<th>June</th>
<th>July</th>
<th>Aug</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>₦</td>
<td>₦</td>
<td>₦</td>
</tr>
<tr>
<td>Salaries and Commission</td>
<td>395,000</td>
<td>355,000</td>
<td>370,000</td>
</tr>
<tr>
<td>Rent</td>
<td>30,000</td>
<td>30,000</td>
<td>30,000</td>
</tr>
<tr>
<td>Miscellaneous expenses</td>
<td>150,000</td>
<td>132,000</td>
<td>168,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>575,000</td>
<td>517,000</td>
<td>568,000</td>
</tr>
</tbody>
</table>

Note: Salaries and commission paid in June, for example, was arrived at as follows:
Half of May's = 210,000
Half of June's = 185,000

395,000

ILLUSTRATION 8-3

The month by month forecasts of profitability of Raphael Company Limited for the five months, July to November, 2008, are given below:

<table>
<thead>
<tr>
<th></th>
<th>July</th>
<th>Aug</th>
<th>Sept</th>
<th>Oct</th>
<th>Nov</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>₦</td>
<td>₦</td>
<td>₦</td>
<td>₦</td>
<td>₦</td>
</tr>
<tr>
<td>Material consumed</td>
<td>600</td>
<td>700</td>
<td>800</td>
<td>1020</td>
<td>900</td>
</tr>
<tr>
<td>Wages</td>
<td>320</td>
<td>320</td>
<td>320</td>
<td>400</td>
<td>320</td>
</tr>
<tr>
<td>Depreciation</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>Factory expenses</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Rent</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Salaries and office expenses</td>
<td>320</td>
<td>320</td>
<td>320</td>
<td>320</td>
<td>320</td>
</tr>
</tbody>
</table>
Advertisement and publicity 120 140 100 160 200
Sales commission 80 90 100 130 110
1,590 1,720 1,790 2,180 2,000
Credit sales 1,600 1,800 2,000 2,600 2,200
Profit 10 80 210 420 200
Closing stock of raw materials 700 800 900 700 600

The following additional information are given:
1. On average, payment is made to suppliers one month after delivery.
2. The lag in payment of wages is one-eighth of a month.
3. Factory expenses are paid during the month incurred.
4. Rent is paid during the month incurred.
5. Salaries and office expenses are paid in the month in which they arise.
6. Advertisement and publicity expenditure is paid one month in advance. December’s expenditure is N220,000.
7. Sales Commission is paid one month in arrears.
8. On average, debtors take two month’s credit.
9. Cash balance at September is N520,000
10. In November, N300,000 will be paid for machinery. A dividend and tax thereon amounting to N60,000 will be paid in October. Investment grants of N200,000 will be received in November.

Required:
(a) Prepare raw material purchases budget for each of the four months to November 30, 2008.
(b) Prepare a cash budget for each of the three months to November 30. *(Adapted from ICAN)*

**SUGGESTED SOLUTION 8-3**

(a) **Raw Materials Purchases Budget**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N’000</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Cost of Material consumed (used)</td>
<td>700</td>
<td>800</td>
<td>1,020</td>
<td>900</td>
</tr>
<tr>
<td>Add: cost of closing stock of raw materials</td>
<td>800</td>
<td>900</td>
<td>700</td>
<td>600</td>
</tr>
<tr>
<td>Total value of raw materials to be available in store</td>
<td>1,500</td>
<td>1,700</td>
<td>1,720</td>
<td>1,500</td>
</tr>
<tr>
<td>Less: Cost of opening stock of raw materials</td>
<td>700</td>
<td>800</td>
<td>900</td>
<td>700</td>
</tr>
<tr>
<td>Cost of raw materials to be purchased</td>
<td>800</td>
<td>900</td>
<td>820</td>
<td>800</td>
</tr>
</tbody>
</table>
### (b) Cash Budget

<table>
<thead>
<tr>
<th></th>
<th>Sept $'000</th>
<th>Oct $'000</th>
<th>Nov $'000</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(i) Receipts</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales incomes</td>
<td>1,600</td>
<td>1,800</td>
<td>2,000</td>
</tr>
<tr>
<td>Investment grants</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td><strong>Total estimated cash receipts</strong></td>
<td><strong>1,600</strong></td>
<td><strong>1,800</strong></td>
<td><strong>2,200</strong></td>
</tr>
<tr>
<td><strong>(ii) Disbursements</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raw materials purchases</td>
<td>800</td>
<td>900</td>
<td>820</td>
</tr>
<tr>
<td>Wages (1/8 + 1/8 of previous month)</td>
<td>320</td>
<td>390</td>
<td>330</td>
</tr>
<tr>
<td>Factory expenses</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Rent</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Salaries and office expenses</td>
<td>320</td>
<td>320</td>
<td>320</td>
</tr>
<tr>
<td>Advert and Publicity</td>
<td>160</td>
<td>200</td>
<td>220</td>
</tr>
<tr>
<td>Sales Commission</td>
<td>90</td>
<td>100</td>
<td>130</td>
</tr>
<tr>
<td>Machinery Purchase</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dividend and tax thereon</td>
<td>-</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total estimated cash disbursements</strong></td>
<td><strong>1,770</strong></td>
<td><strong>2,050</strong></td>
<td><strong>2,200</strong></td>
</tr>
<tr>
<td><strong>(iii) Net Cashflow (i) - (ii)</strong></td>
<td><strong>(170)</strong></td>
<td><strong>(250)</strong></td>
<td><strong>0</strong></td>
</tr>
<tr>
<td>Add: Opening cash balance</td>
<td>520</td>
<td>350</td>
<td>100</td>
</tr>
<tr>
<td>Closing cash balance</td>
<td>350</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

### ILLUSTRATION 8-4

This question illustrates the preparation of all budgets, from functional budgets to master budgets. Hindu K. Textile Company Limited produces and sells textile materials. The company has the budget discipline of budgeting for everything right from inputs needed for production up to the balance sheet.

You are given the following information for budget purposes:

**Materials**

- **No.111** = $12/unit
- **No.112** = $26/unit
- **Direct labour** = $20.50/direct labour hour
- Overhead is applied on the basis of direct labour hours.
Input/Output Relationship:

Cost Elements

<table>
<thead>
<tr>
<th>Contents per unit</th>
<th>Product F</th>
<th>Product G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mat. 111</td>
<td>12 units</td>
<td>12 units</td>
</tr>
<tr>
<td>Mat. 112</td>
<td>6 units</td>
<td>8 units</td>
</tr>
<tr>
<td>Direct labour</td>
<td>14 hours</td>
<td>20 hours</td>
</tr>
</tbody>
</table>

About the finished products:

<table>
<thead>
<tr>
<th>Expected sales in unit</th>
<th>Product F</th>
<th>Product G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5,000 units</td>
<td>1,000 units</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Selling price per unit</th>
<th>Product F</th>
<th>Product G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N1,054</td>
<td>N1,640</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Desired ending inventory</th>
<th>Product F</th>
<th>Product G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1,100 units</td>
<td>50 units</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Beginning inventory</th>
<th>Product F</th>
<th>Product G</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 units</td>
<td>50 units</td>
</tr>
</tbody>
</table>

Direct Materials:

<table>
<thead>
<tr>
<th>Mat. 111</th>
<th>Mat. 112</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning inventory</td>
<td>5,000 units</td>
</tr>
<tr>
<td>Desired ending inventory</td>
<td>6,000 units</td>
</tr>
</tbody>
</table>

The balance sheet for the year just ended is given below:

<table>
<thead>
<tr>
<th>Current Assets</th>
<th>Current Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash</td>
<td>100,000</td>
</tr>
<tr>
<td>A/c Receivables</td>
<td>250,000</td>
</tr>
<tr>
<td>Materials</td>
<td>190,000</td>
</tr>
<tr>
<td>Finished goods</td>
<td>144,800</td>
</tr>
<tr>
<td></td>
<td>684,800</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fixed Assets</th>
<th>Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land</td>
<td>500,000</td>
</tr>
<tr>
<td>Build. &amp; Equip.</td>
<td>3,800,000</td>
</tr>
<tr>
<td>Acc. Depr.</td>
<td>(750,000)</td>
</tr>
<tr>
<td></td>
<td>4,234,800</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At anticipated volume levels, the following cost would be incurred:

| Supplies   | 300,000 |
| Indirect labour | 700,000 |
| Payroll costs | 250,000 |
| Power (variable portion) | 200,000 |
| Maintenance (variable) | 80,000 |
Depreciation 250,000
Property taxes 40,000
Property insurance 5,000
Supervision 200,000
Power (fixed portion) 10,000
Maintenance (fixed portion) 45,000

\[ \text{N}2,080,000 \]

**Selling and Administrative Expenses:**

<table>
<thead>
<tr>
<th>Item</th>
<th>N'000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales Commission</td>
<td>200,000</td>
</tr>
<tr>
<td>Adverts</td>
<td>30,000</td>
</tr>
<tr>
<td>Sales salaries</td>
<td>100,000</td>
</tr>
<tr>
<td>Traveling expenses</td>
<td>50,000</td>
</tr>
<tr>
<td>Clerical wages</td>
<td>100,000</td>
</tr>
<tr>
<td>Supply</td>
<td>10,000</td>
</tr>
<tr>
<td>Aggregated salaries</td>
<td>10,000</td>
</tr>
<tr>
<td>Miscellaneous expenses</td>
<td>50,000</td>
</tr>
</tbody>
</table>

\[ \text{N}550,000 \]

The expected cashflow for the next year are:

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Qtr. 1</th>
<th>Qtr. 2</th>
<th>Qtr. 3</th>
<th>Qtr. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>N'000</td>
<td>N'000</td>
<td>N'000</td>
<td>N'000</td>
<td>N'000</td>
</tr>
<tr>
<td>Collection from customers</td>
<td>1,250</td>
<td>1,500</td>
<td>1,600</td>
<td>2,210</td>
</tr>
<tr>
<td>Disbursements:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For materials</td>
<td>200</td>
<td>350</td>
<td>350</td>
<td>542</td>
</tr>
<tr>
<td>For other costs</td>
<td>250</td>
<td>200</td>
<td>200</td>
<td>170</td>
</tr>
<tr>
<td>For payroll</td>
<td>900</td>
<td>950</td>
<td>950</td>
<td>1,092</td>
</tr>
<tr>
<td>For income tax</td>
<td>50</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>For Machine purchase</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
</tbody>
</table>

The company decides to maintain \text{N}150,000 as the minimum cash balance at the end of each quarter. Money can be borrowed or repaid in multiples of \text{N}5,000 at 10\% per annum. Management does not want to borrow any more cash than necessary and wants to repay as promptly as possible. Interest is computed and paid on the part of the principal that is paid. Assume that borrowing takes place at the beginning and repayment at the end of the quarter in question. Income tax payable next year is \text{N}200,000.

**You are required to prepare:**

(a) Sales budget
(b) Production budget
(c) Direct material purchases budget
(d) Direct labour cost budget
(e) Factory variable and fixed overhead cost budget
(f) Total cost per unit budget
(g) Cost of ending inventory budget
(h) Cost of beginning inventory budget
(i) Cost of Goods sold budget
(j) Selling and administrative expenses budget
(k) Cashflow statement (cash budget)
(l) Budgeted income statement
(m) Budgeted balance sheet.

SUGGESTED SOLUTION 8-4

(a) Sales Budget

<table>
<thead>
<tr>
<th>Product</th>
<th>Units to be sold</th>
<th>SP/unit</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>5,000</td>
<td>1,054</td>
<td>5,270,000</td>
</tr>
<tr>
<td>G</td>
<td>1,000</td>
<td>1,640</td>
<td>1,640,000</td>
</tr>
</tbody>
</table>

(b) Production Budget

<table>
<thead>
<tr>
<th></th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planned sales</td>
<td>5,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Add: Closing units</td>
<td>1,100</td>
<td>50</td>
</tr>
<tr>
<td>Total needed</td>
<td>6,100</td>
<td>1,050</td>
</tr>
<tr>
<td>Less: Opening stock</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Units to be produced</td>
<td>6,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

(c) Direct Material Purchases Budget (in units)

<table>
<thead>
<tr>
<th></th>
<th>111</th>
<th>112</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units needed for production</td>
<td>84,000</td>
<td>44,000</td>
</tr>
<tr>
<td>Add: Desired ending inventory</td>
<td>6,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Less: Opening inventory</td>
<td>5,000</td>
<td>5,000</td>
</tr>
<tr>
<td>Units to be purchased</td>
<td>85,000</td>
<td>40,000</td>
</tr>
</tbody>
</table>

Note:

(i) Units needed for production were arrived at as follows:

For 111:

\[
F = 12 \times 6,000 = 72,000 \\
G = 12 \times 1,000 = 12,000 \\
\text{Total} = 84,000
\]
For 112:
\[ F = 6 \times 6,000 = 36,000 \]
\[ G = 8 \times 1,000 = 8,000 \]
\[ 44,000 \]

(ii) Total cost of materials to be purchased:
\[ 111 = 85,000 \times \text{N12} = \text{N1,020,000} \]
\[ 112 = 40,000 \times \text{N26} = \text{N1,040,000} \]
\[ \text{N2,060,000} \]

(iii) Total cost of materials to be used in production:
\[ 111 = 84,000 \times \text{N12} = \text{N1,008,000} \]
\[ 112 = 44,000 \times \text{N26} = \text{N1,144,000} \]
\[ \text{N2,152,000} \]

(iv) Direct Labour Cost Budget

<table>
<thead>
<tr>
<th>Product</th>
<th>To be produced</th>
<th>DL Hrs/Unit</th>
<th>Total DL Hrs</th>
<th>Rate</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>6,000</td>
<td>14</td>
<td>84,000</td>
<td>20.50</td>
<td>1,722,000</td>
</tr>
<tr>
<td>G</td>
<td>1,000</td>
<td>20</td>
<td>20,000</td>
<td>20.50</td>
<td>410,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>104,000</td>
<td></td>
<td>2,132,000</td>
</tr>
</tbody>
</table>

Note: In the alternative, 104,000 x \text{N20.50} = \text{N2,132,000}

(e) Factory Overhead Cost Budget

(i) Variable Factory Overhead Cost

\[ \text{Supplies} \quad 300,000 \]
\[ \text{Indirect labour} \quad 700,000 \]
\[ \text{Payroll cost} \quad 250,000 \]
\[ \text{Maintenance (variable)} \quad 200,000 \]
\[ \text{Power (variable)} \quad 80,000 \]
\[ \text{Total Variable overhead cost} \quad 1,530,000 \]

(ii) Fixed Factory Overhead Cost

\[ \text{Depreciation} \quad 250,000 \]
\[ \text{Property taxes} \quad 40,000 \]
\[ \text{Property Insurance} \quad 5,000 \]
\[ \text{Supervision} \quad 200,000 \]
\[ \text{Power (fixed portion)} \quad 10,000 \]
\[ \text{Maintenance (fixed)} \quad 45,000 \]
\[ \text{Total fixed overhead cost} \quad 550,000 \]
As overhead is applied on the basis of direct labour hours, overhead per Direct Labour Hrs (DHL) = N2,080,000 / 104,000 = N20/DL Hr.

(f) **Total Cost per Unit Budget**

<table>
<thead>
<tr>
<th>Cost Elements</th>
<th>Rate/Unit (N)</th>
<th>Units Needed</th>
<th>Total Value (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material 111</td>
<td>12.00</td>
<td>12</td>
<td>144</td>
</tr>
<tr>
<td>Material 112</td>
<td>26.00</td>
<td>6</td>
<td>156</td>
</tr>
<tr>
<td>Direct labour</td>
<td>20.50</td>
<td>14</td>
<td>287</td>
</tr>
<tr>
<td>Overhead</td>
<td>20.00</td>
<td>14</td>
<td>280</td>
</tr>
</tbody>
</table>

\[
\text{Total Cost per Unit Budget} = 867 \quad \text{N}
\]

(g) **Cost of Ending Inventory Budget**

(i) **Raw materials Inventory:**

<table>
<thead>
<tr>
<th>Type</th>
<th>Units</th>
<th>Cost/unit (N)</th>
<th>Total Value (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>6,000</td>
<td>12</td>
<td>72,000</td>
</tr>
<tr>
<td>112</td>
<td>1,000</td>
<td>26</td>
<td>26,000</td>
</tr>
</tbody>
</table>

(ii) **Finished goods inventory:**

<table>
<thead>
<tr>
<th>Type</th>
<th>Units</th>
<th>Cost/unit (N)</th>
<th>Total Value (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>1100</td>
<td>867</td>
<td>953,700</td>
</tr>
<tr>
<td>G</td>
<td>50</td>
<td>1,162</td>
<td>58,100</td>
</tr>
</tbody>
</table>

(h) **Cost of Beginning Inventory Budget**

(i) **Raw materials Inventory:**

<table>
<thead>
<tr>
<th>Type</th>
<th>Units</th>
<th>Cost/unit (N)</th>
<th>Total Value (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
<td>5,000</td>
<td>12</td>
<td>60,000</td>
</tr>
<tr>
<td>112</td>
<td>5,000</td>
<td>26</td>
<td>130,000</td>
</tr>
</tbody>
</table>

(ii) **Finished goods inventory:**

<table>
<thead>
<tr>
<th>Product</th>
<th>Units</th>
<th>Cost/unit (N)</th>
<th>Total Value (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>100</td>
<td>867</td>
<td>86,700</td>
</tr>
<tr>
<td>G</td>
<td>50</td>
<td>1,162</td>
<td>58,100</td>
</tr>
</tbody>
</table>

\[
\text{Total Value} = 1,011,800 \quad \text{N}
\]
(i) **Cost of Goods Sold Budget**

<table>
<thead>
<tr>
<th>Product</th>
<th>Units to be sold</th>
<th>Cost/unit</th>
<th>Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>5,000</td>
<td>867</td>
<td>4,335,000</td>
</tr>
<tr>
<td>G</td>
<td>1,000</td>
<td>1,162</td>
<td>1,162,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>5,497,000</strong></td>
</tr>
</tbody>
</table>

Another way of preparing this budget is to sum up the total cost of production, taking into account all the three cost elements and then follow the trading account format, as follows:

**Total Cost of Production:**

- Direct materials to be used: 2,152,000
- Direct labour costs: 2,132,000
- Factory overhead cost: 2,080,000

**Total production (manufacturing) cost:** 6,364,000

**Trading Account Format:**

- Cost of opening stock: 144,800
- Add: Cost of manufacture: 6,364,000
- Less: Cost of closing stock: 1,011,800
- Cost of Goods sold: 5,497,000

(j) **Selling and Administrative Cost Budget**

(i) **Selling Cost Budget:**

- Sales commission: 200,000
- Advertisement: 30,000
- Sales salaries: 100,000
- Traveling expenses: 50,000

**Total:** 380,000

(ii) **Administrative Cost Budget:**

- Wages: 100,000
- Supplies: 10,000
- Salaries: 210,000
- Miscellaneous: 50,000

**Total:** 370,000
Budgeted Cashflow Statement

<table>
<thead>
<tr>
<th></th>
<th>Qtr 1</th>
<th>Qtr 2</th>
<th>Qtr 3</th>
<th>Qtr 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning cash balance</td>
<td>100,000</td>
<td>150,000</td>
<td>150,000</td>
<td>153,250</td>
</tr>
<tr>
<td>Add: Collection from customers</td>
<td>1,250,000</td>
<td>1,500,000</td>
<td>1,600,000</td>
<td>2,210,000</td>
</tr>
<tr>
<td>Less: Disbursements</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For materials</td>
<td>200,000</td>
<td>350,000</td>
<td>350,000</td>
<td>542,000</td>
</tr>
<tr>
<td>For other costs</td>
<td>250,000</td>
<td>200,000</td>
<td>200,000</td>
<td>170,000</td>
</tr>
<tr>
<td>For payroll</td>
<td>900,000</td>
<td>950,000</td>
<td>950,000</td>
<td>1,092,000</td>
</tr>
<tr>
<td>For income tax</td>
<td>50,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>For machinery</td>
<td></td>
<td></td>
<td></td>
<td>200,000</td>
</tr>
<tr>
<td></td>
<td>1,400,000</td>
<td>1,500,000</td>
<td>1,500,000</td>
<td>2,004,000</td>
</tr>
<tr>
<td>Add: Desired ending</td>
<td>150,000</td>
<td>150,000</td>
<td>150,000</td>
<td></td>
</tr>
</tbody>
</table>

Total estimated cash demand

Excess/Deficit (200,000)

Borrowing

Repayment

Interest (10%)

Notes

1. Closing cash balance
   = 359,250 - (110,000 + 11,000) = N238,250

2. The income tax payable of last year is to be paid in the first quarter of the year and that of this year is a current liability to be settled next year.

3. Interest payments are determined as follows:
   Quarter 3 = 10% of 90,000 x 3/4 = 6,750
   Quarter 4 = 10% of 110,000 x 1/4 = 11,000
   Total interest payment = 17,750

Budgeted Income Statement

<table>
<thead>
<tr>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>6,910,000</td>
</tr>
<tr>
<td>Less: Cost of goods sold</td>
<td>5,497,000</td>
</tr>
<tr>
<td>Gross Profit</td>
<td>1,413,000</td>
</tr>
<tr>
<td>Less: Selling expenses</td>
<td>380,000</td>
</tr>
</tbody>
</table>
Admin. expenses 370,000
Int. on borrowing 17,750 767,750
Net Profit Before tax 645,250
Less: Income tax 200,000
Net Profit after tax 445,250

(m) Budgeted Balance Sheet

Current Assets:
- Cash 238,250
- Account receivable 600,000
- Materials 98,000
- Finished goods 1,011,800 1,948,050

Fixed Assets:
- Land 500,000
- Building & Equip. 4,000,000
- Less: Acc. Depr. 1,000,000 3,000,000 3,500,000

Current Liabilities:
- Accounts Payable (on Rm) 700,000
- Income tax payable 200,000 900,000

Capital:
- Ordinary share capital 3,500,000
- Retained earnings 1,048,050 4,548,050 5,448,050

Notes:
1. Account Receivable was computed as follows:

   Opening balance (sundry debtors) 250,000
   Add: credit sales 6,910,000 7,160,000
   Less: Total cash collection from customers 6,560,000 600,000

2. Another way of finding the closing raw materials stock is:

   Opening stock 190,000
   Add: purchases 2,060,000 2,250,000
   Less: raw materials used 2,152,000 98,000
3. Another way of finding the closing finished goods is:

\[
\begin{align*}
\text{Cost of opening stock} & : \text{N}144,800 \\
\text{Add: cost of production} & : \text{N}6,364,000 \\
\text{Cost of stock to be available} & : \text{N}6,508,800 \\
\text{Less: cost of goods sold} & : \text{N}5,497,000 \\
\text{Cost of stock to be available} & : \text{N}1,011,800
\end{align*}
\]

4. Fixed Assets figure was arrived at as follows:

\[
\begin{align*}
\text{Building and Equipment} & : \text{N}3,800,000 \\
\text{New machine} & : \text{N}200,000 \\
\text{Fixed Assets} & : \text{N}4,000,000
\end{align*}
\]

5. Depreciation figure was arrived at as follows:

\[
\begin{align*}
\text{Accumulated Depreciation} & : \text{N}750,000 \\
\text{Depreciation for the year} & : \text{N}250,000 \\
\text{Depreciation} & : \text{N}1,000,000
\end{align*}
\]

6. Accounts Payable was arrived at as follows:

\[
\begin{align*}
\text{Opening Stock of Raw materials} & : \text{N}82,000 \\
\text{Add: Purchases of Raw materials} & : \text{N}2,060,000 \\
\text{Less: Payment for raw materials} & : \text{N}1,442,000 \\
\text{Accounts Payable} & : \text{N}700,000
\end{align*}
\]

7. Retained earnings figure was arrived at as follows:

\[
\begin{align*}
\text{Retained earnings carried forward} & : \text{N}602,800 \\
\text{Profit after tax for the year} & : \text{N}445,250 \\
\text{Retained earnings} & : \text{N}1,048,050
\end{align*}
\]

8.12 FLEXIBLE BUDGET

Flexible budget is a budgeting system which recognises the difference in behaviour between fixed and variable costs in relation to fluctuations in output or turnover. In order to prepare a flexible budget, the cost accountant must be able to separate the cost under consideration into its fixed and variable components. The method is a means of preparing a budget that is tailored to any level of activity. Flexible budgeting is very necessary for responsibility accounting. It allows the assessment of manager based on the activity level achieved. Flexible budget recognises the fact that short-term budgets for
immediate future should be prepared on the basis of various activity levels that are considered capable of achievement.

In flexible budgeting, budget is adjusted to the level of activity level attained. Care must be taken to ensure that activity level achieved is realistic in the light of the prevailing circumstances to enable justifiable comparison of actual figures with the flexed budget.

**ILLUSTRATION 8-5**

The budget of Anuoluwapo Elizabeth Limited for the month of May for 10,000 units of its product are as follows:

<table>
<thead>
<tr>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct material</td>
<td>2,000</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>2,000</td>
</tr>
<tr>
<td>Variable Overheads</td>
<td>1,000</td>
</tr>
<tr>
<td>Fixed Overheads</td>
<td>1,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>6,000</strong></td>
</tr>
</tbody>
</table>

The output of the company in the month of May was 8,000 units with the following actual costs:

<table>
<thead>
<tr>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct material</td>
<td>1,800</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>1,575</td>
</tr>
<tr>
<td>Variable Overheads</td>
<td>800</td>
</tr>
<tr>
<td>Fixed Overheads</td>
<td>1,050</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>5,225</strong></td>
</tr>
</tbody>
</table>

Prepare a flexible budget to show the comparative statement between actual and the level of activity achieved.

**SUGGESTED SOLUTION 8-5**

<table>
<thead>
<tr>
<th></th>
<th>Flexed budget</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variance</strong></td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Direct material</td>
<td>1,600</td>
<td>1,800</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>1,600</td>
<td>1,575</td>
</tr>
<tr>
<td>Variable Overheads</td>
<td>800</td>
<td>800</td>
</tr>
<tr>
<td>Fixed Overheads</td>
<td>1,000</td>
<td>1,050</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>5,000</strong></td>
<td><strong>5,225</strong></td>
</tr>
</tbody>
</table>

If the original budget has been compared with actual, the total variance would have been N775 favourable whereas when the budget
is tailored to the level of activity achieved the variance is $225 adverse.

ILLUSTRATION 8-6

Faith Ltd manufactures one uniform product only and activity levels in the assembly department vary widely from month to month. The following statement shows the departmental overhead budget based on an average level of activity of 20,000 units production per 4 week period, and the actual results for 4 weeks in January, 2005.

<table>
<thead>
<tr>
<th></th>
<th>Budget</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production (units)</td>
<td>20,000</td>
<td>17,600</td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indirect labour - variable</td>
<td>10,000</td>
<td>9,770</td>
</tr>
<tr>
<td>Consumables – variable</td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>Other variable overheads</td>
<td>2,100</td>
<td>1,830</td>
</tr>
<tr>
<td>Depreciation – fixed</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Other fixed overheads</td>
<td>5,000</td>
<td>5,000</td>
</tr>
<tr>
<td></td>
<td>27,500</td>
<td>27,100</td>
</tr>
</tbody>
</table>

You are required to:

(a) Prepare a columnar flexible 4 week budget for January 2005 at 16,000, 20,000 and 24,000 unit levels of production,

(b) Prepare two performance reports, based on production of 17,600 units by the department in January, comparing actual with:

   (i) average 4 week budget, and

   (ii) flexible 4 week budget for 17,600 units of production.

SUGGESTED SOLUTION 8-6

(a) Units of production

<table>
<thead>
<tr>
<th></th>
<th>16,000</th>
<th>20,000</th>
<th>24,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>Indirect Labour – variable</td>
<td>8,000</td>
<td>10,000</td>
<td>12,000</td>
</tr>
<tr>
<td>Consumables – Variable</td>
<td>320</td>
<td>400</td>
<td>480</td>
</tr>
<tr>
<td>Other variable – fixed</td>
<td>1,680</td>
<td>2,100</td>
<td>2,520</td>
</tr>
<tr>
<td>Depreciation – fixed</td>
<td>10,000</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Other fixed overheads</td>
<td>5,000</td>
<td>5,000</td>
<td>5,000</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>27,500</td>
<td>30,000</td>
</tr>
</tbody>
</table>
(b) (i) Performance report January 2005

<table>
<thead>
<tr>
<th></th>
<th>Budget 20,000 units</th>
<th>Actual 17,600 units</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indirect Labour – variable</td>
<td>10,000</td>
<td>9,770</td>
<td>230(F)</td>
</tr>
<tr>
<td>Consumables – Variable</td>
<td>400</td>
<td>500</td>
<td>100(U)</td>
</tr>
<tr>
<td>Other variable Overheads</td>
<td>2,100</td>
<td>1,830</td>
<td>270(F)</td>
</tr>
<tr>
<td>Depreciation – fixed</td>
<td>10,000</td>
<td>10,000</td>
<td>-</td>
</tr>
<tr>
<td>Other fixed overheads</td>
<td>5,000</td>
<td>5,000</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>40,000</strong></td>
<td><strong>39,200</strong></td>
<td><strong>400(F)</strong></td>
</tr>
</tbody>
</table>

(ii) Performance report – January, 2005

<table>
<thead>
<tr>
<th></th>
<th>Budget 17,600 units</th>
<th>Actual 17,600 units</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indirect Labour – variable</td>
<td>8,800</td>
<td>9,770</td>
<td>970(U)</td>
</tr>
<tr>
<td>Consumables – Variable</td>
<td>352</td>
<td>500</td>
<td>148(U)</td>
</tr>
<tr>
<td>Other variable Overheads</td>
<td>1,848</td>
<td>1,830</td>
<td>18(F)</td>
</tr>
<tr>
<td>Depreciation – fixed</td>
<td>10,000</td>
<td>10,000</td>
<td>-</td>
</tr>
<tr>
<td>Other fixed overheads</td>
<td>5,000</td>
<td>5,000</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>26,400</strong></td>
<td><strong>27,500</strong></td>
<td><strong>1,100(U)</strong></td>
</tr>
</tbody>
</table>

Note

(a) (i) This implies that the variances are mostly favourable, whereas (b) (ii) shows that for control purposes the variances are mainly unfavourable.

(b) (ii) It would be more helpful in assessing the production supervisor’s effectiveness, because in this report the actual costs for 17,600 units produced are compared with the budgeted costs of producing 17,600 units. The first performance report (b) (i) compares the budgeted costs of 20,000 units with the actual costs of 17,600 units. This comparison may be useful for judging the effectiveness of planning, and for examining the reasons why 20,000 units were not produced, but this will not always be under the control of the production supervisor. The production supervisor’s main function in this example is cost control and for this purpose the actual costs must be compared with the budgeted costs for that activity level.
ILLUSTRATION 8-7

Peace Manufacturing Company Limited, makers of product X, has prepared its budget for 2004 based on two activity levels of 80% and 100% with a production units of 2,800 and 3,500 units respectively. The budget is as follows:

<table>
<thead>
<tr>
<th></th>
<th>80%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>N224,000</td>
<td>N280,000</td>
</tr>
<tr>
<td>Direct material</td>
<td>N84,000</td>
<td>N105,000</td>
</tr>
<tr>
<td>Direct labour</td>
<td>N50,400</td>
<td>N63,000</td>
</tr>
<tr>
<td>Production overhead</td>
<td>N48,800</td>
<td>N53,000</td>
</tr>
</tbody>
</table>

The company is actual result for the period is as follow:

Sales (Units) 3,150
Sales N252,000
Direct material N94,000
Direct labour N60,000
Overhead (60% of the amount is fixed) N50,000

You are required to:
(a) prepare a flexible budget for a production level of 70% and 90%;
and
(b) compare the flexed budget at 90% level of activity with the actual result and obtain the variances.

SUGGESTED SOLUTION 8-7

(a) The first step is to separate the production overhead into the fixed and variable elements using high and low method.

<table>
<thead>
<tr>
<th>Production Level (Units)</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,500</td>
<td>N53,000</td>
</tr>
<tr>
<td>2,800</td>
<td>N48,800</td>
</tr>
<tr>
<td>700</td>
<td>N4,200</td>
</tr>
</tbody>
</table>

Variable cost per unit
\[
\text{Variable cost per unit} = \frac{4,200}{700} = \text{N}\$6.00
\]

Fixed Cost
\[
\text{Fixed Cost} = N53,000 - (6 \times N3,500) = N32,000
\]
Flexible Budget at 70% and 90% levels of activity:

<table>
<thead>
<tr>
<th></th>
<th>70%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (Units)</td>
<td>2,450</td>
<td>3,150</td>
</tr>
<tr>
<td>Sales</td>
<td>196,000</td>
<td>252,000</td>
</tr>
<tr>
<td>Direct Materials</td>
<td>73,500</td>
<td>94,500</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>44,100</td>
<td>56,700</td>
</tr>
<tr>
<td>Variable Production Overhead</td>
<td>14,700</td>
<td>18,900</td>
</tr>
<tr>
<td>Fixed Production Overhead</td>
<td>32,000</td>
<td>32,000</td>
</tr>
<tr>
<td></td>
<td>164,300</td>
<td>202,100</td>
</tr>
</tbody>
</table>

(b) Comparison of flexed budget at 90% level of activity with the actual

<table>
<thead>
<tr>
<th>Activity level</th>
<th>90%</th>
<th>90%</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (Units)</td>
<td>2,450</td>
<td>2,450</td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>252,000</td>
<td>252,000</td>
<td></td>
</tr>
<tr>
<td>Direct Materials</td>
<td>94,500</td>
<td>94,000</td>
<td>500 (F)</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>56,700</td>
<td>50,000</td>
<td>6,700 (F)</td>
</tr>
<tr>
<td>Variable Production Overhead</td>
<td>18,900</td>
<td>20,000</td>
<td>1,100 (A)</td>
</tr>
<tr>
<td>Fixed Production Overhead</td>
<td>32,000</td>
<td>30,000</td>
<td>2,000 (F)</td>
</tr>
<tr>
<td></td>
<td>202,100</td>
<td>194,000</td>
<td>8,100 (F)</td>
</tr>
</tbody>
</table>

8.13 SUMMARY AND CONCLUSIONS

In this chapter, efforts have been made to explain the principles and practices of budgeting and budgetary control. Various definitions and explanatory notes were used to make the meaning and significance of budgets very clear to the readers. The budgeting framework and budgetary control process have been described. The general purposes of budgets, their limiting factors and the roles of budget committee and budget officer have also been discussed. Three different ways of differentiating budgets - time, activity and quantitative perspectives have been explained. Budgetary improvement techniques like Incremental Budgeting (IB), Zero-Base Budgeting (ZBB), Rolling (Continuous) Budgeting (CB) and Planning, Programming, Budgeting System (PPBS) were discussed as to their underlying assumptions, major steps, advantages, disadvantages and similarities and differences. Finally, the way to prepare company’s functional and master budgets have been illustrated, using some standard questions.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
8.15 REVISION QUESTIONS

8.14.1 MULTIPLE CHOICE QUESTIONS

1. A continuous update of short term budgets to reflect current conditions is:
   A. Zero based budgeting
   B. Activity based budgeting
   C. Rolling budget
   D. Incremental budgeting
   E. Traditional budgeting

2. A budgeting system that recognizes the difference in the behavior between fixed and variable costs in relation to fluctuations in output is:
   A. Kaizen budget
   B. Rolling budget
   C. Flexible budget
   D. Master budget
   E. Functional budget

3. The budget that reflects the day to day activities or operations of an organization is:
   A. A financial budget
   B. A long term budget
   C. A flexible budget
   D. An operating budget
   E. An organisational budget

4. A budgeting method that is based on previous budget or actual results, adjusting for known changes and inflation is:
   A. Zero-based budgeting
   B. Planning, programming budgeting system
   C. Incremental budgeting
   D. Master budgeting
   E. Kaizen budget

5. Activity perspective types of budgeting include:
   A. Operating budget
   B. Financial budget
   C. Incremental budget.
   D. Organisational budget

Which of these is/are true?
I. (a) and (b)
II. (a) and (c)
III. (b) and (c)
IV. (a) and (d)
V. (d) and (c)
8.14.2 SHORT ANSWER QUESTIONS

1. What is a budgeting limiting factor?

2. Briefly explain Kaizen budget.

3. Why is it necessary to consider behavioural aspects of budgeting?

4. A budgetary improvement technique that assumes that current operations will start from zero level is.............

5. What is a cash budget?

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
LEARNING OBJECTIVES

After studying this chapter, readers will be able to understand:

- The objectives of standard costing, types of standards and applications of standard costing;
- Setting of standards, variance analysis, advantages of standard costing; and
- Control ratios.

INTRODUCTION

This aspect of the syllabus deals with the control tools of management. Cost can be ascertained either on historical basis or on a predetermined basis, as we shall illustrate in this chapter. The major application of this technique is control. It does this by computing what a given product or service rendered should have cost under some given efficient operating conditions, and comparing this ‘predetermined’ cost with what it actually cost, so as to identify areas where corrective action could be taken by management.

OBJECTIVES OF STANDARD COSTING

DEFINITIONS

To lead us to an understanding of the objectives of standard costing, it will be useful to first run through some definitions:

(a) **Standard Costing**

Standard Costing is defined by the CIMA Official Terminology as a control technique which compares standard costs and revenues with actual results to obtain variances which are used to stimulate improved performance.
(b) **Standard Cost**

A Standard Cost is defined as “a pre-determined calculation of how much costs should be under specified working conditions. It is built up from an assessment of the value of cost elements and correlates technical specification and quantity of materials, labour other costs to the prices and/or wage rates expected to apply during the period in which the standard cost is intended to be used. Its main purposes are to provide basis for control through variance accounting, for valuation of stock and work-in-progress and, in some cases, for fixing selling prices”. (CIMA Official Terminology).

9.2.2 **SETTING STANDARDS**

From the definition of standard cost given above, certain fundamental issues about Standard Costing can be discerned, that is, the steps involved in setting a standard cost can be appreciated:

(a) It is determined in advance of the actual costs.
(b) It is set based on prescribed set of working conditions.
(c) It is arrived at by correlating quantity of materials and labour with the prices and wage rates.
(d) It is applied to some areas of management needs, that is, controls, stock valuation, and possibly also fixing selling prices.
(e) When used for control purposes, a comparison is made of predetermined costs with actual cost. The result of this comparison is identification of variances which are then reported to management for corrective actions, if necessary.

9.2.3 **TYPES OF STANDARD**

Standards set are always the result of someone’s subjective judgment of efficient working conditions. The validity of the assumed working conditions can also be limited by time. It is, therefore, possible to have different kinds of standards and this in turn, will affect the quality of the variance or the control action. The possible standards are:

(a) Attainable Standard;
(b) Ideal Standard;
(c) Loose Standard;
(d) Basic Standard; and
(e) Current Standard.

These variances are explained as follows:

Attainable Standard: This refers to a standard that can be achieved under normal efficient operating conditions. It makes allowance for normal losses and normal wastages. This kind of standard may appear difficult to attain but not impossible to achieve.

Ideal Standard: This is a standard that can be achieved only under a perfect working condition. No allowance is made for losses or wastages. It does not provide any incentive for workers to work efficiently as it often results in adverse variance.

Loose Standard: This is the other extreme of the ideal standard. It is a standard that can be achieved with minimum effort. It represents a standard that is left unchanged over a long period of time.

Current Standard: Also from the time point of view, it represents a standard that is applicable within a short period of time. In view of the frequent changes in prices and methods of production, standards should be subject to review from time to time, if it is to provide an incentive for workers to work efficiently.

From the above description of the kinds of standards that there are, it would be seen that the standard that should be used for motivation and control purposes, which is the key objective of standard costing, should be the currently attainable standard.

9.2.4 CORRELATION OF QUANTITY & PRICES

Having considered the kinds of standards and concluded that standards should be realistic, the conclusion of the work of setting standards requires a correlation of the quantity and the rates of the specified resources. We will now look at the steps involved in setting a standard cost. Having in mind a prescribed
set of working conditions, estimates are made of the quantity of material and labour prices at which these resources will be procured.

**Information for Setting Standards**

In practice, the following are some of the sources of information for setting standard cost:

**Materials**
Product specification after an intensive study of the input quantity necessary for each unit of output.

**Prices**
These are obtainable from the Purchasing Department after a careful market survey of prices.

**Labour**
Machine Manufacturer’s specification of time required to achieve production.

Test runs of sample production

**Labour Rate**
Company’s Standard Wage rate or negotiated wage rate with Trade Union

**Overhead**
Company’s pre-determined overhead absorption rates

Thus, a Standard cost is arrived at as follows:

<table>
<thead>
<tr>
<th></th>
<th>Qty</th>
<th>Price</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Labour</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Variable overhead</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Fixed production overhead</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Standard production cost</td>
<td>x</td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>

### 9.3 VARIANCE ANALYSIS

The major application of Standard Costing is for controls, through variance analysis and reporting.

A *Variance* is simply the difference between planned or budgeted costs and actual costs and similarly in respect of revenues, while *Variance*
Analysis is the analysis of variances in a standard costing system into their constituent parts. It is the analysis and comparison of the factors which have caused the difference between pre-determined standards and actual results with a view to eliminating inefficiencies.

9.3.1 POSSIBLE CAUSES OF VARIANCES

The following are the more common factors attributed to variances in manufacturing concerns.

(a) Material Price - Buying materials at a price different from the specified buying price.
   (i) Inefficiency of the purchasing department in seeking the most advantageous sources of supply.
   (ii) Changes in market condition causing general price increase.
   (iii) Purchase of inferior (or superior) quality materials.

(b) Material Usage - Using more or less quantities of material than those specified to achieve the actual production
   (i) Careless handling of materials by the production workers.
   (ii) Purchase of inferior quality materials.
   (iii) Changes in method of production.

(c) Labour Rate - Paying labour at a rate different from the agreed rate
   (i) Assignment of work to higher grade labour.
   (ii) Negotiated increase in wage rates not reflected in the standard wage rate.

(d) Labour Efficiency - The work force spending more or less time than allowed for the actual production
   (i) Waste of time due to use of inferior quality materials.
   (ii) Use of different grade of labour from that specified.
(iii) Waste of time due to failure to maintain machines in proper conditions.

(e) Overhead - Since the Recovery Rates are always based on budgeted figures, any deviation from budget will give rise to a variance. Therefore, overhead variance will be caused by the following factors:
(i) Actual expenditure being different from the budgeted expenditure.
(ii) Actual production being different from the budgeted production.

9.3.2 THE PYRAMID OF VARIANCES

The following diagram can be used to depict the various major factors causing variances:

![Profit Variance Diagram]

9.3.3 INTER-RELATIONSHIP OF VARIANCES

It should be noted that the above are only a few of the possible causes of variances. It is also possible for the factor causing a variance in one element of cost to be responsible for a variance
in another element of cost. An example of this is when lower grade labour is employed to save labour cost and this truly yields a favourable labour rate variance but this can be responsible for an adverse material usage variance as well as an adverse overhead cost variance due to wastages of materials and loss of productive hours.

9.4 CALCULATION OF VARIANCES

Calculation of variances is quite often unfortunately dreaded by most readers. In order to avoid the fear, readers are advised to always keep the control objective in mind whenever they are attempting variance analysis questions, and also do two things.

(a) Ascertain two basic information:
(i) the Standard Cost data, and
(ii) the Actual Production for the period.

(b) Keep in mind a pertinent question “Given the actual quantity, what should it have cost?”

This principle will be applied in attempting all the questions that follow. The alternative, which is to use a set of formulae to calculate the variances is not objected to. The formula are, therefore, stated in addition to the explanatory approach.

9.5 MATERIAL COST VARIANCE

This is the difference between the actual cost of materials consumed to achieve a given production level and what it should have cost at standard material cost. It is also given by the formula \((SC - AC)\). This can be analyzed into material price and material usage variances.

<table>
<thead>
<tr>
<th>Material cost variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Variance</td>
</tr>
<tr>
<td>Usage Variance</td>
</tr>
</tbody>
</table>

9.5.1 Material Price Variance

This is the variance that arises from buying materials at a price different from the standard buying price. Its calculation is given by the formula \(\text{Price Variance} = (SP-AP)AQ\).
It should be borne in mind that this is a purchase price variance, that is, given that some quantities of materials were bought in the period at one or more various prices, what should the total purchases have cost, if they were bought at the pre-set standard price?

The understanding of this point is quite fundamental and for those writing professional examinations, some examiners expect the calculation of price variance to be based on quantities used rather than the quantity purchased. This does not mean that the Examiner is wrong because it is possible to calculate this Purchase Price Variance at two different points in time. This alternative approach and its justification will be discussed later.

9.5.2 Material Usage Variance

This is that part of material cost variance arising from the use of more or less quantity of raw materials to achieve the actual production. It is given by the formula Material Usage Variance = (Standard Qty - Actual Qty) Standard Price.

ILLUSTRATION 9-1

Kolly is a product manufactured by ROLMANN Ltd. For every ton of material consumed, it is estimated that 500 units of KOLLY would be produced. The standard price of the material is N18,000 per ton. In March, 210 tons of materials were issued to production, the actual price of which was N17,200 per ton. Production during March was 108,000 units of KOLLY.

You are required to compute the following variances:
(a) Material Cost
(b) Material Price
(c) Material Usage

SUGGESTED SOLUTION 9-1

Standard cost data, that is, Cost per unit
Material = \( \frac{1}{500} \) ton x N18,000 = N36 per unit

Therefore, Material Cost Variance
= Standard Cost of Actual Production - Less Actual Cost of Materials consumed
= (108,000 unit \( \times \) N36) – (210 x N17,200)
= N3,888,000 – 3,612,000
= N276,000 F
Material Price Variance

(Actual Qty purchased @ Actual Price) – (Actual Qty purchased @ Standard Price)
= (210 x ₦17,200) – (210 x ₦18,000)
= ₦3,612,000 – ₦3,780,000)
= ₦168,000 F

Material Usage Variance

Difference between the Actual Quantity of materials used and the quantity that should have been used, valued at the standard price,
that is, (210 – (108,000) x ₦18,000
500
= (210 – 216 tons) x ₦18,000
= ₦108,000 F

Notes

(i) In the above illustration, it is assumed that the quantity of raw material issued was equal to the quantity of raw materials purchased in the period.
(ii) The result of the variance calculation should be indicated by a letter (A) or F following the figure, indicating whether it was an Adverse Variance (A) or a Favourable Variance, (F). This method will be followed in all subsequent examples.

9.5.3 Further Analysis of the Material Usage Variance

The usage variance can be further analysed into the material mix and material yield variance but only where more than one type of raw material is used and which can be substituted one for the other(s). The calculation of these variances will be discussed later to allow for a progressive understanding.

9.6 LABOUR COST VARIANCE

This is the difference between the actual labour cost for the actual production in a given period and what labour should have cost if labour worked at normal efficiency and was remunerated at the standard wage rate. This variance is usually analysed into Labour Rate Variance and Labour Efficiency Variance. Thus:
Labour Rate Variance

This is the variance due to paying labour for hours worked at a rate different from the agreed rate, that is, (SR-AR)AH. We will illustrate the calculation of this variance from the following questions. After using the question to illustrate the Labour Rate and Efficiency Variances, we will use it again to repeat the Material Cost Variance up to the point of Mix and Yield.

ILLUSTRATION 9-2

SALVATION Ltd manufactures and sells SALOME soft drinks. The standard direct post per container is as follows:

**Materials**

- 100 litres concentrated juice at ₦2 per litre
- 200 litres of carbonated water at ₦2.50 per litre
- 10 labour hours at ₦9.00 per hour

The budgeted monthly production and sales is 500 containers and the selling price is ₦1,000 per container.

The following details relate to October, 2003, when 510 containers of soft drinks were produced and sold:

<table>
<thead>
<tr>
<th>Sales</th>
<th>₦506,500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials used:</td>
<td></td>
</tr>
<tr>
<td>Concentrated juice – 51,600 litres</td>
<td>₦102,500</td>
</tr>
<tr>
<td>Carbonated Water – 101,500 litres</td>
<td>₦258,800</td>
</tr>
<tr>
<td>Labour</td>
<td></td>
</tr>
<tr>
<td>5,000 hours cost</td>
<td>₦45,750</td>
</tr>
</tbody>
</table>

**Required**

(a) Compute the price and usage variances for each material.
(b) Calculate the wage rate and efficiency variances.
(c) Comment briefly upon the information revealed by each of the variances you have computed.
Suggested Solution 9-2

Labour Cost Variances

Standard Labour Cost of Actual Production = 510 containers @ N90 labour cost per container, that is, 45,900
Less Actual Wages Cost N45,750

Labour Rate Variances

Variance due to remunerating labour at a different rate

Actual hours paid @ actual rate = 45,750
Actual hours if paid @ standard rate = 45,000
(750)

Labour Efficiency Variance

The variance due to spending more or less hours than should have been spent to produce the actual production; valued at the standard wage rate.

This is (SH-AH)SR.

Actual hours taken to produce output = 5,000 hrs.
Hours expected to have been spent (Standard hours) = 510 x 10, that is, 5,100 hrs.
Hours saved = 100 hrs.
Therefore, Labour Efficiency Variance = 100 hrs x N9 = N900 F

9.7 Material Mix and Yield Variances

Where it is possible to use more or less quantity of a raw material in substitution for another, then any usage in a proportion different from the standard proportion will give rise to a usage variance due to mixture, called Material Mix Variance.

The yield variance is the residual material variance or the absolute quantity difference assuming materials were used at the standard mix. For these variances, a tabular presentation will simplify the exercise, even though an alternative, that is, the average price method, can also be used for the yield variance.

Using illustration 9-2, the material mix and yield variances can be calculated.
**Material Mix Variance**

To calculate the Material Mix Variance, obtained from the question, the standard mix recommended. In this question, it is ratio 1:2, that is, 100 litres of juice to 200 litres water. Then use it to restate the actual quantity of materials used, to obtain actual quantity at standard proportion. Then compare this proportion with the actual proportion used. It is a variance due to the usage of the actual materials used in a proportion different from the standard proportion. Thus Actual Quantity of Raw Materials Used = 153,100 litres.

<table>
<thead>
<tr>
<th></th>
<th>Actual Qty @ Actual proportion</th>
<th>Actual Qty @ Standard proportion</th>
<th>Diff</th>
<th>Standard Price</th>
<th>Mix Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentrated</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Juice</td>
<td>51,600</td>
<td>51,033</td>
<td>(567)</td>
<td>2.00</td>
<td>(1,134) A</td>
</tr>
<tr>
<td>Water</td>
<td>101,500</td>
<td>102,067</td>
<td>567</td>
<td>2.50</td>
<td>1,417.5 F</td>
</tr>
<tr>
<td></td>
<td>153,100</td>
<td>153,100</td>
<td>0</td>
<td>2.83</td>
<td>283.5 F</td>
</tr>
</tbody>
</table>

**Note**

1. It should be noted that the total difference between the quantity used at actual proportion and quantity used at standard proportion will always be zero. This is because you are looking at the same thing in two different ways.

2. The bracket sign for juice in the difference column is an indication that the variance for that input material will be adverse, showing that, as a result of a deviation from the mix specified, we have used more quantity of juice than we should have used, and less water than we ought to have used.

9.7.1 **MATERIAL YIELD VARIANCES**

It is suggested that you begin this variance as a usage variance, that is, comparing standard quantity that should have been used with the Actual Quantity used, except that the actual quantity used will be stated at standard mix, having already isolated the variance due to not using the materials at standard mix above as a mixture variance.
Thus:

<table>
<thead>
<tr>
<th></th>
<th>Standard Qty</th>
<th>Actual Qty</th>
<th>Diff</th>
<th>Standard Price</th>
<th>Material Yield Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>required for production (litres)</td>
<td>and Standard Mix (litres)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Juice</td>
<td>51,000</td>
<td>51,033</td>
<td>(33)</td>
<td>2.00</td>
<td>(66) A</td>
</tr>
<tr>
<td>Water</td>
<td>102,000</td>
<td>102,067</td>
<td>(67)</td>
<td>2.50</td>
<td>(168) A</td>
</tr>
<tr>
<td></td>
<td>153,000</td>
<td>153,100</td>
<td>100</td>
<td></td>
<td>(234) A</td>
</tr>
</tbody>
</table>

We may also calculate the material price usage variances for those materials so as to reconcile the mix and yield variance with the usage.

**Material Price Variance**

<table>
<thead>
<tr>
<th></th>
<th>Actual Quantity Purchased litres</th>
<th>At the Actual price N</th>
<th>At the Standard price N</th>
<th>Price Variance N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentrated Juice</td>
<td>51,600</td>
<td>102,500</td>
<td>103,200</td>
<td>700 F</td>
</tr>
<tr>
<td>Carbonated Water</td>
<td>101,500</td>
<td>258,800</td>
<td>253,750</td>
<td>(5,050) A</td>
</tr>
</tbody>
</table>

**Material Usage Variance**

<table>
<thead>
<tr>
<th></th>
<th>Standard Quantity allowed (litres)</th>
<th>Actual Quantity used (litres)</th>
<th>Diff (litres)</th>
<th>Standard Price</th>
<th>Usage Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentrated Juice</td>
<td>51,000</td>
<td>61,000</td>
<td>(600)</td>
<td>2.00</td>
<td>(1,200) A</td>
</tr>
<tr>
<td>Carbonated Water</td>
<td>102,000</td>
<td>101,500</td>
<td>500</td>
<td>2.50</td>
<td>1,250 F</td>
</tr>
</tbody>
</table>

**Reconciliation**

<table>
<thead>
<tr>
<th></th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Juice</td>
<td>700</td>
<td>(1,134)</td>
<td>(66)</td>
<td>(1,200)</td>
<td>(500)</td>
</tr>
<tr>
<td>Water</td>
<td>(5,050)</td>
<td>1,418</td>
<td>(168)</td>
<td>1,250</td>
<td>(3,800) A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
<th>(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(4,350) A</td>
<td>284 F</td>
<td>(234) A</td>
<td>50 F</td>
<td>(4,300) A</td>
</tr>
</tbody>
</table>

**Check**

(B) + (C) = (D)
(A) - (D) = (E)
9.8 ALTERNATIVE BASIS FOR MATERIAL PRICE VARIANCE

We have mentioned earlier on that Material Price Variance can be calculated based on the quantity of material used rather than quantity purchased. This is the result of an alternative basis that exists for price variance calculation.

In other words, there are two basis for calculating material price variance. Readers are, therefore, advised to watch out for the word used by the examiner in every question.

(a) **At the time of purchase:** This also means that stocks of raw materials are carried at standard prices, or that price variance is written off at the time of purchase.

(b) **At the time of usage:** This means that stocks of raw materials are carried at actual prices, since the price variance is not isolated until there is an issue to production. The examiners might opt for this basis stating that price variance is written off at the time of consumption.

One implication of this latter basis is that an attempt to reconcile the price variance and the usage variance with the material cost variance will show a marked difference. The difference will be equal to the price variance of the raw materials stock carried forward.

9.9 OVERHEAD VARIANCES

Overhead variances are very easy to understand if overhead absorption concept is considered. Other conditions attached may tend to bring in a little confusion but readers should not be troubled but to look for the condition inherent in a particular question. For example, a question may suggest a system that considers overheads in total, or another system where overheads are broken down into fixed and variable elements. The question could also specify that a system of standard marginal costing is in operation or that a system of standard absorption costing is in use. Where overheads are treated in total, this makes it very simple as the total overhead variance is analysed only into Overhead Expenditure Variance; Overhead Efficiency Variance and Overhead Volume Variance, that is:
Under the standard marginal costing, only the variable overhead is absorbed into product cost. Therefore, variable overhead variance only need to be calculated. The fixed overhead variance will be dealt with simply by deducting the actual fixed overhead from the budgeted contribution in the operating statement.

Where standard absorption costing is in use, then both the fixed and variable overheads variances will need to be calculated in full.

This last one which is the most common, will be dealt with first, that is, where the overheads variance is analysed into fixed and variable components.

However, for ease of understanding, we would start with the fixed overhead variances.

This example to be used is specifically designed to assist readers not only to learn overhead variances but also to revise the materials and labour variances and then be introduced to sales margin variances and also preparation of operating statement reconciling Budgeted Profit with Actual Profit.
ILLUSTRATION 9-6

Prudent Business Limited manufactures a single product, which has a standard cost of N80 made up as follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>Quantity/Details</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Materials</td>
<td>15 square metres at N3/sq. mtr.</td>
<td>45</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>5 hours at N4/hour</td>
<td>20</td>
</tr>
<tr>
<td>Variable Overheads</td>
<td>5 hours at N2/hour</td>
<td>10</td>
</tr>
<tr>
<td>Fixed Overheads</td>
<td>5 hours at N1/per hour</td>
<td>5</td>
</tr>
</tbody>
</table>

80

The standard selling price of the product is N100 per unit.
The monthly budget projects production and sales of 1,000 units.
Actual figures for the month of April are as follows:
Sales 1,200 units at N102
Production 1,400 units
Direct materials 22,000 square metres at N4 per square metre
Direct wages 6,800 hours at N5
Variable Overheads N11,000
Fixed Overheads N6,000

You are required to prepare a trading account reconciling actual and budgeted profit and showing all appropriate variances.

9.9.1 Fixed Overhead Cost Variance

This is simply the difference between the Actual fixed overhead incurred and the fixed overhead absorbed using the predetermined absorption rate.

From the Illustration 9-3

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Fixed Overhead incurred</td>
<td>6,000</td>
</tr>
<tr>
<td>Absorbed Fixed Overhead (based on actual production)</td>
<td>7,000</td>
</tr>
<tr>
<td>Therefore, F/ohd cost variance (over-absorption of overhead)</td>
<td>1,000 F</td>
</tr>
</tbody>
</table>
9.9.2 Fixed Overhead Expenditure Variance

Fixed Overhead Expenditure Variance is simply that part of fixed overhead cost variance which was due to the failure to budget the amount of fixed overhead correctly, that is:

\[
\begin{align*}
\text{Budgeted Fixed Expenditure (1,000 units} & \times N5) = 5,000 \\
\text{Actual Fixed Overhead Expenditure} & = 6,000 \\
\text{Difference} & = (1000) \text{ A}
\end{align*}
\]

9.9.3 Fixed Overhead Volume Variance

This is that part of fixed overhead cost variance, which was due to the failure to budget production volume correctly. The reader will recall from the topic Overhead Absorption that the Absorption rate was predetermined by estimating the amount of overhead and the estimated volume. Thus:

Budgeted production volume = 1,000 units
Actual production volume = 1,400 units
Difference = 400 units

Therefore, fixed overhead would be over absorbed @ N5 in the sum of 400 X N5
that is, N2,000 F. This is the Fixed overhead volume variance.

**Fixed Overhead Capacity and Efficiency Variances**

Investigating the volume variance further, it would be discovered that two major factors could be responsible. First, is the capacity budgeted to work. That is, based on a budget of 1,000 units and a working period of 5 hours per unit, the company had planned to work for 5,000 hours.

If the workers fail to work for the 5,000 hours, that is, under utilization of available capacity they might not succeed in producing the 1,000 units except if they worked above normal efficiency. Hence, the efficiency of the labour force could also affect the production volume.

It should, however, be borne in mind that we are working in terms of labour hours and so the fixed overhead rate will have to be expressed in terms of labour hours to obtain the fixed overhead capacity and the fixed overhead efficiency variances.

Thus, in the Illustration 9-3,
**Fixed Overhead Capacity Variance**

Budget Capacity = 1,000 x 5 hours, that is, 5,000 hours
Actual Hours Worked = 6,800 hours
Therefore, Fixed Overhead Capacity Variance = 1,800 hours
= 1,800 x $1
(FOAR per hr)
= $1,800 F

It is favourable, because more hours worked should result in increased production volume.

**Fixed Overhead Efficiency Variance**

The effect of the efficiency of labour on overhead absorption.
Actual hours allowed for actual output = 1,400 unit x 5 hours
= 7,000 hours
Actual hours worked = 6,800 hours
Hours saved = 200 hours

Therefore, Fixed Overhead Efficiency Variance = 200 x $1
(FOAR per hour)
= $200 F

Reconciling, Fixed overhead volume variance:

+ $2,000 F
  
  Fixed Overhead Capacity Variance
  $1,800 F

  Fixed Overhead Efficiency Variance
  $200 F

**Variable Overhead Variances**

The same idea of Overhead absorption would also help simplify the calculation of variable overhead variances. Variable overhead cost variance is simply the difference between the Actual variable overhead expenditure incurred and the Variable overhead absorbed.
From the Illustration 9-3,

Actual Variable Overhead Expenditure was \( N\, 11,000 \)

Absorbed Fixed Overhead would be \( N\, 14,000 \)

Actual production x the predetermined absorption rate (VOAR) that is, \( 1,400 \times N\, 10 \)

Therefore, Variable Overhead Cost Variance = \( N\, 3,000 \) F

The variance is favourable because there was an over-recovery of the overhead.

**The Variable Overhead Cost Sub-Variances**

There is no volume variance in the Variable overhead analysis. This is because, by the very nature of this expenditure it should change when there is a change in volume. For this reason, some cost analysts stop the calculation of the variable overhead variance at the level of the variable overhead cost and call it variable overhead expenditure variance.

However, many analysts also attempt to go further to see the effect of the labour on the overhead recovery. Then variable overhead is analysed into expenditure and efficiency. The calculation at this stage is quite similar to the calculation of the fixed overhead capacity and efficiency variances in that they use labour hours and express the absorption rates in terms of labour hours.

**Variable Overhead Expenditure Variance**

This is based on the assumption that variable overhead varies with actual labour hours worked. Therefore, variable overhead expenditure is the difference between the actual variable overhead expenditure and the 'allowed' variable overhead expenditure based on actual hours worked.

*Hint:* You can first calculate the variable overhead absorption rate per labour hour. In this question, this becomes.

\[
\frac{\text{Budgeted Variable Overhead}}{\text{Budgeted Labour Hour}} = \frac{N\,10,000}{5,000} = N\,2 \text{ per hour}
\]

**Variable Expenditure Variance**

\[
\text{Actual Variable Overhead Expenditure} = N\,11,000
\]

‘Allowed’ Variable Overhead
STANDARD COSTING

(6,800 hours x N2) = N13,600
N2,600 F

Variable Overhead Efficiency Variance

This is the difference between the ‘allowed’ variable overhead and the absorbed variable overhead. If the time-based bonus schemes of remuneration in Labour Costing in Chapter 3 is brought to mind, the variable overhead efficiency variance can be looked at as the labour efficiency indicated by the time saved, but now valued at the variable overhead absorption rate (VOAR) per labour hour.

that is, Allowed Variable Overhead Expenditure

(1,400 units x 5 x N2) 14,000

Absorbed Variable Overhead

(6,800 hours x N2) 13,600

N400 F

Alternatively,

Time Allowed (1,400 x 5) = 7,000 hours
Time Taken = 6,800
Time Saved = 200 hours

Therefore, Variable Overhead Efficiency Variance

(200 hours x N2) = N400 F

Calculation of Sales Margin Total Variance

This is the difference between the Budgeted Margin and the Actual Margin attained. Actual Margin should be understood, to be the Actual sales revenue less the Standard cost of sales.

From Illustration 9-3:

Total Sales Margin Variance

N

Actual Sales Revenue = 1,200 x N102 122,400
Standard Cost of Sales = 1,200 x N80 96,000
Therefore, Actual Margin is 26,400
Budgeted Margin 1,000 units X (100 – 80) 20,000
Therefore, Total Sales Margin Variance 6,400 F
Sales Margin Price Variance

This is the variance due to selling at a price different from the standard selling price. This is similar to the material price and the labour rate variances.

\[
\text{Sales Margin Price Variance} = \text{Actual Quantity Sold @ Actual Selling Price} - \text{Less Actual Quantity Sold @ Standard Selling Price}
\]

\[
\begin{align*}
\text{Actual Quantity Sold @ Actual Selling Price (1,200 x 102)} & = 122,400 \\
\text{Less Actual Quantity Sold @ Standard Selling Price (1,200 x 100)} & = 120,000 \\
& \text{2,400 F}
\end{align*}
\]

Sales Margin Volume Variance

This is Profit Variance due to a change in budgeted volume:

\[
\text{Sales Margin Volume Variance} = \text{Budgeted volume} - \text{Actual volume}
\]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>200</td>
<td>400</td>
</tr>
</tbody>
</table>

\[
\text{Budgeted volume} = 1,000 \text{ units} \\
\text{Actual volume} = 1,200 \text{ units} \\
\text{Difference} = 200 \text{ units} \times \text{Standard Margin of N20} = \text{N4,000 F}
\]

Exception to the variances computed above is sales margin variance where more than one product is for sales, that is, where it is possible or necessary to calculate the sales margin (mix and quantity) variances.

We can now illustrate how the operating statement reconciling budgeted profit with actual profit is prepared.

You are requested to complete the other cost variances on your own, that is, material price, material usage, labour rate and labour efficiency. The operating statement should start with the name of the organisation, and the title of the statement including the relevant period.

**Prudent Business Limited**

**Operating Statement for the month of June 2005**

<table>
<thead>
<tr>
<th></th>
<th>Fav</th>
<th>Adv</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>Budgeted Profit</td>
<td></td>
<td></td>
<td>20,000</td>
</tr>
</tbody>
</table>

Sales Margin Variance

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>2,400</td>
<td></td>
</tr>
<tr>
<td>Volume</td>
<td>4,000</td>
<td>6,400</td>
</tr>
<tr>
<td></td>
<td>26,400</td>
<td></td>
</tr>
</tbody>
</table>

Cost Variances

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials - Price</td>
<td>22,000</td>
<td></td>
</tr>
<tr>
<td>- Usage</td>
<td>3,000</td>
<td></td>
</tr>
<tr>
<td>Labour - Rate</td>
<td>6,800</td>
<td></td>
</tr>
<tr>
<td>- Efficiency</td>
<td>800</td>
<td></td>
</tr>
</tbody>
</table>
Variable Overhead
- Expenditure 2,600
- Efficiency 400

Fixed Overhead
- Expenditure 1,000
- Volume 2,000

\[
\begin{align*}
\text{Expenditure} & : 5,800 \\
\text{Volume} & : 32,800 \\
\text{Actual Loss} & : 27,000 \\
\end{align*}
\]

\[
\text{Actual Loss} = 600
\]

The listing of the variances explains why the Budgeted Profit was not achieved.

The actual loss can be proved as follows:

**Actual Result**

<table>
<thead>
<tr>
<th></th>
<th>(\text{₦})</th>
<th>(\text{₦})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>1,200 x 102</td>
<td>122,400</td>
</tr>
<tr>
<td>Cost of Sales</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>22,000 x 4</td>
<td>88,000</td>
</tr>
<tr>
<td>Labour</td>
<td>6,800 x 5</td>
<td>34,000</td>
</tr>
<tr>
<td>Variable Overhead</td>
<td></td>
<td>11,000</td>
</tr>
<tr>
<td>Fixed Overhead</td>
<td>6,000</td>
<td>139,000</td>
</tr>
<tr>
<td>less: Closing stock at standard cost</td>
<td>200 x 80</td>
<td>(16,000)</td>
</tr>
<tr>
<td><strong>Actual Loss</strong></td>
<td></td>
<td>(600)</td>
</tr>
</tbody>
</table>

**Sales Margin Mix Variance**

Where more than one product is sold, it is possible to isolate the effects on profit of a change in the volume due to proportions in which the products were sold, that is, the mix, and the absolute quantity difference. The sales margin volume variance can be analysed into sales margin mix variance and sales margin quantity (or yield) variance.

Two approaches are possible – the unit approach and the sales value approach. Though, the unit approach will be used in this illustration, it is warned that where the relative sales value of the products are significantly different, it will be advisable to use the sales value approach. This will be considered at the higher level of your study, that is, in the Management Accounting paper.
**ILLUSTRATION 9-4**

The sales budget of Surelocks Ltd for the month of April is as follows:

<table>
<thead>
<tr>
<th>Product</th>
<th>Budgeted Selling Price per unit</th>
<th>Standard Sales</th>
<th>Standard Margin on Budgeted Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>10,000</td>
<td>1,000</td>
</tr>
<tr>
<td>B</td>
<td>1.50</td>
<td>6,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

The Actual Sales in April were:

<table>
<thead>
<tr>
<th>Product</th>
<th>Quantity</th>
<th>Actual Selling Price/unit</th>
<th>Actual Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10,500</td>
<td>1.10</td>
<td>11,500</td>
</tr>
<tr>
<td>B</td>
<td>5,000</td>
<td>1.70</td>
<td>8,500</td>
</tr>
</tbody>
</table>

Reconcile Budgeted Margin with Actual margin for the month disclosing the Sales price and volume variances. Assume actual costs are as standard.

**SUGGESTED SOLUTION 9-4**

The approach is similar to that used in the calculation of the Material Usage, Mix and Yield Variances.

**Total Sales Margin Variances**

\[
\begin{align*}
\text{Actual Sales} & \quad \text{N} \quad 20,000 \\
\text{Budgeted Cost of actual sales} & \\
\text{A} \quad 10,500 \times \text{N}0.90 & \quad 9,450 \\
\text{B} \quad 5,000 \times \text{N}1.25 & \quad 6,250 \\
& \quad 15,700 \\
\text{Actual Margin} & \quad 4,300 \\
\text{Budgeted Margin} & \quad 2,000 \\
\text{Therefore, Total Sales Margin Variance} & \quad 2,350 \text{ F} \\
\end{align*}
\]

**Sales Price Variance**

\[
\begin{align*}
\text{Product} & \quad \text{Actual Qty Sold} \quad \text{Standard Price} \quad \text{Actual Price} \quad \text{Diff.} \quad \text{Sales Margin Price Variance} \\
\text{A} & \quad 10,500 \quad 1.00 \quad 1.10 \quad 0.10 \quad \text{1,050 F} \\
\text{B} & \quad 5,000 \quad 1.50 \quad 1.70 \quad 0.20 \quad \text{1,000 F} \\
& \quad \text{2,050 F} \\
\end{align*}
\]
### Sales Margin Volume Variance

<table>
<thead>
<tr>
<th></th>
<th>Budgeted Volume (units)</th>
<th>Actual Volume (units)</th>
<th>Diff. (units)</th>
<th>Standard Margin</th>
<th>Sales Margin Volume Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10,000</td>
<td>10,500</td>
<td>500</td>
<td>0.1</td>
<td>50 F</td>
</tr>
<tr>
<td>B</td>
<td>4,000</td>
<td>5,000</td>
<td>1,000</td>
<td>0.25</td>
<td>250 F, 300 F</td>
</tr>
</tbody>
</table>

### Sales Margin Mix Variance

<table>
<thead>
<tr>
<th></th>
<th>Actual Qty Sold @ Actual Mix Units</th>
<th>Actual Qty Sold @ Std. Mix Units</th>
<th>Diff.</th>
<th>Sales Margin</th>
<th>Sales Margin Mix Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10,500</td>
<td>11,071</td>
<td>(571)</td>
<td>0.1</td>
<td>(57) F</td>
</tr>
<tr>
<td>B</td>
<td>5,000</td>
<td>4,429</td>
<td>571</td>
<td>0.25</td>
<td>143 F</td>
</tr>
<tr>
<td></td>
<td>15,500</td>
<td>15,500</td>
<td>0</td>
<td></td>
<td>86 F</td>
</tr>
</tbody>
</table>

### Sales Margin Qty Variance

This is the residual sales margin volume variance after the mixture effect has been isolated. In the illustration 10-3

<table>
<thead>
<tr>
<th></th>
<th>Budgeted Volume (Units)</th>
<th>Actual Volume @ Standard Mix (Units)</th>
<th>Diff. (Units)</th>
<th>Standard Margin</th>
<th>Sales Margin Qty Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10,000</td>
<td>11,071</td>
<td>1,071</td>
<td>0.10</td>
<td>107 F</td>
</tr>
<tr>
<td>B</td>
<td>4,429</td>
<td>4,429</td>
<td>0</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>14,429</td>
<td>15,500</td>
<td>1,071</td>
<td></td>
<td>107 F</td>
</tr>
</tbody>
</table>

### 9.10 CONTROL RATIOS

These are ratios developed from the concept of the standard hour and are used in evaluating the performance of the work force without going into variance analysis.

#### 9.10.1 The Standard Hour

This is a hypothetical unit pre-established to represent the amount of work which should be performed in one hour at standard performance.
From this definition of the standard hour, it should be clear that the standard hour is not a measurement of time but of the work content of a clock hour.

The use of the standard hour provides a means for the comparison of the performance of employees engaged in the production of dissimilar products.

These can be expressed in standard hours and from there the following ratios developed:

(a) The Productivity (or Efficiency) Ratio;
(b) The Capacity Ratio; and
(c) The Production Volume Ratio (or Activity Ratio).

(a) *Productivity (or Efficiency) Ratio* is defined as the standard hours equivalent to the production achieved, whether completed or not, divided by (or expressed as a percentage of) the actual direct working hours.

that is, \[
\frac{\text{Standard hours of actual production}}{\text{Actual direct hours worked}} \times 100\%
\]

(b) *The Capacity Ratio* is defined as the actual number of direct working hours divided by (or expressed as a percentage of) the budgeted number of standard hours.

That is, \[
\frac{\text{Actual direct hours worked}}{\text{Budgeted standard hour}} \times 100\%
\]

(c) *The Production Volume Ratio*: This is the number of standard hours equivalent to the production achieved whether completed or not, divided by (or expressed as a percentage of) the budget number of standard hours.

That is,

\[
\frac{\text{Budgeted hours equivalent to actual production}}{\text{Budgeted standard hours}} \times 100\%
\]

9.11 **SUMMARY AND CONCLUSION**

Standard costing is a control technique which compares standards costs and revenues with actual results to obtain variances which are used to stimulate improved performance.

A standard cost is defined as a pre-determined calculation of how much costs should be under specified working conditions.
Possible standards include attainable, ideal, loose, basic and current. Material cost variance is the difference between the actual cost of materials consumed to achieve a given production level and what it should have cost at standard material cost.

Labour cost variance is the difference between the actual labour cost for the actual production in a given period and what labour should have cost if labour worked at normal efficiency and remunerated at the standard wage rate. Control ratios are used in evaluating the performance of a work force without going with variance analysis.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

9.12 REVISON QUESTIONS

9.12.1 MULTIPLE CHOICE QUESTIONS

1. Standard costing can be used for
   A. External reporting
   B. Internal reporting
   C. Internal and external reporting
   D. Stakeholders reporting
   E. Industry reporting

2. Fixed manufacturing overhead budget variance is calculated as the difference between
   A. Actual fixed manufacturing overhead and fixed manufacturing overhead budgeted for the budgeted production level
   B. Actual fixed manufacturing overhead and fixed manufacturing overhead budgeted for the production level achieved
   C. Actual fixed manufacturing overhead and fixed manufacturing overhead applied
   D. Fixed manufacturing overhead budgeted for the production level achieved and fixed manufacturing overhead applied
   E. Actual fixed manufacturing overhead of budgeted level and the budgeted fixed manufacturing overhead

3. If a company produces more units than expected production, there will be
   A. A favourable budget variance
   B. A favourable volume variance
   C. An unfavourable spending variance
   D. An unfavourable volume variance
   E. A favourable expenditure variance

4. During a period 3000kg of raw materials were purchased at a cost of N4 per kg. If there is an unfavourable direct materials price variance of N1500, the standard cost per kg must be
   A. N4.25
   B. N4.5
   C. N3.5
   D. N3.25
5. A standard marginal costing system:
   (i) Calculates fixed overhead variances using budgeted absorption rate per unit
   (ii) Records adverse variances as debit entries in variance accounts within the ledger
   (iii) Values finished goods inventories at standard variable cost of production

Which of the above statements is/are correct?
A. (i) and (ii) only
B. (ii) and (iii) only
C. (ii) only
D. (i) and (iii) only
E. (i), (ii) and (iii)

9.12.2 SHORT ANSWER QUESTIONS

1. The price variance of material AB was N1,000 (F) and usage variance was N300 (A), where standard material usage per unit is 3kg and standard material price N2 per Kg. If 500 units were produced in the period and opening and closing stocks of raw materials were 100kg and 400kg respectively:
   Material purchases in the period were..................kg

2. In a period 11,584 units were made with a standard labour allowance of 6.5 hours per unit at N5 per hour. Actual wages were N6 per hour and there was adverse efficiency variance of N36,000. Labour hours actually worked was ...........hrs.

3. A standard that is based on perfect working conditions is called...................

4. What is attainable standard?

5. What is Sales Margin Variance?

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
10

COST DATA FOR SHORT-RUN TACTICAL DECISION MAKING

10.0 LEARNING OBJECTIVES

After studying this chapter, readers will be able to understand the:

- Nature of managerial decision-making;
- Fundamental differences between absorption costing and marginal costing;
- Various managerial decision areas for marginal costing;
- Concept of Cost-Volume-Profit analysis and how to apply it for managerial decision making; and
- Necessity of cost accounting data in short-term tactical decision making.

10.1 INTRODUCTION

Management of an organisation has the responsibility of utilizing resources to generate revenues that is to be shared, among the stakeholders. To effectively manage the funds, with a view to satisfying all the stakeholders, management must think very seriously and carefully on all the decisions to be made. While the shareholders and creditors to a business – the primary stakeholders – base their decision making exercise on past events, managerial decision making is usually about the future; which is full of risks and uncertainties.

The cost accountant has a duty to supply management with cost figures that would allow for effective decision making in respect of short-term or long-term projects to be executed by the organisation. It is then that management would be able to minimise the negative effects of the risks and uncertainties inherent in the future, when the projects are to be executed.
In supplying the cost figures, the cost accountant has to adopt some techniques of cost accounting that are appropriate to the decision under consideration. The techniques that are mostly used for short-term tactical managerial decision making are marginal costing and cost-volume-profit analysis. This chapter discusses the two techniques, showing how they are to be built into managerial functions of planning, organising, directing and controlling for effective decisions to be made against the future. Sound management is, therefore, required for the costing techniques to be seen as effective decision making tools.

10.2 NATURE OF MANAGERIAL DECISION MAKING

Managerial decision making involves a number of steps necessary for the achievement of desired goals. The problem to be solved and the specific objectives to be achieved by the organisation must be clearly identified. The relevant facts and figures for the solution to the problem must be determined and obtained. The scope of operation and the validity of the factual knowledge must be understood.

Alternative courses of action are to be identified and evaluated based on their costs and benefits. Appraisal or evaluation would suggest ranking of these alternatives in order of importance or in order of decreasing benefit to the organisation.

Based on the ranking made, management can select an alternative project to be executed and be ready to bear its costs. Implementation or execution is to be effectively monitored and supervised by the management. At the end of the project period, management should compare the actual results with predetermined or standard results with a view to take appropriate action on the variance that occurred. Whatever action taken on the variance would amount to feedback for future managerial planning, decision making and control.

10.3 THE CONCEPT OF MARGINAL COSTING

Marginal costing has been described by different names on the two sides of the Atlantic. The term ‘Marginal costing’ is common in the United Kingdom (UK) and other countries of the European continent, while the expression ‘Direct Costing’ or ‘Variable Costing’ is preferred in the United States (US). The technique has been able to generate strong views both for and against in recent times.
'Marginal Cost', derived from the word ‘margin’, is a well-known concept of economic theory. Thus, in tune with the economic connotation of the term, it is described in simple words as the cost which arises from the production of additional increments of output and it does arise only if an additional increment is not produced.

It has been described by the Chartered Institute of Management Accountants, in its publication ‘A Report on Marginal Costing’, as the amount at any given volume of output by which aggregate cost is changed if the volume of output increase or decrease by one unit”. In the words of Chartered Accountants of England and Wales, “Marginal Cost is every expense (whether of production, selling or distribution) incurred by making a particular decision”.

From this point of view, marginal costs will be synonymous with variable cost, that is, prime costs and variable overheads, in the short run but, in a way, would also include fixed costs in planning production activities over a long period of time involving an increase in the productive capacity of the business. Thus, marginal costs are related to change in output under particular circumstance of a case.

According to the Chartered Institute of Management Accountants, “marginal costing is the ascertainment, by differentiating between fixed costs and variable costs, of marginal costs and of the effect on profit or changes in the volume and type of output”.

In this context, marginal costing is not a system of costing in the sense in which other systems of costing like process or job costing are but it has been designed simply as an approach to the presentation of accounting information meaningful to management from the viewpoint of adjudging the profitability of an enterprise by carefully studying the impact of the entire range of costs according to their respective nature.

The concept of marginal costing is a formal recognition of ideas underlying flexible budgets, break-even analysis and/or cost-volume-profit relationships. It is an application of these relationships which involves a change in the conventional treatment of fixed overheads in relation to income determination.

The concept is based on the important distinction between product costs and period costs, the former being related to the volume of output and the latter to the period of time rather than the volume of production.
Marginal costing regards as products costs only those manufacturing costs which have a tendency to vary directly with the volume of output. This is in complete contrast to the conventional system of costing under which all manufacturing costs - fixed as well as variable - are treated as product costs. Thus, marginal costing necessitates the analysis of costs into fixed and variable. Even the semi-variable costs have to be closely and critically analysed in order to classify them into their fixed and variable components, depending upon whether they tend to remain fixed or vary with the output. In this way, marginal costing highlights the effect of costs on the level of output planned.

10.4 COMPARING ABSORPTION AND MARGINAL COSTING

Absorption costing is the technique of income statement preparation where all costs (both fixed and variable) are recognised right from production level up to sales of the goods or services produced. In other words, it is a technique of costing that views fixed cost as relevant for managerial decision making at various levels of operations. On the other hand, marginal costing, as described above, is conceptually the same with variable costing, where only variable costs are recognised in costing units of goods or services produced – fixed costs are assumed to remain unchanged no matter the level of activity.

Some fundamental differences could be identified when the two costing techniques are compared.

First, is the treatment of fixed production cost, where absorption costing recognises the cost as part of the cost of goods sold and part of cost of closing units while marginal costing treats it as a period cost which must be incurred whether or not there is production.

Second, the cost of closing stock under marginal costing would be made up of variable costs only as all fixed costs (production, administration and selling) must have been covered by contribution margin while under absorption costing, the cost of closing units would be inclusive of part of the fixed production costs not absorbed by the units of goods or services sold.

Preparing income statement using absorption costing would amount to the determination of gross profit – which is the difference between sales (turnover) and cost of sales – whereas in marginal costing income statement would amount to the determination of contribution margin – which is the difference between sales and total variable costs. This third difference between the two costing techniques is to show that,
except where all the units produced are sold or planned to be sold, the two income statements would not amount to same net income!

Absorption costing statement is the method being used in preparing income statement at the end of an accounting period by all the three forms of businesses – sole proprietorship, partnership and company. It is a very familiar technique to accounting students at all levels, even though some students might not know the name of the technique being used. Marginal costing technique is only suitable for managerial decision making, especially if the decisions to be made are short-term (tactical) in nature.

Comparative Income Statement Under Marginal Costing and Absorption Costing

**ILLUSTRATION 10-1**

Green-Grass Limited furnishes the following details for the year ended 31 December, 2002 for preparing the income statement of the year:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>1,000 units @ N10 per unit</td>
</tr>
<tr>
<td>Fixed Manufacturing costs</td>
<td>N2,200</td>
</tr>
<tr>
<td>Variable manufacturing cost</td>
<td>N6 per unit</td>
</tr>
<tr>
<td>Inventory at close</td>
<td>100 units</td>
</tr>
<tr>
<td>Fixed selling &amp; admin. exps.</td>
<td>N500</td>
</tr>
<tr>
<td>Variable selling &amp; admin. exps.</td>
<td>N400</td>
</tr>
</tbody>
</table>

**Required:**
Prepare the income statement of the company using:

(a) Absorption costing
(b) Marginal costing.

**SUGGESTED SOLUTION 10-1**

**GREEN-GRASS LIMITED**
**INCOME STATEMENT**
for the year ended 31 December, 2002

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales 1,000 units @ N10 each</td>
<td>N10,000</td>
</tr>
<tr>
<td>Less: Cost of sales:</td>
<td></td>
</tr>
<tr>
<td>Variable manufacturing cost</td>
<td></td>
</tr>
<tr>
<td>1,100 units @ N6 each</td>
<td>N6,600</td>
</tr>
<tr>
<td>Fixed manufacturing costs</td>
<td>N2,200</td>
</tr>
<tr>
<td></td>
<td>N8,800</td>
</tr>
</tbody>
</table>
Les: Inventory at close:
100 units @ ₦8 each  
Gross Margin or profit  
Less: Total selling & admin. exps.  
Net Income  

<table>
<thead>
<tr>
<th>Description</th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inventory at close</td>
<td>800</td>
</tr>
<tr>
<td>Gross Margin</td>
<td>8,000</td>
</tr>
<tr>
<td>Less: Total selling &amp; admin. exps.</td>
<td>900</td>
</tr>
<tr>
<td>Net Income</td>
<td>1,100</td>
</tr>
</tbody>
</table>

**Workings**

The Cost of inventory at close, as well as the cost of each unit of inventory at close, has been calculated as follows:

- Units manufactured: 1,100
- Units of inventory at close: 100

**Ratio of closing inventory to total production**

$$\text{Ratio} = \frac{100}{1,100} = \frac{1}{11}$$

**Cost of Inventory at close**

$$\text{Cost of Inventory at close} = \frac{1}{11} \times ₦8,800 = ₦800$$

Thus, the cost of each unit of inventory at close:

$$\text{Cost per unit} = \frac{₦800}{100 \text{ units}} = ₦8 \text{ per unit}$$

**Under Marginal Costing**

Sales: 1,000 units @ ₦10 each  
Less: variable cost of sales:
Variable manufacturing cost  
Less: Inventory at close:
100 units @ ₦6 each  
Variable Gross Margin  
Less: Variable selling & Admin. exps.  
Operating contribution margin  
Less: Fixed Costs:
Fixed manufacturing costs  
Fixed selling & admin. exps.  
Net Income  

<table>
<thead>
<tr>
<th>Description</th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales:</td>
<td>10,000</td>
</tr>
<tr>
<td>Less: variable cost of sales:</td>
<td></td>
</tr>
<tr>
<td>Variable manufacturing cost</td>
<td>6,600</td>
</tr>
<tr>
<td>Less: Inventory at close:</td>
<td>600</td>
</tr>
<tr>
<td>Variable Gross Margin</td>
<td>4,000</td>
</tr>
<tr>
<td>Less: Variable selling &amp; Admin. exps.</td>
<td>400</td>
</tr>
<tr>
<td>Operating contribution margin</td>
<td>3,600</td>
</tr>
<tr>
<td>Less: Fixed Costs:</td>
<td></td>
</tr>
<tr>
<td>Fixed manufacturing costs</td>
<td>2,200</td>
</tr>
<tr>
<td>Fixed selling &amp; admin. exps.</td>
<td>500</td>
</tr>
<tr>
<td>Net Income</td>
<td>900</td>
</tr>
</tbody>
</table>

**Workings**

The cost of inventory at close, as well as the cost of each unit of inventory at close, have been calculated as follows:

**Ratio of closing inventory to total production**

$$\text{Ratio} = \frac{100}{1,100} = \frac{1}{11}$$
Cost of Inventory at close = \( \frac{1}{11} \times \text{₦6,600} = \text{₦600} \)

Thus, the cost of each unit of inventory at close

\[
= \frac{600}{100 \text{ units}} = \text{₦6 per unit}
\]

Note

The difference of ₦200 in the net income calculated under the two methods is due to the difference between the cost of closing inventory which, under absorption costing, is ₦800 and, under marginal costing, is ₦600.

10.5 THE CONCEPT OF CONTRIBUTION

Contribution is of vital important under marginal costing technique. The rationale of contribution lies in the fact that, where a business manufactures more than one product, the profit realized on individual products can not possibly be calculated due to the problem of apportionment of fixed costs to different products which is unaffected under marginal costing. Therefore, some methods are required for the treatment of fixed costs and marginal costing’s answer to the challenge is ‘contribution’, which covers the whole of fixed costs (production, administrative and selling expenses) in the course of determining net profit or net loss of the company as a whole.

Contribution is the difference between sales and the total variable cost and is, therefore, sometimes referred to as a ‘gross margin’. It is conceived as some sort of a ‘fund’ or ‘pool’ from which all fixed costs, irrespective of their nature, are to be covered and to which each product has to contribute its share. The difference between contribution and fixed costs is either net profit or net loss as the case may be.

The following illustration is on the concept of contribution and how it is applied in deciding as to whether or not to eliminate/close a product from a range. By extension, therefore, the illustration shows one of the short-term managerial decision making areas where marginal costing is applied.
ILLUSTRATION 10-2

The following information, organised on departmental basis, has been gathered from the accounts of Chuwa-Chuwa Engineering Limited:

<table>
<thead>
<tr>
<th>Departments</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable costs</td>
<td>90,000</td>
<td>100,000</td>
<td>150,000</td>
</tr>
<tr>
<td>Fixed costs: (apportioned on the basis of sales)</td>
<td>20,000</td>
<td>30,000</td>
<td>50,000</td>
</tr>
<tr>
<td>Total Costs</td>
<td>110,000</td>
<td>130,000</td>
<td>200,000</td>
</tr>
<tr>
<td>Sales</td>
<td>100,000</td>
<td>150,000</td>
<td>250,000</td>
</tr>
<tr>
<td>Profit (Loss)</td>
<td>(10,000)</td>
<td>20,000</td>
<td>50,000</td>
</tr>
</tbody>
</table>

With the object of doing away with departments incurring a loss, the management asks for your opinion on:

(a) The closure of Department X on the basis of the above information.
(b) The comparative profitability of different departments if specific fixed costs are ascertained to be ₹5,000 for Department X, ₹55,000 for Department Y and ₹30,000 for Department Z, the remaining ₹10,000 being general fixed costs.

Prepare appropriate statements so as to help management in arriving at a decision on the above points. Also give your comments, explaining the position presented in the statements.

SUGGESTED SOLUTION 10-2

(a) All fixed costs are regarded as general fixed costs:

**Marginal Cost Statement**

<table>
<thead>
<tr>
<th>Departments</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>100,000</td>
<td>150,000</td>
<td>250,000</td>
<td>500,000</td>
</tr>
<tr>
<td>Less: Variable cost</td>
<td>90,000</td>
<td>100,000</td>
<td>150,000</td>
<td>340,000</td>
</tr>
<tr>
<td>Contribution</td>
<td>10,000</td>
<td>50,000</td>
<td>100,000</td>
<td>160,000</td>
</tr>
<tr>
<td>Less: Fixed Costs</td>
<td>100,000</td>
<td></td>
<td></td>
<td>100,000</td>
</tr>
<tr>
<td>Profit</td>
<td></td>
<td></td>
<td></td>
<td>60,000</td>
</tr>
</tbody>
</table>
Comment

The loss of ₦10,000 shown by Department X has been due to the arbitrary basis of the apportionment of fixed overheads to different departments. The fact has been underlined by the marginal costing statement which does not attempt to allocate fixed costs to the various departments with the result that, in the event of the closure of Department X, the total contribution as well as profit would be reduced by ₦10,000. Therefore, the company would not stand to gain anything by closing Department X; rather, it would lose. The rule is simply that if a product is making positive contribution, it should not be eliminated.

(b) On the basis of specific fixed costs:

<table>
<thead>
<tr>
<th>Departments</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>₦100,000</td>
<td>₦150,000</td>
<td>₦250,000</td>
<td>₦500,000</td>
</tr>
<tr>
<td>Less: Variable cost</td>
<td>₦90,000</td>
<td>₦100,000</td>
<td>₦150,000</td>
<td>₦340,000</td>
</tr>
<tr>
<td>Contribution</td>
<td>₦10,000</td>
<td>₦50,000</td>
<td>₦100,000</td>
<td>₦160,000</td>
</tr>
<tr>
<td>Less: Specific fixed costs</td>
<td>₦5,000</td>
<td>₦55,000</td>
<td>₦30,000</td>
<td>₦90,000</td>
</tr>
<tr>
<td>Net contribution</td>
<td>₦5,000</td>
<td>₦55,000</td>
<td>₦30,000</td>
<td>₦90,000</td>
</tr>
<tr>
<td>Less: General Fixed costs</td>
<td>₦5,000</td>
<td>(₦5,000)</td>
<td>₦70,000</td>
<td>₦70,000</td>
</tr>
<tr>
<td>Profit</td>
<td>₦10,000</td>
<td></td>
<td></td>
<td>₦10,000</td>
</tr>
</tbody>
</table>

Comment

Although, the second statement shows the same profit of ₦60,000 as is shown by the first statement, it tells an entirely different story. From the above statement, it is Department Y which incurs a loss of ₦5,000 instead of Department X. This position has been brought about by the availability of fixed costs specifically assigned to different departments. It also highlights the anomalies created by the apportionment of fixed costs on some arbitrary basis, as according to sales in this example.

It is also evident that, if Department Y is closed or eliminated, the total contribution as well as profit would increase by ₦5,000. Advice on these lines would be tendered only if either the working of Department Y can not be improved so as to show a profit, or at least to break-even, or the price of its products can not be increased. Ideally, it is better from the point of view of business to offer a wider range of choice to its
customers. It is assumed that Product Y can be eliminated together with its specific fixed costs.

The position on the elimination of Department Y can be shown as follows:

<table>
<thead>
<tr>
<th>Statement of Comparative Profitability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departments</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>X</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Sales</td>
</tr>
<tr>
<td>Less: Variable costs</td>
</tr>
<tr>
<td>Contribution</td>
</tr>
<tr>
<td>Less: Specific fixed costs</td>
</tr>
<tr>
<td>Net Contribution</td>
</tr>
<tr>
<td>Less: General Fixed costs</td>
</tr>
<tr>
<td>Profit</td>
</tr>
</tbody>
</table>

Thus, profit increases by ₦5,000 on the elimination of Department Y, the amount being exactly equal to the loss incurred by that Department.

10.6 OTHER MANAGERIAL AREAS OF MARGINAL COSTING

10.6.1 Make or Buy Decisions

This type of situation arises when a manufacturer is faced with the decision as to whether:

(a) to manufacture one of its components in-house, or
(b) to buy such components from an outside supplier. The decision maker will be interested in the difference between the suppliers quotation and the cost of producing in-house. The cost of in-house production of the components is made up of:

(i) The incremental cost of production; and

(ii) Any opportunity cost that may arise from producing in-house.

The following qualitative factors will also be considered in deciding whether or not to produce in-house:
(a) The quality of the product that will be bought from outside;
(b) The reliability of the outside supplier;
(c) The effect of future market prices;
(d) Ability of management to cope with the present and future production capacity;
(e) The possible adverse effects of revealing trade secrets;
(f) The possible problems of transport and handling costs; and
(g) Government regulations, especially on import from overseas.

Decision Criteria

(a) If the outside supplier’s quotation is greater than the total variable cost of producing the component in-house, then the component should be manufactured in-house.

(b) If the total variable cost of in-house production is higher than the quotation of the outside supplier, then it will be quite discrete for the management to purchase the component from the supplier.

ILLUSTRATION 10-3

Nakowa Nigeria Plc is considering an opportunity to subcontract the production of a certain component part of its product to another manufacturer. The subcontractor has offered to produce the part for N1.80 per piece. The management accountant makes a study of the savings of cost resulting from subcontracting as follows:

(a) The standard unit cost of material is reduced by 50k
(b) The direct labour cost per unit is reduced by 25k
(c) The reduction in variable overhead is estimated at N1.00 per unit.

It is believed by the plant superintendent that supervision costs to the extent of N2,000 per year can be saved if the job is subcontracted and that the cost of special tools will be reduced by N500 per year.

The other relevant cost data in connection with the manufacture of this product are as follows:
Units sold 14,000

Net sales - 14,000 @ N25 each 350,000

Standard cost of goods sold:
- Direct materials 70,000
- Direct labour 28,000
- Factory overheads 140,000
Total Standard cost of goods sold 238,000

Standard Gross Margin 112,000
Less: Unabsorbed factory overheads 36,000
Actual Gross margin 76,000
Less: Selling and distribution expenses 100,000
Net Loss (24,000)

The break-up of factory overheads into their fixed and variable components at normal capacity level of 20,000 units is as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>Per Unit</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>4</td>
<td>80,000</td>
</tr>
<tr>
<td>Fixed</td>
<td>6</td>
<td>120,000</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>200,000</td>
</tr>
</tbody>
</table>

The company is working at less than normal capacity which is manufacture of 20,000 units of this product.

You are required to advise management on whether to subcontract the production of the component or to produce it internally.
SUGGESTED SOLUTION 10-3

NAKOWA NIGERIA PLC

Solution to this problem may best be presented in form of differential cost analysis as follows:

<table>
<thead>
<tr>
<th></th>
<th>Make</th>
<th>Difference</th>
<th>Buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net sales</td>
<td>N 350,000</td>
<td></td>
<td>N 350,000</td>
</tr>
<tr>
<td>Dir. Materials</td>
<td>70,000</td>
<td>(14,000 x N 0.50), i.e. – 7,000</td>
<td>63,000</td>
</tr>
<tr>
<td>Purchased parts</td>
<td>–</td>
<td>(14,000 x N 1.80), i.e. + 25,200</td>
<td>25,200</td>
</tr>
<tr>
<td>Direct labour</td>
<td>28,000</td>
<td>(14,000 x N 0.25), i.e. – 3,500</td>
<td>24,500</td>
</tr>
<tr>
<td>Var. overheads</td>
<td>56,000</td>
<td>(14,000 x N 1), i.e. – 14,000</td>
<td>42,000</td>
</tr>
<tr>
<td>Fixed ohd's</td>
<td>120,000</td>
<td>(N 2,000 + N 500), i.e. – 2,500</td>
<td>117,500</td>
</tr>
<tr>
<td>Total manuf. Cost</td>
<td>274,000</td>
<td>– 1,800</td>
<td>272,200</td>
</tr>
<tr>
<td>Selling &amp; Admin. cost</td>
<td>100,000</td>
<td></td>
<td>100,000</td>
</tr>
<tr>
<td></td>
<td>374,000</td>
<td>– 1,800</td>
<td>372,200</td>
</tr>
<tr>
<td>Net sales</td>
<td>(24,000)</td>
<td>– 1,800</td>
<td>(22,200)</td>
</tr>
</tbody>
</table>

It is to be noted that no change in volume or selling price is assumed so that revenue is not a relevant factor. The benefit lies purely in cost saving which at this level of volume is N 1,800. Despite the fact that the company is operating at less than normal capacity, the small monetary advantage offered by this opportunity may not be considered sufficient to offset the ill-effects of laying off more of the labour force due to the non-manufacture of the component. Inspite of the impact of non-quantitative factors, the differential cost analysis remains at the core of make or buy decision.

10.6.2 Mix of Sales

Usually, business enterprises have a variety of product lines, each making its own contribution to the coverage of fixed expenses. Changes in the operating profit can result from shifts in the mixture of products sold inspite of the fact that sales prices are unchanged and the total volume of sales expressed in terms of money remains the same. Such a situation may lead to changes in distribution channels, or sales to different classes of customers, as this affects the
quantum of contribution over variable costs. It is in this context that marginal costing is applicable in informing management regarding the most profitable mix of sales from the entire range of selected alternatives.

**ILLUSTRATION 10-4**

<table>
<thead>
<tr>
<th></th>
<th>Per unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct materials:</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>₦8</td>
</tr>
<tr>
<td>Y</td>
<td>₦6</td>
</tr>
<tr>
<td>Direct wages:</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>24 hours @ 25k per hour</td>
</tr>
<tr>
<td>Y</td>
<td>16 hours @ 25k per hour</td>
</tr>
<tr>
<td>Variable overheads</td>
<td>150% of wages</td>
</tr>
<tr>
<td>Fixed overheads</td>
<td>₦750</td>
</tr>
<tr>
<td>Selling price:</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>₦25</td>
</tr>
<tr>
<td>Y</td>
<td>₦20</td>
</tr>
</tbody>
</table>

The directors want to be advised on the desirability of adopting any one of the following alternative sales mixes in the budget for the next period:
(a) 250 units of X and 250 units of Y;
(b) 400 units of Y only;
(c) 400 units of X and 100 units of Y; and
(d) 150 units of X and 350 units of Y.

State which of the alternative sales mixes you would recommend to management.

*(Adopted I.C.W.A., London)*
SUGGESTED SOLUTION 10-4

Marginal Cost Statement
(Per Unit)

<table>
<thead>
<tr>
<th></th>
<th>Product X</th>
<th>Product Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct materials</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Direct wages</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Variable overheads</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Marginal cost</td>
<td>23</td>
<td>16</td>
</tr>
<tr>
<td>Contribution</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Selling price</td>
<td>25</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Selection of Sales Alternative</th>
<th>X</th>
<th>Y</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 250 units of X and</td>
<td>500</td>
<td>1,000</td>
<td>1,500</td>
</tr>
<tr>
<td>250 units of Y:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contribution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less: Fixed overheads</td>
<td></td>
<td></td>
<td>750</td>
</tr>
<tr>
<td>Profit</td>
<td></td>
<td></td>
<td>750</td>
</tr>
<tr>
<td>(b) 400 units of Y only:</td>
<td>1,600</td>
<td>1,600</td>
<td>750</td>
</tr>
<tr>
<td>Contribution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less: fixed overheads</td>
<td></td>
<td></td>
<td>750</td>
</tr>
<tr>
<td>Profit</td>
<td></td>
<td></td>
<td>850</td>
</tr>
<tr>
<td>(c) 400 units of X and</td>
<td>800</td>
<td>400</td>
<td>1,200</td>
</tr>
<tr>
<td>100 units of Y;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contribution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less: Fixed overhead</td>
<td></td>
<td></td>
<td>750</td>
</tr>
<tr>
<td>Profit</td>
<td></td>
<td></td>
<td>450</td>
</tr>
<tr>
<td>(d) 150 units of X and</td>
<td>300</td>
<td>1,400</td>
<td>1,700</td>
</tr>
<tr>
<td>350 units of Y:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contribution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less: Fixed overheads</td>
<td></td>
<td></td>
<td>750</td>
</tr>
<tr>
<td>Profit</td>
<td></td>
<td></td>
<td>950</td>
</tr>
</tbody>
</table>

As the fourth alternative of manufacturing 150 units of X and 350 units of Y yields the maximum profit and contribution, it is the best sale alternative and would, therefore, be recommended for adoption by the management.
10.6.3 Acceptance/Rejection of Special Order

This type of situation arises when a company receives an order from a customer at a price lower than its normal selling price. The company, if working below capacity, may be advised to accept the offer after taking into consideration the marginal cost of its production. Therefore if the answer to the following questions are in affirmative, the special price quoted by the customer should be accepted:

(a) Does the price quoted by the special customer cover the marginal cost of production?
(b) Does the company have excess capacity?

ILLUSTRATION 10-5

Shukura Nigeria Limited manufactures a special product for ladies called ‘the slimming stick’. A stick sells for 20k per unit. Current output is 400,000 sticks which represents 80% level of activity. A customer Gyaranya Stores Limited, recently, placed an order for 100,000 sticks at 13k per unit. The total cost for the period were ₦56,000 of which ₦16,000 were fixed costs. This represents a total cost of 14k per slimming stick.

Required:

(a) Based on the above information, advise Shukura Nigeria Limited whether or not to accept the offer.
(b) What other factors may be taken into consideration in taking this type of decision?

SUGGESTED SOLUTION 10-5

(a) SHUKURA NIGERIA LIMITED

Decision to accept/reject the offer:

<table>
<thead>
<tr>
<th>Description</th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (400,000 units)</td>
<td>80,000</td>
</tr>
<tr>
<td>Less: marginal cost (400,000 x 10k)</td>
<td>40,000</td>
</tr>
<tr>
<td>Contribution</td>
<td>40,000</td>
</tr>
<tr>
<td>Less: Fixed costs</td>
<td>16,000</td>
</tr>
<tr>
<td>Net Profit</td>
<td>24,000</td>
</tr>
</tbody>
</table>
(ii) If the order from Gyaranya Stores is accepted:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (100,000 x 13k)</td>
<td>N13,000</td>
</tr>
<tr>
<td>Less: marginal cost (100,000 x 10k)</td>
<td>N10,000</td>
</tr>
<tr>
<td>Contribution</td>
<td>N3,000</td>
</tr>
</tbody>
</table>

**Recommendation**

The Order from Gyaranya Stores should be accepted, since it will increase profit by N3,000.

(b) Other factors to consider include:

(i) Will the acceptance of one order at a lower than market price make other customers to ask for price reductions?

(ii) Is the so-called special price the most profitable way of utilizing the excess capacity?

(iii) If accepted, will the special order not block the acceptance of offers which may be at true market price?

(iv) Will fixed cost remain constant?

10.6.4 **Price Determination Based on Capacity to Sale**

The determination of prices of products manufactured or services rendered, by a business is often considered to be a difficult problem generally faced by management of an enterprise. However, the basic problem involved in pricing is the matching of demand and supply.

The marginal costing technique shows a simple relationship between specific products costs and the different possible selling prices being considered. This is due to the fact that contribution margin is unaffected by the allocation of indirect costs.

**ILLUSTRATION 10-6**

Mashasha Industries Limited produces and markets industrial containers and packing cases. Due to competition, the company proposes to reduce its selling price. As the present level of profit is to be maintained, indicate the number of units to be sold if the proposed reduction in selling price is 5%, 10% and 15%. The following information is available:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present sales turnover</td>
<td>N300,000</td>
</tr>
<tr>
<td>(30,000 units)</td>
<td></td>
</tr>
<tr>
<td>Variable cost</td>
<td>N180,000</td>
</tr>
<tr>
<td>(30,000 units)</td>
<td></td>
</tr>
<tr>
<td>Fixed cost</td>
<td>N70,000</td>
</tr>
<tr>
<td>Net profit</td>
<td>N50,000</td>
</tr>
</tbody>
</table>
MASHASHA INDUSTRIES LIMITED

Marginal Cost Statement

<table>
<thead>
<tr>
<th></th>
<th>Present Price</th>
<th>Price Reduction of 5%</th>
<th>Price Reduction of 10%</th>
<th>Price Reduction of 15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>N 300,000</td>
<td>N 285,000</td>
<td>N 270,000</td>
<td>N 255,000</td>
</tr>
<tr>
<td>Less: variable costs</td>
<td>N 180,000</td>
<td>N 180,000</td>
<td>N 180,000</td>
<td>N 180,000</td>
</tr>
<tr>
<td>Contribution</td>
<td>N 120,000</td>
<td>N 105,000</td>
<td>N 90,000</td>
<td>N 75,000</td>
</tr>
<tr>
<td>Less: Fixed costs</td>
<td>N 70,000</td>
<td>N 70,000</td>
<td>N 70,000</td>
<td>N 70,000</td>
</tr>
<tr>
<td>Net Profit</td>
<td>N 50,000</td>
<td>N 35,000</td>
<td>N 20,000</td>
<td>N 5,000</td>
</tr>
<tr>
<td>Contribution per unit</td>
<td>N 4.00</td>
<td>N 3.50</td>
<td>N 3.00</td>
<td>N 2.50</td>
</tr>
</tbody>
</table>

As profit is to be maintained at the present level of N 50,000, the contribution has to be N 120,000 as at present when the total sales proceeds from 30,000 units are N 300,000.

The number of units required to be sold at different levels of price reduction would be calculated as follows:

\[
\text{Total Contribution Required} = \frac{\text{Contribution per unit sold}}{
\]

Therefore:
- At 5% Reduction: \( \frac{\text{N120,000}}{\text{N3.50}} = 34,286 \text{ units (approximately)} \)
- At 10% Reduction: \( \frac{\text{N120,000}}{\text{N3}} = 40,000 \text{ units} \)
- At 15% Reduction: \( \frac{\text{N120,000}}{\text{N2.50}} = 48,000 \text{ units} \)

**Verification**

The result obtained at 10% price reduction can be verified as follows:

\[
\begin{align*}
\text{Sales (40,000 x 270,000)} &= 360,000 \\
\text{Less: variable costs (40,000 x 180,000)} &= 240,000 \\
\end{align*}
\]
COSTING AND QUANTITATIVE TECHNIQUES

<table>
<thead>
<tr>
<th>Contribution</th>
<th>120,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less: Fixed Costs</td>
<td>70,000</td>
</tr>
<tr>
<td>Profit</td>
<td>50,000</td>
</tr>
</tbody>
</table>

The other results can also be similarly verified. It should be noted, however, that the whole analysis depends on the capacity of the company to produce and sell at any of the three levels of production (34,286; 40,000 or 48,000).

10.6.5 Allocation of Scarce Resources

This is another area where marginal analysis may be adopted to good effect. All company resources – materials, plant capacity, management’s time, machine time or money – are limited (scarce) in supply when compared against their needs and some may be very scarce so much so that decisions affecting their allocation are important. Bad managerial decisions on their allocation may result in poor performance and reduced profitability. Therefore, if management is faced with scarcity of a factor of production (termed a limiting or key factor), it must ensure that the affected factor of production is used in such a way that profits per unit of the factor automatically secure maximum profit.

Scarcity of resources is an economic fact of life which is being faced by all the components of an economy – individuals, firms and governments. Marginal analysis or contribution analysis is the cost and management accounting response to this problem and, so, decisions on how to allocate any scarce resource should be based on marginal analysis otherwise the allocation would not amount to optimum result of performance.

ILLUSTRATION 10-7

Dadin Kowa Ltd manufactures and sells three products: P, Q and R. For the period ended 31 December, 2004, the following data were obtained from the company:

<table>
<thead>
<tr>
<th>Output/Sales volume units</th>
<th>P 4,000 units</th>
<th>Q 3,000 units</th>
<th>R 3,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling Price/unit</td>
<td>₦50</td>
<td>₦30</td>
<td>₦20</td>
</tr>
<tr>
<td>Variable cost/unit</td>
<td>₦15</td>
<td>₦20</td>
<td>₦15</td>
</tr>
</tbody>
</table>

Fixed cost of ₦100,000 are absorbed on the basis of labour hours which were: P (2,000), Q(1,500) and R(1,500).
Assuming the company is faced with reduced machine hours since its plant is getting old. It has been estimated that it takes:
5 machine hours to produce a unit of P
2 machine hours to produce a unit of Q
0.5 machine hour to produce a unit of R

The company is planning to replace its plant with a new one, but this can only be ready for operation in January, 2006. Before then the plant can only be used for 30 hours in a week instead of the normal 45 hours per week. Management wants to reconsider the allocation of this scarce resource. Advise management.

SUGGESTED SOLUTION 10-7

DADIN KOWA LIMITED

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output/Sales (in units)</td>
<td>4,000</td>
<td>3,000</td>
<td>3,000</td>
</tr>
<tr>
<td>Sales (Turnover)</td>
<td>N200,000</td>
<td>N90,000</td>
<td>N60,000</td>
</tr>
<tr>
<td>Less marginal (variable) costs</td>
<td>N60,000</td>
<td>N60,000</td>
<td>N45,000</td>
</tr>
<tr>
<td>Contribution</td>
<td>N140,000</td>
<td>N30,000</td>
<td>N15,000</td>
</tr>
<tr>
<td>Contribution per unit</td>
<td>= N140,000/4,000 = N35</td>
<td>N30,000/3,000 = N10</td>
<td>N15,000/3,000 = N5</td>
</tr>
<tr>
<td>Contribution per limiting factor (machine hour)</td>
<td>= N35/5 = N7</td>
<td>N30/2 = N5</td>
<td>N15/0.5 = N10</td>
</tr>
<tr>
<td>Ranking</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Comment

From the above relevant data, it is clear that if resources are concentrated on product R (other things being equal), the company will maximise profit. Let’s check. 30 hours in a week will provide the following results:

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contribution per machine hours</td>
<td>N7</td>
<td>N5</td>
<td>N10</td>
</tr>
<tr>
<td>Total Contribution</td>
<td>30 x N7 = N210</td>
<td>30 x N5 = N150</td>
<td>30 x N10 = N300</td>
</tr>
</tbody>
</table>

In the absence of ceiling (maximum level of production), the whole of the machine hours are to be allocated for the production of R if the company is to maximise profit.
10.7 IMPORTANCE OF MARGINAL COSTING

From the discussion and analyses in 12.5 and 12.6, it should be clear that marginal costing and contribution analyses are extremely useful tools for management in decision making and cost control, providing the decision makers with a very high degree of flexibility and scope for application in the areas of pricing, costing and resource allocation.

Companies producing two or more products or services and assessing the performance of the products or services based on profitability, might sometimes be contemplating as to whether to eliminate one product or the other or close one department or the other as a result of loss shown, using absorption costing income statement. As shown in illustration 10-2, the best guide to that decision area is to base the decision on marginal costing income statement, which would show the contribution of each of the products/services or departments. Any department or product/service that is making positive contribution should not be eliminated/closed. The contribution is assisting the company to cover its fixed costs and to make whatever profit being realised.

When a company receives quotation for the supply of a component being produced internally, the company may be contemplating as to whether to stop its manufacture and be buying from that outside supplier, especially if the supplier is ready to meet all the specifications of the company in terms of quality, quantity, lead time, compliance with government regulations, etc. The technique of marginal costing is to be used in getting relevant figures to be compared before taking this type of decision, as shown in illustration 10-3. On a simpler analysis, total variable cost of making part or component (fixed cost being a period cost), is to be compared against the total actual cost of purchasing it from outside and the option giving lower figure is to be adopted.

To select sales mix alternative, total contribution margins to be realised based on different proposals are to be determined. Since fixed costs to be covered are the same no matter the level of activity or the sales mix, the net profit/loss to be realised from the proposed sales mix would easily suggest the sales mix to be adopted, depending on capacity. This is shown in illustration 10-4.
To decide on the acceptance or rejection of a special order from a supplier, marginal costing and contribution analysis should be applied. The company may not have the capacity to cover existing demand of its products and satisfy the special order as well. It may have to cut down present level of production for it to meet the special order. This decision area has been demonstrated in illustration 10-5.

Sometimes, competition may force a business to reduce its selling price for it to survive in the market. In reducing price to make its products/services competitive, a business has to be conscious of the need for it to have the capacity to maintain its present level of contribution if not improve on it. Reduced selling price would amount to increase output and sales if contribution margin and profits are to be sustained. This short term tactical managerial decision area is the subject of discussion in illustration 10-6.

All resources to all the three components of an economy (individuals, firms and governments) are scarce when compared against their needs. Business have to adopt appropriate strategies of managing the money, material, time, etc. Resources available to them in their efforts to achieve set objectives. Budgeting and Budgetary control and marginal costing are techniques of managing scarcity, marginal costing is another. Illustration 10-7, shows how allocation of scarce resources should be done with a view to maximising profit by a business organisation.

10.8 COST-VOLUME-PROFIT ANALYSIS

Cost-Volume-Profit (CVP) analysis involves the analysis of how total costs, total revenues and total profits are related to sales volume, and is therefore concerned with predicting the effects of changes in costs and sales volume on profit.

CVP analysis seeks to find out answers to the following questions:

(a) What would be the cost of production under different circumstances?
(b) What has to be the volume of production?
(c) What profit can be earned?
(d) What is the difference between the selling price and cost of production?

If carefully used the technique may be helpful in:
(a) **Budget planning**: The volume of sales required to make a profit, that is, break-even point and the “safety margin” for profits in the budget can be measured;
(b) Pricing and sales volume decisions;
(c) In sales mix decisions, that is, in what proportions should each product be sold?
(d) Decisions affecting the cost structure and production capacity of the company.

An understanding of the inter-relationship between the three forces of cost, volume and profit, and of the probable effect that any change in sale volume would have on the business, is extremely helpful to management in a broad variety of issues involving planning and control. The relationship between cost, volume and profit makes up the profit structure of an enterprise. It is only through the knowledge and intelligent use of such information that the prediction of the probable effect of any number of contemplated actions is made possible. This makes Cost-Volume-Profit relationship to be of primary importance for budgeting and profit-planning.

The data to be used in the review of such relationships may come from several sources and may differ considerably in adaptability and usefulness. In companies where a rather complete sales analysis is made and flexible budgets and standard costs are available, the records would provide the necessary information in readily usable form. Costs, in all probability, will have been segregated into the fixed and variable elements. In case such sources are not available, the conventional historical records might be utilized. Quite detailed analysis may be necessary to isolate the effects of changes in volume, selling prices and variable cost. However, if cost control has been lax, the relationship between volume and cost will be difficult to trace and the margin of error will be high. Thus, the accuracy of results would depend greatly on the reliability of the data and the validity of assumptions. Very often, for investment and credit purposes, published financial statements are used as source data in studying the effects of volume on the business.

The study of Cost-Volume-Profit relationship is frequently referred to as ‘break-even analysis’. The term ‘break-even analysis’ may be interpreted in two senses - narrow sense and broad sense. In its narrow sense, it refers to a system of determining that level of operations where total revenues equal total expenses, that is, the point of zero profit. Taken in its broad sense, it denotes a system of analysis that can be used to determine the probable profit at any level of operations. This last view is what is considered throughout this chapter, as profit could be positive, negative or zero – depending on the trading report.
Cost-Volume-Profit data are based upon certain assumed conditions which are to be rarely found in practice. Some of these basic assumptions are as follows:

(a) The principle of cost variability is valid.
(b) Costs can be analysed into their fixed and variable components.
(c) Fixed costs remain constant.
(d) Variable costs vary proportionally with volume.
(e) Selling price does not change as volume changes.
(f) There is only one product or, in the case of multiple products, sale mix remains constant.
(g) There will be no change in general price level.
(h) Productivity per worker remains mostly unchanged.
(i) There is synchronization between production and sales.
(j) Revenue and costs are being compared with a common activity base, for example, sales value of production or units produced.
(k) The efficiency of plant can be predicted.

A change in any one of the above factors will alter the break-even point so that profits are affected by changes in factors other than volume. Thus, break-even chart must be interpreted in the light of above limitations and underlying assumptions, especially with respect to price and sale mix factors.

10.9 PRESENTATION OF BREAK-EVEN ANALYSIS

(a) Usually, ‘break-even analysis’ is presented graphically as this method of visual presentation is particularly well-suited to the needs of business owing to the manager being able to appraise the situation at a glance. Thus, it removes the danger accompanying many accounting reports, a danger that the reader would get bogged down with unnecessary details in such a way that he may never come to grip with the heart of the matter. The graphical break-even analysis eliminate the details and presents the information in a simplified way. To that extent, it is especially attractive for a person of non-accounting background. When presented graphically the break-even analysis takes the shape of ‘break-even’ chart or charts.

A break-even chart shows the profitability (or otherwise) of an undertaking at various levels of activity and, as a result, indicates the point at which neither profit is made nor loss is incurred. Break-even charts are frequently used and needed where a business is new or where it is experiencing trade
difficulties. In these cases, the chart assists management in considering the advantages and disadvantages of marginal sales. However, in a highly profitable enterprise, there is little need of break-even charts except when studying the implications of a major expansion scheme involving a heavy increase in fixed charges.

**ILLUSTRATION 10-8**

A company makes and sells a single product. The variable cost of production is N3 per unit, and the variable cost of selling is N1 per unit. Fixed costs totaled N6,000 and the unit sales price is N6. The company budgets to make and sell 3,600 units in the next year.

**Required:**
A break-even chart and a Profit-Volume graph, each showing the expected amount of output and sales required to break-even, and the safety margin in the budget.

**SUGGESTED SOLUTION 10-8**

A break-even chart records the amount of fixed costs, variable costs, total costs and total revenue at all volumes of sales and at a given sales price as follows:

**Figure 10.1  Break-even Chart**
The break-even point is where revenues and total costs are exactly the same, so that there is neither profit nor loss. It may be expressed either in terms of units of sales or in terms of sales revenue. Reading from the graph (above), the break-even point would be determined as 3,000 units of sale and N18,000 in sales revenue.

The Profit-Volume graph is shown in figure 10-2

**Margin of Safety (MOS)**

The margin of safety is the amount by which actual output/sales may fall short of the budget without incurring a loss, usually expressed as percentage of the budgeted sales volume. It is, therefore, a crude measure of the risk that the company might make a loss if it fails to achieve budget. In this illustration, the margin of safety is:

<table>
<thead>
<tr>
<th>Units</th>
<th>Budgeted sales 3,600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Break-even point</td>
<td>3,000</td>
</tr>
<tr>
<td>Margin of safety (MOS)</td>
<td>600</td>
</tr>
</tbody>
</table>

As a percentage of budgeted sales:

the MOS = \(\frac{600}{3,600 \times 100}\%\) = 16.7%

A high margin of safety indicates a good expectation of profit, even if budget is not achieved. In our illustration, it would not be clear without further investigation whether the safety margin in the budget is sufficient to indicate a good prospect of making profits. In other words, it would not be clear whether the forecast of 3,600 units of sale might be over optimistic by more than 16.67%.

**Profit Volume Chart (P/V)**

The profit/volume (P/V) chart, is similar to the break-even chart, and records the profit or loss at each level of sales at a given sale price. It is a straight line graph, drawn most simply by recording:

(a) The loss at zero sales, which is the full amount of fixed costs; and
(b) The profit (or loss) at the budgeted level of sales; and joining up the two points.
In our illustration, the profit volume graph would be:

**Figure 10.2  P/V Chart**

(a) The break-even point may be read from the graph as £18,000 in sales revenue, and the margin of safety is £3,600 in sales volume of 16.67% budgeted sales revenue.

(b) Another way of presenting information pertaining to cost volume profit relationship is by using simple formulae. This could take two forms:

(i) Equation technique, or
(ii) Contribution margin technique.

**The equation technique**

This technique uses the formulae which also expresses the relationship of the items of income statement.

\[
\text{Sales} = \text{Variable Expenses} + \text{Fixed Expenses} + \text{Profit}
\]

This simple equation may be adapted to any break-even or profit estimate situation.

**Contribution Margin or Marginal Income Technique**

This is obviously based on the concept of marginal costing. Contribution margin is the difference between sales and variable
expenses. Where break-even point is desired, sales and expenses are analysed thus:

Unit selling price – Unit variable expenses = Unit contribution to cover fixed expenses.

*This unit contribution is divided into total fixed expenses to secure the number of units which have to be sold to break-even.*

That is, \[
\text{Fixed expenses} = \frac{\text{Contribution per unit}}{}
\]

These two techniques can be illustrated by:

**ILLUSTRATION 10-9**

Alhaji Gazali plans to sell a toy rocket at Kano International Trade Fair. He may purchase these rockets at ₦5 each, with the privilege of returning all unsold rockets. The booth rent at the fair is ₦2,000 payable in advance. The rockets will be sold at ₦9 each.

Determine the number of rockets, which must be sold, to break-even as well as the number of rockets to be sold to yield a 20 percent operating margin on sales.

**SUGGESTED SOLUTION 10-9**

**ALHAJI GAZALI**

(a) **DETERMINATION OF ROCKETS TO BREAKEVEN**

Equation Technique

\[
\text{Sales} = \text{Variable costs} + \text{Fixed Costs} + \text{Profit}.
\]

Assuming that A is the number of units to be sold to break-even. The values in the above formula can be substituted as follows:

\[
\begin{align*}
9A &= 5A + ₦2,000 + 0 \\
9A - 5A &= ₦2,000 \\
\text{Therefore } 4A &= ₦2,000 \\
\text{Therefore } A &= ₦2,000/4 \\
\text{Therefore } A &= 500 \text{ units.}
\end{align*}
\]
(b) DETERMINATION OF ROCKETS TO YIELD 20% OF MARGIN OF SALES

Contribution Margin Technique
Unit selling price – Unit variable cost = Unit contribution to cover the fixed charges.

Substituting the values, we have:
₦9 – ₦5 = ₦4

Then:
\[
\frac{\text{Fixed cost}}{\text{Contribution per unit}} = \frac{\text{No of units to be sold to break-even}}{\text{₦2,000}} = 500 \text{ units.}
\]

Determination of volume at 20% profit on sales.
Sales = Variable costs + Fixed costs + Profit

Assuming that X is the number of units to be sold to yield desired profit, the values in the above formula can be substituted as:

\[
\begin{align*}
9X &= 5X + \text{₦2,000} + 0.2(9X) \\
9X &= 5X + \text{₦2,000} + 1.8X \\
9X - 1.8X - 5X &= \text{₦2,000} \\
2.2X &= \text{₦2,000} \\
X &= \text{909.09 units approximately.} \\
\text{or } X &= \text{910 units.}
\end{align*}
\]

OTHER FORMULA TO DETERMINE COST VOLUME PROFIT

Other simple formula that can be used to determine cost-volume-profit relationship are as follow:

(a) Break-even point (in ₦)

\[
\begin{align*}
&= \frac{\text{Fixed cost} \times \text{Sales price/unit}}{\text{Contribution per unit}} \\
&\text{or} \\
&\text{Fixed cost} \times \frac{1}{\text{C/S ratio}}
\end{align*}
\]
(b) \[ \text{C/S ratio} = \frac{\text{Contribution per unit}}{\text{Sales price per unit}} \times 100 \]

(c) \[ \text{Level of sales to result in target profit (in units)} = \frac{\text{Fixed cost} + \text{Target profit}}{\text{Contribution per unit}} \]

(d) \[ \text{Level of sales to result in target profit (in N)} = \frac{\text{Fixed cost} + \text{Target profit}}{\text{Contribution per unit}} \times \text{Sales price/unit} \]

(e) \[ \text{Level of sales to result in target profit after tax (in units)} \]

\[ \text{Target profit} = \frac{\text{Fixed cost} + (1 - \text{tax rate})}{\text{Contribution per unit}} \]

**Note**

The above formula are applicable only to single product firms or one with constant mix of sales. In case of a multi-product firms, the break-even point is calculated as follows:

\[ \text{Break-even point} = \frac{\text{Fixed cost}}{\text{Contribution}} \times \text{Sales value} \]

**ILLUSTRATION 10-10**

Umar Ibrahim Nigeria Limited manufactures and sells a unique product. The selling price of which is N20.00.

The summarized profit and loss statement for last year is as follows:

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>800,000</td>
<td></td>
</tr>
<tr>
<td>Direct materials</td>
<td>120,000</td>
<td></td>
</tr>
<tr>
<td>Direct wages</td>
<td>160,000</td>
<td></td>
</tr>
<tr>
<td>Variable production overhead</td>
<td>80,000</td>
<td></td>
</tr>
<tr>
<td>Fixed production overhead</td>
<td>100,000</td>
<td></td>
</tr>
<tr>
<td>Administration overhead</td>
<td>75,000</td>
<td></td>
</tr>
<tr>
<td>Selling and Distribution Overhead</td>
<td>60,000</td>
<td>595,000</td>
</tr>
<tr>
<td>Net Profit before tax</td>
<td>205,000</td>
<td></td>
</tr>
<tr>
<td>Less: provision for taxation (40%)</td>
<td>82,000</td>
<td>123,000</td>
</tr>
</tbody>
</table>
Required

(a) Calculate the break-even point in naira and in units for last year.

(b) What do you understand by the terms profit volume ratio and margin of safety. Illustrate using last year’s result.

(c) Determine the number of units to sell in the current year to achieve an after-tax profit of N150,000.

(d) Calculate the sales value required to achieve a net profit before tax of 15% of total revenue.

(e) Assuming no change in unit selling price and cost structure, calculate the percentage increase in sales volume required in the current year to produce a profit before tax of 20% higher than last year’s results.

(f) Calculate the selling price per unit that the company must charge in the current year to cover a potential increase of 12% in variable production costs this current year and still maintain last year’s contribution margin ratio.

(g) Determine the volume of sales (in N) that the company must achieve in the current year to maintain the same net profit of last year, if the selling price remains at N20 and variable cost per unit increases by 12%.

(h) Recalculate last year’s result if salesmen commission of 10% is introduced, selling price is reduced by 13% and volume increases by 30%.

SUGGESTED SOLUTION 10-10

(a) UMAR IBRAHIM NIGERIA LIMITED

(i) Break-even point (in N)

\[
\text{B/E point} = \frac{\text{Fixed Costs}}{\text{Contribution/unit}} \times \text{selling price/unit}
\]

\[
= \frac{\text{N235,000}}{11} \times \text{N20}
\]

\[
= \text{N427,273}
\]

(ii) Break-even point (in units)

\[
= \frac{\text{Fixed costs}}{\text{Contribution/unit}}
\]

\[
= \frac{\text{N235,000}}{11}
\]

\[
= 21,364 \text{ units}
\]
Workings

1. Calculation of fixed costs:
   - Production overhead: N100,000
   - Administration overhead: N75,000
   - Selling & Distribution overhead: N60,000
   - Total fixed costs: N235,000

2. Contribution per unit:
   - Selling price per unit - Variable cost per unit: N20 - N9 = N11

(b)(i) A profit volume (P/V) ratio indicates the relationship between contribution and revenue. It is otherwise referred to as contribution margin ratio (CMR) or contribution sales ratio (CSR).

It is calculated in the following ways:

- \( P/V \) ratio = \( \frac{\text{SP} - \text{VC}}{\text{SP}} \)
- \( P/V \) ratio = \( \frac{\text{CM}}{\text{SP}} \)
- \( P/V \) ratio = \( \frac{\text{TS} - \text{TVC}}{\text{TS}} \)
- \( P/V \) ratio = \( \frac{\text{Total contribution}}{\text{Total sales}} \)
- \( P/V \) ratio = \( \frac{\text{FC} + \text{Profit}}{\text{Total Sales}} \)

Calculation of P/V ratio:

\[
P/V \text{ ratio} = \frac{\text{CM}}{\text{SP}} = \frac{\text{N11}}{\text{N20}} = 0.55
\]

Interpretation

The P/V ratio of 0.55 means that for every one N sale generated 55 kobo would accrue as contribution margin.

(ii) **Margin of safety:** It shows the difference between the sales level and the break-even point. It could be expressed either in units or in monetary terms. It could also be expressed in relation to sales (in percentage term).
Calculation of Margin of Safety

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (B)</td>
<td>800,000</td>
<td>40,000</td>
</tr>
<tr>
<td>Break-even point</td>
<td>427,273</td>
<td>21,364</td>
</tr>
<tr>
<td>Margin of safety (A)</td>
<td>372,727</td>
<td>18,636</td>
</tr>
<tr>
<td>in percentage term:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A/B x 100%)</td>
<td>47%</td>
<td>47%</td>
</tr>
</tbody>
</table>

The margin of safety of 47% means that sales must fall by more than 47% before a loss is sustained.

(c) Level of sales required to achieve a target profit (after tax) of N150,000 (in units).

The answer is given by:

Target profit

\[
\frac{\text{FC} + \left( \frac{\text{Target Profit}}{(1 - \text{tax rate})} \right)}{\text{Contribution per unit}}
\]

\[
= \frac{N235,000 + \left( \frac{150,000}{1 - 0.4} \right)}{11}
\]

\[
= \frac{235,000 + 250,000}{11}
\]

\[
= 44,091 \text{ units}
\]

(d) Level of sales to result in net profit before tax of 15% of total revenue.

\[
Y = \frac{\text{Fixed Cost} + \text{Target profit}}{\text{Contribution margin ratio}}
\]

\[
Y = \frac{235,000 + 0.15Y}{0.55}
\]

\[
0.55Y = 235,000 + 0.15Y
\]

\[
0.55Y - 0.15Y = 235,000
\]

\[
0.40Y = 235,000
\]

\[
Y = \frac{235,000}{0.4}
\]

\[
= N587,500
\]
Workings

Contribution margin ratio is determined as follows:

\[
\text{Contribution margin ratio} = \frac{\text{Selling price} - \text{Variable cost}}{\text{Selling price}}
\]

\[
= \frac{20 - 9}{20}
\]

\[
= \frac{11}{20}
\]

\[
= 0.55
\]

\(e\) Level of sales to result in target profit.

\[
\text{Level of sales} = \frac{\text{Fixed costs} + \text{Target profit}}{\text{Contribution per unit}}
\]

\[
= \frac{\text{₦235,000} + (120\% \text{ of } \text{₦205,000})}{11}
\]

\[
= \frac{\text{₦235,000} + \text{₦246,000}}{11}
\]

\[
= \text{43,727 units}
\]

Percentage increase:

\[
= \frac{43,727 \text{ units} - 40,000 \text{ units} \times 100\%}{40,000 \text{ units}}
\]

\[
= 9.32\%
\]

\(f\) Contribution margin ratio of last year = contribution margin ratio of this year

\[
0.55 = \frac{\text{Revised SP} - \text{Revised VC per unit (RSP - RVC/U)}}{\text{Revised SP} \ (\text{RSP})}
\]

\[
0.55 \text{ RSP} = \text{RSP} - \text{RVC/U}
\]

\[
0.55 \text{ RSP} = \text{RSP} - (3 + 4 + (2 \times 1.12))
\]

\[
0.55 \text{ RSP} = \text{RSP} - 9.24
\]

\[
0.55 \text{ RSP} - \text{RSP} = -9.24
\]

\[
-0.45 \text{ RSP} = -9.24
\]

\[
\text{RSP} = -9.24
\]

\[
-0.45
\]

\[
= \text{₦20.53}
\]
(g) Sales value required to achieve a target profit of N205,000

\[ \text{Sales value} = \frac{\text{FC} + \text{TP}}{\text{Contribution/unit}} \times \text{Unit selling price} \]

\[ = \frac{N235,000 + 205,000 \times N20}{9.92} \]

\[ = N887,097 \]

**Workings**

Contribution per unit is determined as follows:

\[ \text{Contribution per unit} = \text{SP} - \text{RVC/U} \]

\[ = N20 - (N9 \times 1.12) \]

\[ = N20 - N10.08 = N9.92 \]

(h) Revised Income Statement

<table>
<thead>
<tr>
<th>Sales ((N20 x 0.87) x (40,000 x1.30))</th>
<th>904,800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less: Variable production</td>
<td>468,000</td>
</tr>
<tr>
<td>overhead (52,000 x 9)</td>
<td></td>
</tr>
<tr>
<td>Salesmen Comm. (10% of 904,800)</td>
<td>90,480</td>
</tr>
<tr>
<td>Total Contribution</td>
<td>558,480</td>
</tr>
<tr>
<td>Less: Fixed Costs</td>
<td>235,000</td>
</tr>
<tr>
<td>Net Profit before tax</td>
<td>111,320</td>
</tr>
<tr>
<td>Less: 30% tax</td>
<td>33,396</td>
</tr>
<tr>
<td>Net Profit after tax</td>
<td>77,924</td>
</tr>
</tbody>
</table>

10.10 **SUMMARY AND CONCLUSIONS**

In this chapter, to make readers would appreciate the nature of short-term tactical decisions that management of businesses are expected to make as managers of the funds entrusted to them. The decisions are futuristic and have to contend with the inherent risks and uncertainties to be faced. The fundamental differences between absorption costing and marginal costing – two techniques of income statements preparation – have discussed and illustrated. Short-term tactical decision areas like make or buy, allocation of scarce resources, closure or elimination of department/product, acceptance/rejection of offer, selling price determination, etc. that management of a business organisation must address were discussed, showing how marginal costing and contribution analysis should be applied as those decision areas are treated by the management. The concepts of Cost-Volume-
Profit Analysis and Break-Even Analysis were discussed and illustrated, graphically and mathematically.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

10.12 REVISION QUESTIONS

10.11.1 MULTIPLE CHOICE QUESTIONS

1. Which of the following cannot feature where others are present?
   A. Cash Account
   B. Cost ledger control account
   C. Debtors account
   D. Work in progress accounts.
   E. Stock account

2. During the last period, the profit figures of the cost accounts and financial accounts did not agree. The following were the stated differences:

<table>
<thead>
<tr>
<th>Cost Accounts</th>
<th>Financial Accounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening raw materials</td>
<td>13,000</td>
</tr>
<tr>
<td>Closing raw materials</td>
<td>17,000</td>
</tr>
<tr>
<td>Interest charged</td>
<td>-</td>
</tr>
<tr>
<td>Notional charges</td>
<td>5,500</td>
</tr>
<tr>
<td>Dividend income</td>
<td>-</td>
</tr>
</tbody>
</table>

   If the accounts were well prepared, the differences in gross profit should be:
   A. N\text{nil}
   B. N\text{5,500}
   C. N\text{14,000}
   D. N\text{19,500}
   E. N\text{16,000}

3. Notional charges in cost accounts such as interest on capital and charges in lieu of rent are meant to:
   A. Be kept in a bank account for future use
   B. Differentiate the cost accounts from the financial accounts
   C. Reduce distributable profit figure.
   D. Reward the capital providers and owners of building.
   E. Provide for replacement costs

4. Which account is not included in interlocking accounts?
   A. Finished Goods Account
   B. Debtors accounts
   C. Stock accounts
   D. Work in progress account.
   E. Raw material control account
5. Items treated in both the costing system and the financial system but at different amounts are all but:
   A. Differences in valuation of stock
   B. Under/Over recoveries when it is thought to prepare interim accounts.
   C. Differences in bases of charging depreciation.
   D. Interest received on bank deposits.
   E. Increases or Decreases in valuation of raw materials

10.11.2 SHORT ANSWER QUESTIONS

1. Reconciling the profits of both costing and financial accounts, mention two notional costs which may be found in the costing records only.

2. Highlight the fundamental differences between:
   a. integrated cost accounts
   b. Interlocking cost accounts.

3. State briefly the advantages and disadvantages of integrated cost accounts.

4. Mention the ledger contained in a control account.

5. Why is it necessary to keep a separate cost accounting system from financial accounting system?

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
11.0 LEARNING OBJECTIVES

After studying this Chapter, readers will be able to understand:

° The difference between cost control and cost reduction;
° The mechanisms for controlling and reducing costs; and
° Recent developments in cost accounting.

11.1 INTRODUCTION

For organisation goals to be achieved, activities must be controlled. If business entities must be profitable, costs must be maintained within acceptable limits. Cost control and cost reduction have been put forward by organisations in response to the considerable pressures of competition. Cost control and cost reduction, however, are different and so are their objectives. This distinction forms an important part of this chapter.

11.2 COST CONTROL

This refers to measures put in place by management to monitor operations such that actual costs remain within acceptable limits, that is, to ensure that cost incurred do not deviate significantly from those pre-planned or expected.

Cost control is an important objective of management to monitor operation as escalating costs may impair achievement of the targeted levels of profit.
11.2.1 Tools for cost control

Cost control involves instilling measures that continually monitor costs and indicates need for control action for effective management of resources. The process of cost control usually follows this sequence:

(a) Setting the acceptable or expected level of costs for various activities;
(b) Measuring actual cost of activities as they unfold;
(c) Comparing actual costs with those pre-determined; and
(d) Taking corrective action where necessary.

The settling of pre-determined level of costs to be compared with actual costs gives room for two basic tools to be employed in cost control:

(a) Budgetary control; and
(b) Standard costing.

These tools have been considered in earlier chapters, thus the discussion here is limited to a comparison of the two.

Differences Between Standard Costing and Budgetary Control

(a) Budgets are almost always stated in financial terms while standards need not be monetary. Example of non-financial standards, include material usage standards, standard hours etc.

(b) Budgets are usually focused on cost centres, that is, they aggregate the costs of cost centres while standards are usually set for the various activities.

Similarities between Standard Costing and Budgetary Control

(a) They both seek to maintain costs within stated limits.

(b) They involve careful estimates of how much cost should be in the relevant periods.

Although, it has been stated that budgetary control and standard costing are different, it should be noted that they are complementary rather than conflicting measures. Thus, they are
usually combined and used jointly for effective cost control in organisations.

11.3 COST REDUCTION

This is a planned positive action aimed at reducing costs of products or services without adversely affecting their quality or usability. While cost control is about keeping actual costs within acceptable limits, cost reduction maintains that even those pre-determined amount might be too high. Cost reduction is usually focused on the expected costs with a view to reducing the eventual cost of operations.

Organisations are constantly being faced with pressures from competition which tend to limit their sales revenue. For them to remain profitable, they are being forced to reduce costs.

11.3.1 TOOLS FOR COST REDUCTION

(a) Value Analysis

This is a systematic interdisciplinary examination of the factors affecting the cost of a product or service, with the aim of devising a means of achieving the desired purpose most economically, at the required standard of quality and reliability. In value analysis, the products or services are judged against three standards of value:

(i) Cost value: The objective of the exercise is to reduce costs. This affects all aspects of cost from production to distribution.

(ii) Usage value: This refers to the function and performance of the product, that is, what purpose was it meant to serve. Consideration is also given here to the required quality of the product. Cost reduction exercises are not expected to impair the usage value of the concerned products or services.

(iii) Esteem Value: This is value of aesthetics. It is the value placed on the product that is not derived from the function or performance of the product. For example, a consumer will value a gold plated wristwatch higher than the copper plated though they both accurately tell time. The esteem value of
products is also important in cost reduction and efforts are usually made to maintain or enhance the esteem value of products.

In some circumstances, however, an attempt to reduce cost may threaten the esteem value and management has to decide which is more important. The decision should be based on what the consumer is willing to pay for. There is no point spending money on the beauty of a product if its consumer will not pay extra for the beauty. In those cases where the consumer pays for the aesthetics, the esteem value should be maintained.

(b) **Total Quality Management (TQM)**

This is a term used to describe a management philosophy based on the continuous improvement of quality. It is an idea focused on the reduction or eventual elimination of the cost of re-working. TQM has at its roots two fundamental ideas:

(i) It is cheaper to get it right the first time; and

(ii) It may be possible to improve the current level of quality.

TQM as a cost reduction tool aims to reduce cost by reducing cost of damages and re-working.

(c) **Just-In-Time (JIT) Processes**

Just In Time processes have been discussed in an earlier chapter. Its aim is to reduce inventory levels and its attendant costs. JIT requires that products or materials are not acquired in anticipation of sales or use. Rather the materials and products are acquired only when needed. The use of JIT requires that production processes and suppliers are reliable. In some organisations, TQM is applied in conjunction with JIT.

(d) **Work Study**

This is a critical analysis of the method of work carried out by examining the various stages of a production line with the aim of finding the most efficient means of production
and ensuring that only that means is employed. In this age of competition, inefficient organisations are clearly at a loss. Work study seeks out efficient operations. Efficiency in operations will ultimately reduce cost of operations. Thus work study may be viewed as a cost reduction technique.

Work study has two (2) categories

(i) **Method Study**

This is an examination of the various methods and sequence of tasks to identify the best combination of steps or stages in the production process.

(ii) **Work Measurement**

Having identified the required combination of stages, work measurement sets out to measure the time and cost required to carry out the various steps.

(e) **Others**

Any real cost reduction exercise is a combination of various ideas. Other principles that could be applied to achieve the cost reduction objective include:

(i) Organisation and methods;
(ii) Standardisation;
(iii) Simplification;
(iv) Production planning and control; and
(v) Rationalisation.

It should be noted that cost reduction exercises should consider the organisation as a whole to ensure that attempts to reduce costs in one area do not ultimately increase costs for the entire organisation as this will be suboptimal.

11.4 DEVELOPMENTS IN COST ACCOUNTING

(a) **Back flush Accounting**

Traditional accounting for stock traces material items from purchases through work in progress (WIP) to finished goods and
costs collected with documents generated through the processes. Back flush accounting attempts to avoid this strict process and works back from output allocating costs between cost of sales and inventories.

The principles of back flush accounting are ideal for those situations where inventory valuation is less important. A good example could be where the Just In Time philosophy has been adopted. The elimination of the strict valuation rules from acquisition to work in progress to finished goods will translate to cost savings for the organisation.

(b) **Life Cycle Costing**

Life cycle costing involves estimation of the total costs as well as the expected revenues of a product over its entire life so as to determine whether or not the costs to be committed now and in the future would be recovered. Life cycle costing assesses the profitability of a product over its market life. It is recognized that in product design certain amount of costs may be committed into research and development. Life cycle costing attempts to link this cost to the other incidental costs of the product and then comparing with the expected revenue to enhance a decision as to whether or not a product is worth producing at all in the first place.

(c) **Target Costing**

Target costing sets the acceptable limits of cost of a product even before the production is embarked upon. Target costing usually involves the following processes:

(i) Identify a product that may be appealing to the market;
(ii) Estimate possible market price based on closely related products adjusting for the special features of the products; and
(iii) Deduct expected profit margin from expected market price to arrive at target cost.

Production is thus embarked upon when expected production costs looking at the production plan is within the ambits of its target cost. This is the only way such product could reasonably be expected to generate the required profit margin. Target costing could be employed in those industries characterized by
innovation, quick changes in taste and short product lives. Examples of such industries include fashion and clothing industry and the automobile industry.

(d) **Advanced Manufacturing Techniques (AMT)**

Advanced manufacturing techniques generally refer to the use of advanced technology and automation in production systems. It involves the use of computers in product design, production control and various stages of the production process. AMT involves the adoption of various other techniques which include:

(i) Computer aided design (CAD).
(ii) Computer aided manufacturing (CAM), etc.

(e) **Business Process Re-engineering**

This challenges the status quo in the organisation, attempting to re-design a new production system by identifying what the production system should be, given the level of quality of product or service. It is carried on by asking ‘why’ and ‘what if’ questions for every activity throughout an organisation. Its object is to design a more efficient production system thus requiring improvements on the current system.

11.5 SUMMARY AND CONCLUSIONS

Cost control refers to measures put in place by management to monitor operations such that actual costs remain within acceptable limits.

Tools for cost reduction include value analysis, total quality management, just in time processes and work study.

Developments in cost accounting include Back flush accounts, life cycle costing, target costing, advanced manufacturing technique and business process re-engineering.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

11.6 REVISION QUESTIONS

11.6.1 MULTIPLE CHOICE QUESTIONS

1. The systematic inter disciplinary examination of the factors affecting the cost of a product for the achievement of desired purpose most economically is:
   A. Total Quality Management
   B. Total Cost Management
COST CONTROL AND COST REDUCTION

C. Value Analysis
D. Just-in-Time purchases.
E. Job Grading

2. The Critical analysis of method of work carried out by the examination of the various production line for the determination of the most efficient means is:
   A. Materials Resource planning
   B. Materials Requisition planning
   C. Work study
   D. Enterprise Resource planning
   E. Job Measurement

3. The system that sets the acceptable limit of cost of a product even before the production is embarked on is:
   A. Target Costing
   B. Budget
   C. Standard Costing
   D. Activity Based Costing
   E. Process Costing

4. The use of high technology and automation in production system which involves the use of computers in product design and production control is:
   A. Program Code Analysis
   B. Business Process Re-engineering
   C. Back flush Accounting
   D. Advanced Manufacturing Technique.
   E. Value Analysis

5. The system of maintaining costs within acceptable limit which involves all measures taken and all methods used to regulate cost of operating a business is:
   A. Cost Reduction
   B. Value Analysis
   C. Budgetary Control
   D. Cost Control
   E. Cost Minimisation

11.6.2 SHORT ANSWER QUESTIONS

1. What is Backflush Accounting?
2. What is life cycle costing?
3. Briefly describe materials requirement planning.
4. Explain Business Process Re-engineering
5. Explain the differences between Standard Costing and Budgetary Control.

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
12.0 LEARNING OBJECTIVES

After studying this Chapter, readers will be able to understand:

- Interlocking System of Accounts, and integral or Integrated System of Account;
- Reconciliation of Cost and Financial Accounts profits;
- Re-creation of the Cost Records; and
- Treatment of Notional Charges.

12.1 INTRODUCTION

The double-entry principles applied in financial accounting can also be extended to the cost accounts system. It will, however, be observed that information required to implement a cost accounting system are quite often more detailed than those available from the financial accounting system. That is why it is necessary to maintain a cost accounting system separate from the financial accounting system (called the Interlocking System) or to have a more sophisticated system where a single record is maintained for both financial and cost accounting transactions known as the integrated accounting system. This latter system is facilitated by the technological advancement in information technology.

The operation of either interlocking or integral system is quite simple if the double entry principles of accounting are kept in mind, except that under the interlocking system, the cost ledger is independent of the financial ledger while under the integrated system, there is only one ledger. The level of integration of the cost and financial ledgers can also be restricted.
12.2 INTERLOCKING SYSTEM OF ACCOUNTS

Under this system of cost accounting, separate ledgers are maintained for the Financial accounting system and the Cost accounting system.

The cost accounting system makes use of the information from the financial accounting system but at greater level of details. For example, whereas the financial accounting system may simply debit raw materials consumed to a WIP account (or even just a purchases accounts as some systems do), the cost accountant will obtain more details about the materials through the Materials Requisition Notes to debit WIP account and the individual Job accounts in the cost ledger. It will also be recorded in such a way that the total of the individual Job accounts can be reconciled with the Control accounts for all the jobs (the WIP account). Transfers between jobs are also recorded by the cost accounting system while this may be ignored by the financial accounting system.

The same principles are also extended to labour cost through payroll analysis, time sheet analysis, etc, and also especially to overheads through the use of overhead recovery rates.

12.2.1 THE RECORDS

The principal Ledger in an interlocking system is the Cost Ledger. A Control Account as defined by CIMA is a total account inserted in a ledger (or section of accounts) to make it self-balancing. When debits or credits are posted to individual ledger accounts, their total is posted to the control account. The balance of the control account should equal the total of balances on the individual accounts.

This ledger will contain the following Control Accounts:
(a) Stores Ledger Control Account;
(b) W-I-P Control Account;
(c) Finished Goods Control Account; and
(d) Overhead Control Account.

Additional Accounts which will be closed at the end of each accounting period, by transfer to other accounts or the costing Profit and Loss account are:
(a) Wages control a/c;
(b) Sales control a/c;
(c) Cost of sales control a/c; and
(d) Notional Expense a/c.
A Cost Ledger Control Account or Cost Ledger Adjustment account is also maintained to make the Cost Ledger self-balancing.

**Principal Ledger, that is, Cost Ledger, contains:**

| Cost Ledger Adjustment Account | Raw Materials or Stores Ledger Control Account |
| W – I – P Control count | Finished Goods Control Account |

**Other Accounts are:**

| Sales Control Account | Cost of Sales Control Account |
| Wages Control Account | Production Overhead Control Account |
| Notional Expense Account | Admin Overhead Control Account |
| Costing P&L Account | Selling & Distribution Overhead |
The focus of a cost accounting system under the interlocking system is to determine the cost of production, valuation of stock, and the determination of profit on manufacture. Hence, personal accounts (like debtors and creditors) and real accounts (like fixed assets) are not maintained with the cost accounting system. Where, therefore, there is a transaction affecting those accounts, the corresponding entry to complete the double entry will be made through the cost ledger adjustment account.

### 12.2.2 TYPICAL ACCOUNTING ENTRIES

(a) **Raw Materials Purchased for cash** N300,000

<table>
<thead>
<tr>
<th>Dr</th>
<th>Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Material Stock Account 300,000</td>
<td>Cost Ledger Adjustment A/c 300,000</td>
</tr>
</tbody>
</table>

(b) From analysis of materials issued, it was discovered that Raw Materials worth N220,000 were issued to production, entries will be passed thus:

<table>
<thead>
<tr>
<th>Dr</th>
<th>Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work-In-Progress Control a/c 220,000</td>
<td>Raw Materials Control a/c 220,000</td>
</tr>
</tbody>
</table>

If a mental review is carried out at this stage, it will be seen that the Raw materials will have a balance of N80,000 representing the value of Raw Materials in stock.

(c) **Wages paid through Bank** N120,000

<table>
<thead>
<tr>
<th>Dr</th>
<th>Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages Control a/c 100,000</td>
<td>Cost Ledger Adj. A/c 100,000</td>
</tr>
</tbody>
</table>

(d) Upon analysis of the Wages paid, it was discovered that 80% related to production, while the balance related to Administration

Entries will be to:

<table>
<thead>
<tr>
<th>Dr</th>
<th>Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-I-P Account 80,000</td>
<td>Admin Overhead Account 20,000</td>
</tr>
<tr>
<td>Wages Control Account 100,000</td>
<td></td>
</tr>
</tbody>
</table>
(e) Upon determination of the appropriate overhead to be absorbed into product cost using the pre-determined absorption rates, it was discovered that overhead to be absorbed is ₦78,750.

\[
\begin{array}{ll}
\text{Dr} & \text{W-I-P Account} \quad 78,750 \\
\text{Cr} & \text{Production Overhead Control} \quad 78,750 \\
\end{array}
\]

The production overhead account will now be left with a balance of ₦1,250 representing under-absorbed overhead. This may be carried forward as a balance in that account.

(f) If the WIP account is extracted from these entries, it will appear as follows:

**W-I-P Control account**

\[
\begin{array}{ll}
\text{₦} \\
\text{Raw Material} & 220,000 \\
\text{Wages} & 80,000 \\
\text{Production. Overhead} & 78,750 \\
\end{array}
\]

If all the goods processed in the period were completed, then the total on this account will represent the production value of the goods produced, and a transfer made from this account to the Finished Goods Accounts, thus:

\[
\begin{array}{ll}
\text{Dr} & \text{Finished Goods Stock Account} \\
\text{Cr} & \text{WIP Control Account}. \\
\end{array}
\]

(g) When these goods are sold, or a proportion thereof, a transfer will be made from the finished goods stock account to the cost of sales account, and from there to the Costing Profit and Loss account.

(h) A sales account will be opened only for the purposes of ascertaining the profit for the period, as it will be closed at the end of the period to the costing profit and loss account. The costing profit and loss account will similarly be closed by a transfer to the cost ledger control (Adjustment) account, thus leaving us with the four principal control accounts earlier on mentioned.
From the above explanations, it should be clear that the cost accounting system is only a record in double-entry terms of the same principles of cost ascertainment, valuation of stock and profit measurement.

12.3 NOTIONAL ITEMS

Quite often, in many organisations, especially the small ones, certain items are provided by the entrepreneur, for example, capital, instead of borrowed capital or bank loan, and own building instead of a rented structure. Consequently, no payment is made for the use of these factors of production.

In the financial accounting system, no expense will be recorded in the accounts for these items as the accounting concept of objectivity would not permit recording of such notional expenses. However, the objective of the cost accounting systems is to determine a realistic cost of production and should include these notional expenses, otherwise true cost and managerial efficiency cannot be truly or objectively ascertained and inter-firm comparison will also be of limited value.

12.4 RECONCILIATION OF COST AND FINANCIAL ACCOUNTS PROFITS

As earlier mentioned under the interlocking system, two separate ledgers are maintained. One, for the cost accounting system and another, for the financial accounting system.

It is also important to say that certain transactions are not recorded in the cost accounting system, even though, recorded in the financial accounting system. So, in the cost accounting system, certain items – notional charges – are recorded but not in the financial ledger. We are, however, not interested in all the differences except those that affect the reported profits. For example, since notional charges are both reported as charges and as income within the cost ledger, they have no impact on profit except as regards stock valuation.

We can, therefore, summarise those items that do cause a disagreement between the Financial Accounts profit and the Cost Ledger profit, as follows:
(a) **Items treated in both the Costing System and the Financial system but at different amounts:**

(i) Differences in valuation of stock and work-in-progress
(ii) Under/over recoveries of overhead when it is thought to prepare interim accounts but leave the treatment of over/under-absorbed overhead till the end of the period.
(iii) Differences in basis of charging depreciation between the cost accounts and the financial accounts.

(b) **Purely Financial Items**

These could be expenses or incomes that are regarded purely as non-operating because they are not related to the manufacturing/trading operation, for example:

(i) Interest received on bank deposits;
(ii) Rent receivable;
(iii) Profit or loss on sale of assets;
(iv) Fines and penalties paid;
(v) Donations;
(vi) Dividends paid;
(vii) Provision for bad debts; and
(viii) Appropriations of profits, such as, company income tax, transfers to general reserves and preliminary expenses written off.

It is, therefore, possible to simplify the topic of reconciliation of cost and financial accounts profits by suggesting the following format:

<table>
<thead>
<tr>
<th>XYZ COMPANY LTD</th>
</tr>
</thead>
<tbody>
<tr>
<td>RECONCILIATION OF COST AND FINANCIAL ACCOUNTS</td>
</tr>
<tr>
<td>PROFITS FOR THE PERIOD ENDED</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Add</th>
<th>Subtract</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

Profit per cost accounts  X
Items treated on both book that differ  X
Items treated in financial accounts  X
Profit per financial accounts  X
ILLUSTRATION 12-1

Onilari Manufacturing Company Ltd operated an interlocking system of accounting. For the month just ended the Financial Accountant produced an operating statement showing a profit of N12,000 while the statement prepared by the Cost Accountant showed a profit of N16,000.

During your investigation you ascertained the following:

a) Stock valuation

<table>
<thead>
<tr>
<th></th>
<th>Financial Account</th>
<th>Cost Account</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw materials</td>
<td>- Opening 20,000</td>
<td>16,000</td>
</tr>
<tr>
<td></td>
<td>- Closing 24,000</td>
<td>22,000</td>
</tr>
<tr>
<td>Finished good</td>
<td>- Opening 50,000</td>
<td>51,000</td>
</tr>
<tr>
<td></td>
<td>- Closing 60,000</td>
<td>61,000</td>
</tr>
</tbody>
</table>

b) The following items do not appear in the cost accounts:

<table>
<thead>
<tr>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Donation: Apapa Boat Club</td>
<td>700</td>
</tr>
<tr>
<td>SOS children school</td>
<td>300</td>
</tr>
<tr>
<td>Profit on sale of Motor vehicle</td>
<td>2,000</td>
</tr>
<tr>
<td>Exchange rate losses</td>
<td>2,500</td>
</tr>
<tr>
<td>Bad debts written off</td>
<td>500</td>
</tr>
<tr>
<td>Discounts allowed</td>
<td>500</td>
</tr>
<tr>
<td>Discounts received</td>
<td>400</td>
</tr>
<tr>
<td>Rents receivable</td>
<td>600</td>
</tr>
</tbody>
</table>

You are required to prepare a statement reconciling the two profit figures.

SUGGESTED SOLUTION 12-1

Onilari Manufacturing Company Ltd.
Reconciliation of Financial Profit and Costing Profit

<table>
<thead>
<tr>
<th></th>
<th>Add</th>
<th>Less</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit per financial accounts</td>
<td>12,000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Differences in Stock Valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Materials: Opening 4,000</td>
</tr>
<tr>
<td>Closing 2,000</td>
</tr>
</tbody>
</table>
Finished Good: 

<table>
<thead>
<tr>
<th></th>
<th>Opening</th>
<th>Closing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1,000</td>
<td>2,500</td>
</tr>
<tr>
<td></td>
<td>5,500</td>
<td>3,000</td>
</tr>
<tr>
<td></td>
<td>14,500</td>
<td></td>
</tr>
</tbody>
</table>

**Items treated in financial accounts only:**

- Donations (700+300) 1,000
- Exchange rate losses 2,500
- Discounts allowed 500
- Bad debts 500
- Profit on sale of motor vehicle 2,000
- Discounts received 400
- Rents receivable 600

**Profit per cost accounts** 16,000

---

### 12.5 INTEGRATED ACCOUNTS

The integrated accounts system (also called the integral system) is one in which only one ledger is maintained for both cost and financial accounts transactions, that is, the cost ledger and the financial ledger are integrated into one single ledger.

The Control Accounts, that is, for raw materials, work-in-progress, finished goods stock, production overhead, etc, would still be maintained, but the Real and Personal Accounts which were hitherto not maintained in the Cost Ledger under the Interlocking systems would now be maintained since the two systems have been integrated. Consequent upon this, the entries that were correspondingly passed to the Cost Ledger Control (or Adjustment) account would now be passed through those Real or Personal Accounts as appropriate that is, Bank, Debtors, Creditors, etc. It will no longer be necessary for the Cost Ledger Adjustment Account.

#### 12.5.1 ILLUSTRATION ENTRIES

A re-visit to the sample entries under the Interlocking System would be presented as follows in integrated system:

- **(a)** Purchases of raw material by cash will be recorded as:
  - Raw materials stock account DR
  - Cash account CR

- **(b)** Raw materials issued to production would still be recorded as:
W-I-P Control account   DR
Raw materials control account   CR

(c) Wages paid through Bank will be recorded as:
Wages control account   DR
Bank account   CR

Full accounting for overheads on the absorption basis would still be maintained with disposition of the over/under-absorbed overheads.

ILLUSTRATION 12-2

In the absence of the Accountant you have been asked to prepare a month’s cost accounts for Reid Limited which operates a batch costing system, fully integrated with the financial accounts. The cost clerk has provided you with the following information, which he thinks is relevant:

Balances at beginning of month:

<table>
<thead>
<tr>
<th>Account</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stores Ledger control account</td>
<td>24,175</td>
</tr>
<tr>
<td>Work in progress control account</td>
<td>19,210</td>
</tr>
<tr>
<td>Finished goods control account</td>
<td>34,164</td>
</tr>
<tr>
<td>Prepayments of production overheads brought forward from previous month</td>
<td>2,100</td>
</tr>
</tbody>
</table>

Transactions during the month:

<table>
<thead>
<tr>
<th>Transaction</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials purchased</td>
<td>76,150</td>
</tr>
<tr>
<td>Materials issued: to production</td>
<td>26,350</td>
</tr>
<tr>
<td>For factory maintenance</td>
<td>3,280</td>
</tr>
<tr>
<td>Material transferred between batches</td>
<td>1,450</td>
</tr>
<tr>
<td>Direct wages paid: net employee’s deductions</td>
<td>17,646</td>
</tr>
<tr>
<td>Indirect workers</td>
<td>3,342</td>
</tr>
<tr>
<td>Total wages paid: net employee’s deductions</td>
<td>4,364</td>
</tr>
<tr>
<td>Indirect workers</td>
<td>890</td>
</tr>
</tbody>
</table>

| Direct wages charged to batches from work tickets     | 15,236 |
| Recorded non-productive time of direct workers        | 5,230 |
| Direct wages incurred on production of capital equipment for use in the factory | 2,670 |
| Selling and distribution overheads incurred           | 5,240 |
| Other production overheads incurred                   | 12,200 |
| Sales                                                | 75,400 |
Cost of finished goods sold 59,830
Cost of goods completed and transferred into finished goods store during the month 62,130
Physical stock value of work in progress at end of month 24,360

The production overhead absorption rate is 150% of direct wages and it is the policy of the company to include a share of production overheads in the cost of capital equipment in the factory.

**Required:**

(a) Prepare the following accounts for the month:
   (i) stores ledger control account
   (ii) work in progress control account
   (iii) finished goods control account
   (iv) production overhead control account
   (v) profit/loss account.

(b) Identify any aspects of the accounts which you consider should be investigated.

(c) Explain why it is necessary to value a company’s stocks at the end of each period and also why in a manufacturing company, expense items such as factory rent, wages of direct operatives, power costs, etc are included in the value of work in progress and finished goods stocks.

**SUGGESTED SOLUTION 12-2**

**REID LIMITED**

<table>
<thead>
<tr>
<th></th>
<th>₹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening balances b/f</td>
<td>24,175</td>
</tr>
<tr>
<td>Creditors – materials</td>
<td>76,150</td>
</tr>
<tr>
<td>Purchased</td>
<td>100,325</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>₹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials issued</td>
<td>26,350</td>
</tr>
<tr>
<td>Work in progress control</td>
<td>3,280</td>
</tr>
<tr>
<td>Production overhead control</td>
<td>70,695</td>
</tr>
<tr>
<td>Closing stock c/f</td>
<td>100,325</td>
</tr>
</tbody>
</table>
### (ii) WORK IN PROGRESS CONTROL ACCOUNT

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening balance b/f</td>
<td>19,210</td>
</tr>
<tr>
<td>Stores ledger - control</td>
<td></td>
</tr>
<tr>
<td>Materials issued</td>
<td>26,350</td>
</tr>
<tr>
<td>Wages control - cost</td>
<td></td>
</tr>
<tr>
<td>Direct wages</td>
<td>15,236</td>
</tr>
<tr>
<td>Production overhead control - overhead absorbed</td>
<td>22,854</td>
</tr>
<tr>
<td>P&amp;L account – stock gain</td>
<td>2,840</td>
</tr>
<tr>
<td>Closing stock c/f</td>
<td>24,360</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>86,490</strong></td>
</tr>
</tbody>
</table>

### (iii) FINISHED GOODS CONTROL ACCOUNT

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening balance b/f</td>
<td>34,164</td>
</tr>
<tr>
<td>Work in progress – cost of good sold</td>
<td>62,130</td>
</tr>
<tr>
<td>P&amp;L account – stock gain</td>
<td>2,840</td>
</tr>
<tr>
<td>Closing stock c/f</td>
<td>36,464</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>96,294</strong></td>
</tr>
</tbody>
</table>

### (iv) PRODUCTION OVERHEAD CONTROL ACCOUNT

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prepayments b/f</td>
<td>2,100</td>
</tr>
<tr>
<td>Stores ledger – materials issued for repairs</td>
<td>3,280</td>
</tr>
<tr>
<td>Wages control – idle time of direct workers</td>
<td>5,230</td>
</tr>
<tr>
<td>Wages control – indirect workers’ wages</td>
<td>(3,342 + 890)</td>
</tr>
<tr>
<td>Cash/creditors – other overheads incurred</td>
<td>12,200</td>
</tr>
<tr>
<td><strong>Overhead (balance)</strong></td>
<td><strong>183</strong></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>27,042</strong></td>
</tr>
</tbody>
</table>

### (v) PROFIT AND LOSS ACCOUNT

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finished goods control</td>
<td></td>
</tr>
<tr>
<td>Cost of goods sold</td>
<td>59,830</td>
</tr>
<tr>
<td>Gross profit c/f</td>
<td>45,570</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>75,400</strong></td>
</tr>
</tbody>
</table>

**Note:** All amounts are in Nigerian Naira (₦).
<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling and distribution overheads</td>
<td>5,240</td>
</tr>
<tr>
<td>Production overhead control – under absorbed overhead</td>
<td>183</td>
</tr>
<tr>
<td>Stock gain</td>
<td>–</td>
</tr>
<tr>
<td>WIP control</td>
<td>2,840</td>
</tr>
<tr>
<td>Net profit c/f</td>
<td>12,987</td>
</tr>
</tbody>
</table>

\[ \text{Gross profit b/f} + \text{Stock gain} - \text{WIP control} = \text{Net profit c/f} \]

\[ 15,570 + 0 - 2,840 = 12,987 \]

\[ \text{Net profit c/f} = 18,410 \]

(b) (i) Closing stock increased in all the three stages of production but prominent in the case of raw materials. The company must ensure that demand in the near future is sufficient to justify this high stock level or that measures are taken to reduce it.

(ii) Direct workers incurred idle time costing ₦5,230 in the month, nearly one-quarter of the total direct wages cost. Management must investigate the reasons for the non-productive time and prevent its recurrence.

(iii) The company must ensure that the value of the capital equipment on completion is not over-stated by the inclusion of production overheads.

(iv) The wages information does not reconcile. The reasons for this should be obtained. Possibly, too much time is being charged to batches or indeed, the payroll department has miscalculated wages payable.

(c) It is necessary to value a company’s stock at the end of each period for the following reasons:

(i) To derive a stock figure for the balance sheet;

(ii) To calculate the cost of stocks used or sold in the period and hence to calculate profit;

(iii) To have a value which can be compared with expected sales demand, budgeted production, etc to see if levels should be built up or run down;

(iv) To see if too much of a company’s capital is being tied up in its stocks; and

(v) To carry out stock calculation based on the stock-take, which will enable management to check the accuracy of its stock records in the cost accounts, or amend the stock records accordingly, transferring a stock gain or write-off to the profit and loss account to balance the accounts.
12.5.2 ADVANTAGES AND DISADVANTAGES OF THE INTEGRAL SYSTEMS

Advantages of Integral Systems
Some of the major advantages of the integral systems include the following:

(a) Duplication of records is avoided;
(b) There is no need to reconcile financial and cost accounts profits as there is now only one profit figure from one single system;
(c) It saves the administrative effort of maintaining two separate systems; and
(d) It will be unwise to stick to an old method when the advancement in information technology now makes it easier to combine with ease.

Disadvantages of Integral System

It could, however, still suffer from the disadvantage of not being able to satisfy the requirement for external reporting, for example, where stocks are valued on a basis different from historical costs.

12.6 RECREATING THE COST RECORDS

Sometimes, examination questions require accounts (could be under interlocking or integral) to be written from incomplete information. This only requires a proper understanding of the entries required under the relevant system.

However, some questions, based on the interlocking accounts system could give the information from one system – it could be from the cost ledger or from the financial ledger – and then the memorandum Reconciliation statement, and require the other accounts to be built up.

This does not require any special principle other than the ability to substitute, given a proper understanding of how to treat differences arising when cost ledger entries are being compared with the financial ledger entries in an interlocking system.

12.7 SUMMARY AND CONCLUSIONS

An integrated accounting system is a sophisticated system where a single record is maintained for both financial and cost accounting
transactions. Notional items provided by entrepreneur include capital instead of bank loan.

Advantages and disadvantage of integral systems include avoidance of duplicate records, no need to reconcile financial and cost accounts profits, saves administrative efforts of main training two systems and inability to satisfy external reporting requirements.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

12.8 REVISION QUESTIONS

12.8.1 MULTIPLE CHOICE QUESTIONS

1. If variable costs per unit decrease, sales volume at break even will:
   A. Decrease
   B. Remain the same
   C. Increase
   D. Remain the same; however contribution margin per unit will decrease.
   E. Fluctuate

2. The margin of safety in Naira is:
   A. Expected sales minus expected profit.
   B. Expected sales minus sales at breakeven.
   C. Expected profit minus profit at breakeven.
   D. Expected costs minus cost at breakeven.
   E. Sales value at break-even point

3. Which of the following costs are not relevant for special decisions?
   A. Avoidable costs
   B. Incremental costs
   C. Sunk cost
   D. Marginal cost
   E. Variable costs

4. Which of the following costs is not relevant to a special order decision?
   A. The direct labour cost to manufacture the special order units.
   B. The variable manufacturing overhead incurred to manufacture the special order units.
   C. The portion of the cost of leasing the factory that is allocated to the special order.
   D. The variable overhead incidental to the project
   E. The additional conversion costs

5. When there is one scarce resource, the product that should be produced first is the product:
   A. With the highest sales price per unit of scarce resources.
   B. With the highest demand.
   C. With the highest contribution margin per unit
   D. With the highest contribution margin per unit of scarce resource
   E. With the lowest contribution per unit of scarce resource.
12.8.2 SHORT ANSWER QUESTIONS

Use the information given below to answer questions 1 and 2.
A business sells a single product at a selling price of N20 and a contribution margin of 30%. The fixed costs for the period are N210, 000.

1. What is the number of units to break even?

2. If the business wishes to make a profit of N60,000, the number of units that must be sold is...............................

3. Shade Ltd is preparing a quotation for a special order that requires 1000kg of Material L. Tolu Ltd has 600kg of material L in stock (original cost N5 per kg). Material L is used in the company’s main product, BAM. The resale value of material L is N4.00 per Kg. The present replacement price of Material L is N6.00 per kg. Material L is readily available in the market. What is the relevant cost of the 1000kg of material L to be included in the quotation?

4. The considerations one will have in deciding whether or not to accept a special order are.................... and ....................

5. What is the criterion for a make or buy decision?

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
13.0 LEARNING OBJECTIVES

After studying this chapter, readers should be able to:

- Define an index number;
- Compute and interpret simple index number;
- Compute price relative, quantity relatives and value relatives;
- Compute and interpret weighted index number;
- Compute and interpret quantity weighted index number; and
- Use and interpret the following special index numbers:
  - Laspeyre’s index
  - Paache’s index
  - Marshall Edgeworth’s index
  - Fisher’s index.

13.1 INTRODUCTION

(a) Definition of an index number

An index number is a representation of a current phenomenon, like price, quantity or volume in one period as a percentage of the same phenomenon in a fixed base period.

Some examples are:

(i) Consumer price index;
(ii) Index of industrial production;
(iii) All share price index on April 20, 2005;
(iv) All share quantity (traded) index on April 17, 2005; and
(v) Agricultural production index.

In all these cases, a base reference period must be specified.
ILLUSTRATION 13-1

Table 13.1 shows the volume of annual wages paid to all the employees of PAT (Nig.) Ltd Between 1998 and 2004.

Table 13-1 Total workers’ wages of PAT (Nig.) Ltd by year

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total wages in Nm</td>
<td>10.5</td>
<td>9.8</td>
<td>11.2</td>
<td>12.4</td>
<td>14.8</td>
<td>13.4</td>
<td>15.0</td>
</tr>
</tbody>
</table>

Compute the average indexes using year 2000 as base.

SUGGESTED SOLUTION 13-1

Suppose year 2000 was a year with very reasonable inflation rate and favourable economic condition. Using this as the fixed base period, then the wage indexes for the company with year 2000 as the base year are computed as follows:

\[ I_k = \frac{V_k}{V_0} \times 100 \]

1998: \[ \frac{10.5}{11.2} \times 100\% = 93.75\% \]

1999: \[ \frac{9.8}{11.2} \times 100\% = 87.5\% \]

2000: 100\% i.e. \[ \frac{11.2}{11.2} \times 100\% \]

2001: \[ \frac{12.4}{11.2} \times 100\% = 110.71\% \]

2002: \[ \frac{14.8}{11.2} \times 100\% = 132.14\% \]

2003: \[ \frac{13.4}{11.2} \times 100\% = 119.64\% \]

2004: \[ \frac{15.0}{11.2} \times 100\% = 133.93\% \]
These results are tabulated in the following table

13-2: Table Total wages and volume of wages index for PAT (Nig.) Ltd

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages (₦m)</td>
<td>10.5</td>
<td>9.8</td>
<td>11.2</td>
<td>12.4</td>
<td>14.8</td>
<td>13.4</td>
<td>15.0</td>
</tr>
<tr>
<td>Wages index (%)</td>
<td>94</td>
<td>88</td>
<td>100</td>
<td>111</td>
<td>132</td>
<td>120</td>
<td>134</td>
</tr>
</tbody>
</table>

Year 2000 = 100

Interpretation

The amount of wages in 1998 was 6.25% lower than the wages in 2000, while the amount of wages in 2004 was 33.93% higher than the wages in 2000. Thus, index numbers enable us to find the percentage change (in this case) between the wages in the base year and any other year of interest.

The following are some indexes routinely generated by the Federal Office of Statistics in collaboration with the Central Bank of Nigeria.

Consumer Price Index comprises:

(i) Composite consumer price index;
(ii) Urban consumer price index;
(iii) Rural consumer price index;
(iv) Retail price indexes of selected consumer items in the Lagos area
(v) Summary of price indexes of ordinary shares listed on the Nigeria Stock Exchange; and
(vi) Indexes of average World prices of the Cost of Insurance and Freight (CIF) of Nigeria’s major agricultural commodities.

All the above can be found in the Central Bank of Nigeria Quarterly Statistical Bulletin publication, e.g., Volume 6, No. 2, December 1995.
There is also the series “Consumer Price Indexes” published annually by the Federal Office of Statistics. (Most of the indexes in the publications referred to have 1985 as the base year).

(b) **Choice of the Base Year**

The base year (reference period) is usually a year when conditions (economic, market, business) were regarded as stable. The situations in all other years are computed relative to this base year.

(c) **Changing the Base Year**

Let $V_k$ be the volume (value) for the $k$th year, $k = 0, 1, 2,$

Then, Index ($I_k$) for year $k$ is $I_k = \frac{V_k}{V_0} \times 100\%$; $k = 0, 1, 2, \ldots$

Notice that $I_0 = \frac{V_0}{V_0} \times 100\% = 100\%$, as required. If the base year is changed to year $r$, reference to this new base year, ...

$$I_{k}^{(\text{new})} = \frac{I_k}{I_r} \times 100\%; \ k = 0, 1, 2, \ldots$$

For illustration, take the wage indexes as follows:

<table>
<thead>
<tr>
<th>Wage indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Year</strong></td>
</tr>
<tr>
<td>index (%)</td>
</tr>
<tr>
<td>2000 = 100(%)</td>
</tr>
<tr>
<td>$I_k^{(\text{new})}$</td>
</tr>
<tr>
<td>2001 = 100(%)</td>
</tr>
<tr>
<td>= 85</td>
</tr>
</tbody>
</table>

These indexes can also be computed directly from the annual wages as:

$$I_k = \frac{W_k}{W_0} \times 100\%,$$ where 2001 = 100, the base year.
For example,

(i) With 2001 as base year, index for 1998 is

\[ I_1 = \frac{0.5}{12.4} \times 100 = 85\% \] - as obtained directly for indexes with 2000 as the base year.

(ii) With 2001 as base year, index for 1999 is

\[ I_2 = \frac{9.8}{12.4} \times 100 = 79\% \] (etc)

13.2 SOME SIMPLE USEFUL INDEXES

The idea of index numbers started from prices, like the consumer price indexes previously referred to.

(a) **Price relative**: This is the price relative introduced in Section 8.1. If \( P_1 \) is the price of a commodity in the required year and \( P_0 \) is the price in the base year, the price relative (price index) is

\[ \frac{P_1}{P_0} \times 100 \% . \]

(b) **Quantity relative**: If \( q_1 \) and \( q_0 \) are the quantities of a commodity needed in the required year and base year respectively, the quantity relative is

\[ \frac{q_1}{q_0} \times 100 . \]

(c) **Value relative** (or Quantity relative): If \( v_1 \) and \( v_0 \) are the two values of a commodity purchased in the required year and base year respectively, volume relative is

\[ \frac{v_1}{v_0} \times 100 . \]

But \( v_1 = p_1q_1 \) and \( v_0 = p_0q_0 \), so value relative (or value index)

\[ = \frac{p_1q_1}{p_0q_0} \times 100 \% . \]

ILLUSTRATION 13-2

The following are the prices (in Naira) per kilogram of cow meat purchased in January of each of the indicated years and the quantity (kg) purchased by a family of the indicated years.
Table 13-2: Price and quantity of family consumption of low meat

<table>
<thead>
<tr>
<th>Year</th>
<th>Price(₦)</th>
<th>Quantity(kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>50</td>
<td>8</td>
</tr>
<tr>
<td>1990</td>
<td>70</td>
<td>12</td>
</tr>
<tr>
<td>1995</td>
<td>120</td>
<td>15</td>
</tr>
</tbody>
</table>

Use 1990 as the base year to compute

(i) Price indexes
(ii) Quantity indexes
(iii) Value indexes

**SUGGESTED SOLUTION 13-2**

Price quantity and volume indexes

1990 = 100

<table>
<thead>
<tr>
<th></th>
<th>1985</th>
<th>1995</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price index</td>
<td>$\frac{50}{70} \times 100 = 71.4%$</td>
<td>$\frac{120}{70} \times 100 = 171.4%$</td>
</tr>
<tr>
<td>Quantity index</td>
<td>$\frac{8}{12} \times 100 = 66.7%$</td>
<td>$\frac{15}{12} \times 100 = 125%$</td>
</tr>
<tr>
<td>Value index</td>
<td>$\frac{50 \times 5}{70 \times 2} \times 100 = 47.6%$</td>
<td>$\frac{120 \times 5}{70 \times 2} \times 100 = 214.3%$</td>
</tr>
</tbody>
</table>

From the above table, while the price of meat has decreased by 28.6% from 1985 to 1990, the quantity of meat consumed has also decreased by 33.3% over the same period while the volume has witnessed a dramatic decline of 52.4%.

While the price of meat witnessed a 71.4% increase between 1990 and 1995, the quantity recorded a 25% increase and value recorded a considerable increase of 114.3% over the same period.

### 13.3 WEIGHTED AGGREGATE INDICES

(a) **Price Index**

In the last example, meat is the only household consumption that was considered. In practice, it needs not be so as these may include meat, gari and palm oil. Suppose gari is sold in units of
tins and palm oil in units of gallons. To get a single aggregate price index for the 3 commodities, we need a weighted average of the price of the commodities together with a set of weights to reflect the household’s need of each commodity. This is illustrated for one year as follows:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Prices (₦)</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat</td>
<td>120</td>
<td>$w_1$</td>
</tr>
<tr>
<td>Gari</td>
<td>1100</td>
<td>$w_2$</td>
</tr>
<tr>
<td>Palm oil</td>
<td>500</td>
<td>$w_3$</td>
</tr>
</tbody>
</table>

In general, the choice of appropriate weights is problematic. We must ensure that the weights reflect the importance of each commodity purchased by the household.

**ILLUSTRATION 13-3**

Table 13.3 shows the price and weights attached to each commodity consumed by a family for the first quarter of the years 2002 and 2003. Compute the 2003 simple aggregate index using 2002 as the base year.

**Table 13.3 Weights and prices of selected commodities in 2002 & 2003**

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Weights</th>
<th>$p_0$:2002 price (₦)</th>
<th>$p_n$:2003 price (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat</td>
<td>15</td>
<td>10 / kg</td>
<td>150 / kg</td>
</tr>
<tr>
<td>Gari</td>
<td>5</td>
<td>1100 / tin</td>
<td>1426 / tin</td>
</tr>
<tr>
<td>Palm oil</td>
<td>6</td>
<td>500/gallon</td>
<td>800/gallon</td>
</tr>
</tbody>
</table>

**SUGGESTED SOLUTION 13-3**

Table 13.3.1 Weights and prices of selected commodities in 2002 and 2003

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Weights</th>
<th>$p_0$:2002 price (₦)</th>
<th>$w_p_0$</th>
<th>$p_n$:2003 price (₦)</th>
<th>$w_x_p_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat</td>
<td>15</td>
<td>10 / kg</td>
<td>C_3</td>
<td>150 / kg</td>
<td>C_5</td>
</tr>
<tr>
<td>Gari</td>
<td>5</td>
<td>1100 / tin</td>
<td>C_4</td>
<td>1426 / tin</td>
<td></td>
</tr>
<tr>
<td>Palm oil</td>
<td>6</td>
<td>500/gallon</td>
<td></td>
<td>800/gallon</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>8650</td>
<td>Total</td>
<td>14180</td>
</tr>
</tbody>
</table>
The calculations for $\sum wp_0$ and $\sum w_n p_o$ are shown in columns C_4 and C_6 respectively in table 13.3.1. The price index for 2003 using 2002 as base year is then computed as

$$I = \frac{\sum w_n p_o}{\sum wp_0}$$

$$I = \frac{4180}{8650} \times 100\% = 163.93\%$$

The above set of weights means that to this household, meat is 3 times as important as gari and 2 times as important as palm oil while gari and palm oil are of approximately equal importance to the household. It is important to note that the quantity of each commodity consumed by the household is not a reflection of the importance of each of the commodities to the household. Thus, except we are in a situation where we have no direct way of assigning the weights, the quantity consumed should rarely be used as weights.

(b) **Aggregate Price Relative**

This is another type of commonly used price index, defined as

$$\text{Aggregate Price Relative} = \frac{\sum p_n w}{\sum w} \times 100\%$$

**ILLUSTRATION 13-4**

Use the relevant part of the data in the table of illustration 13.3.1 for the selected commodities to compute a weighted price index of the price relative for 2003 using 2002 as the base year.
SUGGESTED SOLUTION 13-4

The given data can be summarised as follows:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Weights</th>
<th>Price 2002 ( (p_0) )</th>
<th>Price 2003 ( (p_a) )</th>
<th>( \frac{p_a}{p_0} )</th>
<th>( \frac{w_p_a}{p_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat</td>
<td>15</td>
<td>120</td>
<td>150</td>
<td>1.25</td>
<td>18.75</td>
</tr>
<tr>
<td>Gari</td>
<td>5</td>
<td>1100</td>
<td>1426</td>
<td>1.3</td>
<td>6.5</td>
</tr>
<tr>
<td>Palm oil</td>
<td>6</td>
<td>500</td>
<td>800</td>
<td>1.6</td>
<td>9.6</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>26</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>34.85</strong></td>
</tr>
</tbody>
</table>

Year 2002 = 100
i.e base year

Single price relative (index) for 2003 with 2002 as base year is calculated as:

\[
\frac{\sum p_a w}{\sum w} \times 100 = \frac{34.85}{26} \times 100\% = 134\%
\]

This is a weighted average of price relative.

13.4 QUANTITY WEIGHTED INDICES

There are other methods of calculating weighted indexes, particularly using quantities as weights. Consider the following:

(a) \( I_L = \text{Laspeyre’s index} \)
(b) \( I_P = \text{Paasche’s index} \)
(c) \( I_M = \text{Marshall Edgeworth’s index} \)
(d) \( I_F = \text{Fisher’s index} \)

The formula for each of these is stated thus:

\[
I_L = \text{Laspeyre’s index} = \frac{\sum p q_0}{\sum p q_0} \times 100
\]

In this formula, it is assumed that the same quantity purchased in the base year is also purchased in the current year to get the numerator.
**THEORY OF INDEX NUMBERS**

\[ I_p = \text{Paasche's index} = \frac{\sum p q_1}{\sum p q_i} * 100 \]

Here, the quantity purchased in the current year is assumed to have been purchased in the base year.

\[ I_M = \text{Marshall Edgeworth's index} = \frac{\sum p \left( g_0 + q_1 \right)}{\sum p \left( g_0 + q_i \right)} * 100 \]

\[ I_F = \text{Fisher's index} = \sqrt{I_L * I_P} \]

This is a geometric mean of the product of Laspeyre's and Paasche's indices.

**ILLUSTRATION 13-5**

The following table shows the commodity price and quantity purchased for the year in January of each of the indicated years by a family:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Price/Tin</th>
<th>Quantity for the years (Tins)</th>
<th>Price/Tin</th>
<th>Quantity for the years (Tins)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Year 2000</td>
<td>Year 2002</td>
<td></td>
</tr>
<tr>
<td>A (Gari)</td>
<td>250</td>
<td>26</td>
<td>420</td>
<td>32</td>
</tr>
<tr>
<td>B (Rice/Tin)</td>
<td>528</td>
<td>13</td>
<td>820</td>
<td>12</td>
</tr>
<tr>
<td>C (Beans/Tin)</td>
<td>820</td>
<td>12</td>
<td>1030</td>
<td>10</td>
</tr>
</tbody>
</table>

Construct an aggregate index using:

(a) Laspeyre’s method
(b) Paasche’s method
(c) Marshall and Edgeworth’s method
(d) Fisher’s method.

with year 2000 as the base year.
**SUGGESTED SOLUTION 13-5**

<table>
<thead>
<tr>
<th>commodity</th>
<th>Price 2000 ($p_0$)</th>
<th>Qty 2000 ($q_0$)</th>
<th>Price 2002 ($p_1$)</th>
<th>Qty 2002 ($q_1$)</th>
<th>$p_0q_0$</th>
<th>$p_0q_1$</th>
<th>$p_1q_0$</th>
<th>$p_1q_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (Gari)</td>
<td>250</td>
<td>26</td>
<td>420</td>
<td>32</td>
<td>6500</td>
<td>8000</td>
<td>10920</td>
<td>13440</td>
</tr>
<tr>
<td>B (Rice)</td>
<td>528</td>
<td>13</td>
<td>820</td>
<td>12</td>
<td>6864</td>
<td>6336</td>
<td>10660</td>
<td>9840</td>
</tr>
<tr>
<td>C (Beans)</td>
<td>820</td>
<td>12</td>
<td>1030</td>
<td>10</td>
<td>9840</td>
<td>8200</td>
<td>12360</td>
<td>10300</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>23204</td>
<td>22536</td>
<td>33940</td>
<td>33580</td>
</tr>
</tbody>
</table>

Index number for 2002 with 2000 as the base year, is given by:

(i) **Laspeyre’s method**

\[
\frac{\sum p_i q_0}{\sum p_0 q_0} \times 100
\]

\[
\frac{33940}{23204} \times 100 = 46.27
\]

(ii) **Paasche’s method**

\[
\frac{\sum p_i q_i}{\sum p_0 q_i} \times 100
\]

\[
\frac{33580}{22536} \times 100 = 49.01
\]

(iii) **Marshall and Edgeworth’s method**

\[
\frac{\sum p_i (q_0 + q_i)}{\sum p_0 (q_0 + q_i)} \times 100
\]

\[
\frac{33940 + 3580}{23204 + 2536} \times 100 = 47.62
\]

(iv) **Fisher’s method**

\[
\sqrt{I_L \times I_P} = \sqrt{146.27 \times 149.01} = 147.63
\]
ILLUSTRATION 13-6

Construct index number of prices from the following data:

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Year 1960</th>
<th></th>
<th>Year 1970</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
<td>Quantity</td>
<td>Price</td>
<td>Quantity</td>
</tr>
<tr>
<td>A</td>
<td>6</td>
<td>50</td>
<td>10</td>
<td>56</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>100</td>
<td>2</td>
<td>120</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>60</td>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>30</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>E</td>
<td>8</td>
<td>40</td>
<td>12</td>
<td>36</td>
</tr>
</tbody>
</table>

Using:
(a) Laspeyre’s
(b) Paasche’s
(c) Marshall and Edgeworth’s method
(d) Fisher’s method
taking 1960 as the base year.

SUGGESTED SOLUTION 13-6

<table>
<thead>
<tr>
<th>commodity</th>
<th>Price 1960 (p₀)</th>
<th>Qty 1960 (q₀)</th>
<th>Price 1970 (p₁)</th>
<th>Qty 1970 (q₁)</th>
<th>p₀q₀</th>
<th>p₀q₁</th>
<th>p₁q₀</th>
<th>p₁q₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
<td>50</td>
<td>10</td>
<td>56</td>
<td>300</td>
<td>336</td>
<td>500</td>
<td>560</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>100</td>
<td>2</td>
<td>120</td>
<td>200</td>
<td>240</td>
<td>200</td>
<td>240</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>60</td>
<td>6</td>
<td>60</td>
<td>240</td>
<td>240</td>
<td>360</td>
<td>360</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>30</td>
<td>12</td>
<td>24</td>
<td>300</td>
<td>240</td>
<td>360</td>
<td>288</td>
</tr>
<tr>
<td>E</td>
<td>8</td>
<td>40</td>
<td>12</td>
<td>36</td>
<td>320</td>
<td>288</td>
<td>480</td>
<td>432</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1360</td>
<td>1344</td>
<td>1900</td>
<td>1880</td>
</tr>
</tbody>
</table>

Index number for 1970 taking 1960 as the base year, by

(a) Laspeyre’s method

\[ \text{Index number} = \frac{\sum pₐ qₐ}{\sum pₐ q₀} \times 100 \]

\[ = \frac{900}{1360} \times 100 = 39.7 \]
(b) Paasche’s method

\[ \frac{\sum p_i q_i \times 100}{\sum p_i q_i} = \frac{1880}{1344} \times 100 = 39.9 \]

(c) Marshall and Edgeworth’s method

\[ \frac{\sum p_i (q_0 + q_1) \times 100}{\sum p_i (q_0 + q_1)} = \frac{1900 + 880}{1360 + 344} \times 100 = 39.8 \]

(d) Fisher’s method

\[ \sqrt{I_p \times I_L} = \sqrt{139.7 \times 139.9} = 139.8 \]

13.5 SUMMARY AND CONCLUSIONS

Index numbers are introduced as the magnitude of a numerical quantity in one period of time relative to the same quantity in another period of time that is, the base period when the quantity of interest is assumed to be reasonably stable. Simple price and quantity indices (for a single item) are first introduced. The technique of change of the base period of index numbers is treated with examples.

The concepts of a single aggregate index for a group of items are then treated with numerical examples. The discussion ends with the computation of Laspeyre’s, Paache’s, Fisher’s and Marshall and Edgeworth’s indices.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
13.7  REVISION QUESTIONS

13.7.1  MULTIPLE CHOICE QUESTIONS

1. A price index is a representative of the current price of an item as a percentage of the
   A. quantity of a base year
   B. price in a base year
   C. total purchases in the base year
   D. total number of items purchased
   E. the current price of the item in a different town.

2. Which of the following is an index number?
   A. Urban consumer index
   B. Index of industrial production
   C. All shares index
   D. Variety of rice index
   E. Index of agricultural production.

3. Consider the following quantities of an item purchased in the given month:
   
   \[
   \begin{array}{cccccc}
   \text{Month} & \text{May} & \text{June} & \text{July} & \text{Aug} & \text{Sept} \\
   \text{Quantity} & 120 & 80 & 150 & 180 & 75
   \end{array}
   \]

   An index number for July is given to be 125%. The base month is:
   A. May
   B. June
   C. July
   D. August
   E. September

13.6.2  SHORT ANSWER QUESTIONS

1. If the consumer price index for 2004 relative to 2000 is 180%. What is the percentage change in the consumer price from 2000 to 2004?

   Use the following information to answer questions 5 – 7

   Production indices of soft drink

   \[
   \begin{array}{ccccccc}
   \text{Index} & 89 & 105 & 100 & 125 & 90 & 110
   \end{array}
   \]

   2. What is the base year?
   3. Calculate the index for 2004 using 2000 as the base year.
   4. Use 2002 as base year to calculate the percentage change in production from 2000 to 2004
Use the following data to answer questions 5 – 7

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>25</td>
<td>12</td>
<td>40</td>
</tr>
</tbody>
</table>

5. Find Paasche’s index number.
7. A Laspeyre’s and Paasche’s indexes are 120% and 98% respectively. Find the Fisher’s index.

*Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.*
14.0 LEARNING OBJECTIVES
After studying this chapter, readers should be able to:

- Demonstrate good understanding of events and sample space;
- Distinguish between an event and the sample space;
- Compute probability of the occurrence of an event defined on a finite sample space;
- Apply the addition and multiplication laws of probability;
- Apply the concept of independence in solving probability problems;
- Know the concept of conditional probability; and
- Compute the probability of an event happening knowing that another event has occurred.

14.3 INTRODUCTION
PROBABILITY AS A CHANCE TAKING
There are many phenomena in real life that are subject to chance fluctuation. When a businessman decides to invest some huge amount of money in a new business venture, he is taking a risk because there is an element of chance that the new venture will be profitable. Before the businessman therefore goes into the new venture, he must thoroughly study the risk involved in the proposed investment. Such a situation makes probability a very useful tool in business, accounting and indeed in most aspects of human endeavour.

14.4 HOW PROBABILITY IS MEASURED
The concept of probability is intended to provide a numerical measure of the likelihood of an event’s occurrence. Probability is measured on a scale of zero to 1. At the extremes of this range, a probability of zero
implies that the event is impossible i.e. it is certain NOT to occur, whereas a probability of 1 implies that the event is certain to occur.

For uncertain events, we want to attach a probability between zero and 1 such that the more likely the event is to occur, the higher the probability. In practice, such ideas are frequently met: it is known that rain is more likely under certain meteorological conditions than others; an experienced manager may judge that one product is more likely to achieve substantial market penetration than another.

Therefore probability can be regarded as the language in which we discuss uncertainty. Thus, before we can communicate with one another in this language, we need to acquire a common vocabulary. Moreover, as in any other language, rules of grammar are needed so that clear statements can be made with our vocabulary.

14.4.1 SOME BASIC CONCEPTS

Processes whose outcomes are uncertain include:
(a) Tossing a coin: outcome is head or tail;
(b) Throwing a die: outcome is any of 1, 2, 3, 4, 5, and 6; and
(c) Asking a consumer which of the two products Zain or Globacom he or she prefers: outcome is either Zain or Globacom or none at all.

Each of these examples involves a random experiment, Thus: a random experiment is a process leading to at least two possible outcomes with uncertainty as to which will occur.

In each of the above three experiments, it is easy to see that it is possible to specify before hand the outcomes that might arise. In each of the above three cases, the different possible outcomes are known as basic outcomes and they are as listed above. The set of all these basic outcomes exhausts all possibilities and is called the sample space of the random experiment.

14.4.2 CHARACTERISTICS OF AN OUTCOME

These are:
(a) No two outcomes can occur simultaneously;
(b) The random experiment must necessarily result in the occurrence of the basic outcomes.
Often times, interest is not particularly in the basic outcomes themselves, but in some subset of all the outcomes in the sample space. For example, if a die is an event that might be of interest is whether the resulting basic outcome or number is EVEN (i.e. a result that will occur if one of the basic outcomes 2, 4, or 6 arises) such sets of basic outcomes are called events that is Event is a part of the points in S defined by a given rule.

Therefore, probability of an event A is defined as:

\[ P(A) = \frac{\text{No of points in } A}{\text{No of points in } S} = \frac{n(A)}{n(S)} \]  \hspace{1cm} \text{14.2.1} \]

where \( n(A) \) denotes the number of points in A and \( n(S) \) the number of points in the sample.

**ILLUSTRATION 14-1**

ICAN has just requested submission of tender for 3 aspects of production of the Study Pack in Costing and Quantitative Techniques: Word processing (WP), Artwork (AT) and final printing (PT). A chartered accountant who is also an industrialist decides to bid for the three contracts.

(a) List all the possible bids he can make.
(b) List the points in each of the following events:
   (i) He wins all the 3 bids = A
   (ii) He wins only 2 bids = B
   (iii) He wins none of the 3 bids = C

Calculate the probability of each event (use W for winning a bid and L for losing a bid).
SUGGESTED SOLUTION 14-1

(a) 

<table>
<thead>
<tr>
<th>WP</th>
<th>AT</th>
<th>PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>W</td>
<td>W</td>
</tr>
<tr>
<td>W</td>
<td>L</td>
<td>W</td>
</tr>
<tr>
<td>L</td>
<td>W</td>
<td>W</td>
</tr>
<tr>
<td>W</td>
<td>W</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
<td>W</td>
</tr>
<tr>
<td>L</td>
<td>W</td>
<td>L</td>
</tr>
<tr>
<td>W</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
<td>L</td>
</tr>
</tbody>
</table>

so, \( S = \{WWW, WLW, LWW, WWL, LLW, LWL, WLL, LLL\} \)

\( n(S) = 8 \)

A = \{he wins all 3 bids\}

\[ A = \{WWW\} \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{1}{8} \]

B = \{he wins exactly 2 bids\}

\[ B = \{WLW, LWW, WWL\} \Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{3}{8} \]

C = \{He wins none of the 3 bids\}

\[ C = \{LLL\} \Rightarrow P(C) = \frac{n(C)}{n(S)} = \frac{1}{8} \]

It should be noted that had there been seven (7) contracts the man can bid for, it would have been difficult to list the (35) points corresponding to the event: \( E = \{\text{He wins exactly 4 bids}\} \). It will be recalled that in our definition of probability of an event A, the required pieces of information are \( n(A) \) and \( n(S) \). In other words, we need only the number of points in both the event and the sample space regardless of whether or not the points can be listed.
14.3 RELATIVE FREQUENCY APPROACH TO PROBABILITY

When an unbiased coin is tossed once, the sample space is $S = \{H, T\}$, where $H$ stands for head and $T$ for tail. If we want the probability that the toss results in a head, then we define the event $A = \{H\}$. Hence by definition $P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$. However the law of averages states that if an unbiased coin is tossed $n$ times (where $n$ is large) the proportion of the tosses that would result in a head is very close to $\frac{1}{2}$. The larger the number of tosses $n$, the closer the proportion of heads to $\frac{1}{2}$. Hence we can then define the probability of a head to be

\[ P(H) = \frac{\text{Number of tosses resulting in a head}}{\text{Total number of tosses}}; \text{ for a large number of tosses.} \]

Suppose there are 1000 Small and Medium Scale Enterprises (SME) in a community. Of these, 400 are into foundering and 600 into repairs and services (R&S). The probability that an enterprise randomly selected from the 1000 is engaged in repairs and services is $P(R \& S) = \frac{600}{1000}$.

Similarly, if we have a group of 10 friends, 3 of whom are accountants, 5 insurance brokers and the rest doctors, the probability of the event $A$ that a randomly selected person in the group is an accountant is:

\[ P(A) = \frac{\text{Number of Accountants}}{\text{Number in the group}} = \frac{3}{10} \]

14.6 LAWS OF PROBABILITY

14.4.1 Basic laws

(a) Law 1: If $\emptyset$ is the null event (impossible event), then $P(\emptyset) = 0$
(b) Law 2: If $S$ is the sample space, then $P(S) = 1$
(c) Law 3: If $A$ is any event defined on $S$, then $0 \leq P(A) \leq 1$.
(d) Law 4: Let $A^c$ be all the points in $S$ which are not in the event $A$. We call $A^c$ the complement of $A$. Pictorially, we have:

Thus $P(A^c) = 1 - P(A)$
ILLUSTRATION 14-2
A group of friends consists of 6 accountants and 4 doctors. If one of them is selected at random, compute the probability that the chosen person is:
(i) A doctor (ii) Not a doctor (iii) A lawyer (iv) A doctor or an accountant.

SUGGESTED SOLUTION 14-2
(i) \( P(\text{A doctor selected}) = P(D) = \frac{4}{10} \)
(ii) \( P(\text{selected person is not a doctor}) = P(D^c) = 1 - P(D) = 1 - \frac{4}{10} = \frac{6}{10} \)
(iii) \( P(\text{selected person is lawyer}) = P(\emptyset) = 0, \) since there is no lawyer in the group.
(iv) \( P(\text{selected person is a doctor or an accountant}) = P(S) = 1 \)
since each person in the group is either a doctor or an accountant. In other words, the event that the selected person is either a doctor or an accountant is the same as the sample space i.e. it is a sure event.

14.4.2 Some important definitions

(a) **The union of two events:** Let A and B be two events. The union of A and B is another event such that every member of the union is either a member of A or a member of B. The union is written as \( A \cup B \) which is said to occur when A occurs OR B occurs. The key word defining the union of two sets is ‘OR’.

(b) **The intersection of two events:** Let A and B be two events. The intersection of A and B is another event such that every element in the intersection belongs to both A and B. The intersection is written as \( A \cap B \) which occurs when A and B simultaneously occur together. The key word for intersection is ‘AND’.

(c) **Mutually exclusive (or disjoint) events:** Events A and B are said to be mutually exclusive (or disjoint) if A and B cannot occur together. In other words, \( A \cap B = \emptyset \) which is the same as saying that \( P(A \cap B) = 0 \).
These are illustrated in the following diagrams:

14.4.3 More laws of probability:

(a) **Laws 5**: if A and B are mutually exclusive events; then \( P(A \cup B) = P(A) + P(B) \). In other words, the probability that A or B occurs is the sum of the probability that A occurs alone and the probability that B occurs alone when A and B are mutually exclusive. However, when A and B are not mutually exclusive \( P(A \cup B) \neq P(A) + P(B) \). The exact expression is stated next.

(b) **Law 6**: When A and B are not mutually exclusive (not disjoint), then:

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

This is the probability law of addition.

This says that the probability that A or B occurs when they are not mutually exclusive is the probability of A occurring + the probability of B occurring – the probability of both occurring simultaneously. The use of these laws is illustrated in the following example:
ILLUSTRATION 14-3

A local chapter of the Nigerian Institute of Estate Surveyors and Valuers has 60 members, 25 of whom are males. Out of the 60 members, 35 have invested in the shares of companies in the banking sector, 15 in shares of companies in the Food and beverages sector, and the rest in companies in the petroleum sector. Of the male members, 10 invested in companies in the Banking sector while 8 invested in Food and Beverages sector.

(a) Put the information in this problem in tabular form with gender as the row and investment sector in the column:

(b) A surveyor is randomly picked from the community. Find the probability that the selected person
(i) is a male;
(ii) has invested in the banking sector;
(iii) has invested in the petroleum sector;
(iv) has invested in the banking sector or the food and beverage sector;
(v) is either a female or has invested in the petroleum sector; and
(vi) is a male and has invested in the banking sector.

SUGGESTED SOLUTION 14-3

<table>
<thead>
<tr>
<th>Gender</th>
<th>Banking sector</th>
<th>Food and beverages</th>
<th>Petroleum sector</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>10*</td>
<td>8*</td>
<td>(7)</td>
<td>25*</td>
</tr>
<tr>
<td>Female</td>
<td>(25)</td>
<td>(7)</td>
<td>(3)</td>
<td>(35)</td>
</tr>
<tr>
<td>Total</td>
<td>35*</td>
<td>15*</td>
<td>(10)</td>
<td>60*</td>
</tr>
</tbody>
</table>

*These figures are directly given in the problem

The figures in the brackets are obtained by calculation.

(b)(i) \[ \text{P (selected person is a male)} = \frac{\text{No. of Males}}{\text{No. of Persons}} = \frac{25}{60} \]

(ii) \[ \text{P (selected person has invested in the banking sector)} = \frac{\text{No of invested in banking sector}}{\text{No. of persons}} = \frac{35}{60} \]
(iii) \[ P(\text{selected person has invested in the petroleum sector}) = \frac{\text{no invested in petroleum sector}}{\text{no of persons}} = \frac{10}{60} \]

(iv) Note that nobody has invested in both the banking sector and in the food and beverages sector. Hence \[ P(\text{selected person has invested in both the banking and the food and beverages sector}) = 0, \] because the two events are mutually exclusive.

Hence, \[ P(\text{selected person has invested in the banking sector or the food and beverages sector}) = P(\text{person invested in banking sector}) + P(\text{person invested in food and beverages sector}) = \frac{35}{60} + \frac{15}{60} = \frac{50}{60} = \frac{5}{6} \]

(v) \[ P(\text{selected person is either a female or has invested in the petroleum sector}) = P(\text{Person is female}) + P(\text{selected person has invested in the petroleum sector}) - P(\text{selected person is a female who has at the same time invested in the petroleum sector}) = \frac{35}{60} + \frac{10}{60} - \frac{3}{60} = \frac{42}{60} = \frac{7}{10} \]

Note the two events involved are not mutually exclusive.

(vi) \[ P(\text{selected person is a male and has invested in the banking sector}) = 10/60. \]

Thus: \[ P(A\cup B) = P(A) + P(B) - P(A\cap B) \] holds always, while \[ P(A\cup B) = P(A) + P(B) \] holds when A and B are mutually exclusive.
14.7 CONDITIONAL PROBABILITY AND INDEPENDENCE

(a) **Conditional Probability**: Consider the data of Illustration 14-3. Suppose the randomly selected estate surveyor is a female, what is the probability that such a person invested in the petroleum sector? This is a conditional probability which is the probability that a selected person has invested in the petroleum sector given (when it is already known) that the person is a female. This is written as $P(\text{selected person has invested in the petroleum sector/ the person is a female})$.

The symbol $/$ is read as given. By definition, the probability that an event $B$ occurs when it is known that $A$ has already occurred is:

$P(B/A)$ which is defined as:

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

In other words, the conditional probability that $B$ occurs when it is known that $A$ has previously occurred is the quotient of the probability that $A$ and $B$ occur simultaneously and the unconditional probability that $A$ occurs.

Thus, in the last problem, the probability that the person has invested in the petroleum sector given that the person is a female.

$$P(\text{Selected Person has invested in the petroleum sector AND the person is also a female}) = \frac{3}{60} = \frac{3}{35}$$

Notice that the conditional probability is $3/35$ while the unconditional probability that the selected person has invested in the petroleum sector is $10/60$, showing that in general $P(B/A) \neq P(B)$.
(b) **Independence**

Suppose $P(B/A) = P(B)$. In other words, the conditional probability of $B$ given that $A$ has previously occurred is the same as the probability that $B$ occurs whether or not $A$ has previously occurred. When this happens, it is clear that $A$ and $B$ are occurring independently. Further, when $P(B/A) = P(B)$, it follows from the definition of conditional probability that

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

14.5.1

When the two events occur independently

Hence, $P(A \cap B) = P(A) P(B)$  

14.5.2

Also then $P(B/A) = P(B)$  

14.5.3

This is the general definition of independence.

Definition: Events $A$ and $B$ are said to be independent if:

$$P(A \cap B) = P(A) P(B), \text{ otherwise}$$

$A$ and $B$ are dependent or not independent.

Further, since $P(B/A) = \frac{P(A \cap B)}{P(A)}$, then

$$P(A \cap B) = P(A) P(B/A)$$

14.5.4

The expression 14.5.2 holds when $A$ and $B$ are independent and 14.5.4 holds when $A$ and $B$ are dependent. Expression 14.5.4 is called the multiplication law of probability.

(c) **Multiplication Law**

$$P(A \cap B) = P(A) P(B/A)$$

14.5.5

for dependent events $A$ and $B$

But when $A$ and $B$ are independent, then the law simplifies to

$$P(A \cap B) = P(A) P(B)$$

as obtained in 14.5.2 above.
ILLUSTRATION 14-4

There are 7 companies manufacturing table salt (S) and 8 different companies manufacturing beverages (B). Two companies are to be selected one after the other as training centers for food vendors. Compute the probability that the two randomly selected companies are:

(i) Salt manufacturers

(ii) Manufacturers of the same products

(iii) Manufacturers of different products, when:

(a) The selection process is without replacement

(b) The selection is with replacement

SUGGESTED SOLUTION 14-4

(a) Selection is without replacement

(i) Required: the probability that both companies selected are salt manufacturers.

= P(1\textsuperscript{st} company selected is a manufacturer of salt) \times P (2\textsuperscript{nd} company selected is a manufacturer of salt/1\textsuperscript{st} chosen is a manufacturer of salt)

P(SS) = P(S) \times P(S/S), where

P(S) = P (1\textsuperscript{st} chosen company is a manufacturer of salt)

P(S/S) = P(2\textsuperscript{nd} selected company is a manufacturer of salt given that the 1\textsuperscript{st} selected is a manufacturer salt)

P(S) = 7/15

If a salt manufacturing company is selected first, then we have only 6 salt manufacturing companies left for the 2\textsuperscript{nd} choice. Hence:

P(S/S) = \frac{6}{14} = \frac{6}{15}

P(SS) = P(S) \times P(S/S)

= \frac{7}{15} \times \frac{6}{14} = 0.2
(ii) **Required:** The probability that the two selected companies manufacture the same product

\[
= P(SS) + P(BB)
\]

\[
= P(S) P(S/S) + P(B) P(B/B)
\]

\[
= \frac{7}{15} \times \frac{6}{14} + \frac{8}{15} \times \frac{7}{14} = 0.47
\]

Based on similar argument to the one used in (3) above.

(iii) **Required:** P(The two selected companies are manufacturers of different products)

\[
= P(SB) + P(BS)
\]

\[
= P(S) P(B/S) + P(B) P(S/B)
\]

\[
= \frac{7}{15} \times \frac{8}{14} + \frac{8}{15} \times \frac{7}{14} = 0.53
\]

(b) **Selection is with replacement**

(i) P(Both Selected companies are salt manufacturers)

\[
= P(SS) = P(S) P(S/S). 
\text{However when selection is with replacement}
\]

\[
P(S/S) = \frac{7}{15} \text{ which is the same as } P(S)
\]

Thus with replacement \(P(S/S) = P(S)\);

\[
P(S/S) = P(S); P(B/S) = P(B) \text{ and } P(B/B) = P(B)
\]

Hence: (i) \(P(SS) = P(S) P(S) = \left(\frac{7}{15}\right)^2 = 0.22\)

(ii) \(P(SS \text{ or } BB) = P(SS) + P(BB)\)

\[
= P(S) P(S/S) + P(B) P(B/B)
\]

\[
= \left(\frac{7}{15}\right) \times \left(\frac{7}{15}\right) + \left(\frac{8}{15}\right) \times \left(\frac{8}{15}\right) = 0.5
\]

(iii) \(P(SB \text{ or } BS) = P(SB) + P(BS)\)

\[
= P(S) P(B/S) + P(B) P(S/B)
\]

\[
= \left(\frac{7}{15}\right) \times \left(\frac{8}{15}\right) + \left(\frac{8}{15}\right) \times \left(\frac{7}{15}\right) = 0.498
\]
(d) **Extended Probability law**
Let A be an event on a sample space S. Suppose $B_1$, $B_2$ and $B_3$ are a set of mutually exclusive events on S such that at least one of them must occur, then

$$P(A) = P(B_1) P(A/B_1) + P(B_2) P(A/B_2) + P(B_3) P(A/B_3)$$

We illustrate with the following example

**ILLUSTRATION 14-5**

An industrialist operates in a community where a year’s business environmental condition can be classified as favorable (F), Normal (N) or unfavorable (U). Over the years, favorable business environmental condition has prevailed for 40% of the time, while the proportion of years with normal business environmental condition is 35%. The probability that the industrialist will expand the scope of his business in a year with favorable environmental condition is 15%, while the probabilities of expansion in a year with normal or unfavorable environmental condition are 10% and 2% respectively. What is the probability that the industrialist will expand the scope of the business in year 2006?

**SUGGESTED SOLUTION 14-5**

Required: $P(\text{Expand business})$

$$= P(\text{Favorable condition}) P(\text{Expansion/Favorable condition}) + P(\text{Normal condition}) P(\text{Expansion/Normal condition}) + P(\text{Unfavorable condition}) P(\text{Expansion/Unfavorable condition})$$

From the given information in the question,

$P(\text{favorable condition}) = 40\% = 0.4$

$P(\text{expansion / favorable condition}) = 0.15$

$P(\text{Normal condition}) = 35\% = 0.35$

$P(\text{expansion / Normal condition}) = 10\% = 0.1$

$P(\text{unfavorable condition}) = 1 - 0.4 - 0.35 = 0.25$

$P(\text{Expansion / unfavorable condition}) = 2\% = 0.02$

Substituting these in the expression above we get

$$P(\text{expansion in 2006}) = (0.40) (0.15) + (.35) (.10) + (.25) (.02)$$

$$= 0.06 + 0.035 + 0.005 = 0.1$$
The data provided for the solution of the problem can be presented as a probability tree as shown in the following diagram.

**Note**

(i) For example \( P(E^c / F) = P(\text{No expansion given favorable condition}) \)

\[ = 1 - P(\text{expansion given favorable condition}) \]

In other words 

\[ P(E^c / F) = 1 - P(E/F). \]

(ii) Following our last calculation 

\[ P(E) = P(F) P(E/F) + P(N) P(E/N) + P(U_F) P(E/U_F) \]

\[ = (0.40)(0.15) + (0.35)(0.10) + 0.25(0.02) = 0.06 + 0.035 + 0.005 = 0.1 \]
which is the sum of the product of probabilities leading to E. Note that to get to E one can go via F or N or U.

Similarly the probability that the industrialist will not expand is the sum of the products of probabilities leading to E (No expansion).

Thus \( P(E^c) = (0.40)(0.95) + (0.35)(0.90) + (0.25)(0.98) = 0.380 + 0.315 + 0.245 = 0.940 \)

(F) **Further treatment of independence**

Note that two events A and B are independent if \( P(A \cap B) = P(A)P(B) \).

This definition is extendable to 3 or more events. For example 3 events A, B and C are independent if \( P(A \cap B \cap C) = P(A)P(B)P(C) \). The following is a practical example of the application of independence.

**ILLUSTRATION 14-6**

PANFOOD (Nig) LTD is a food processing company located in an area with irregular PHCN power supply.

A day when the company has less than 8 hours power supply is classified as an unproductive day. Past records show that the proportion of unproductive days is 40%.

Assuming that day-by-day power supplies are independent, compute the probability that from Monday to Wednesday of a given week the company will have unproductive days on

(i) Monday only
(ii) Tuesday and Wednesday only
(iii) one day only
(iv) two days only
(v) all the 3 days
(vi) none of the 3 days
(vii) at least one of the 3 days.

**SUGGESTED SOLUTION 14-6**

Let A denote the event that the company has an unproductive day. From the data provided:

\[ P(A) = 0.40; \text{ Hence } P(A^c) = 1 - P(A) = 1 - 0.4 = 0.6 \]

Thus probability of an unproductive day = 0.4
And probability of a productive day = 0.6

(i) Probability that only Monday is unproductive.

The situation is as shown below:

<table>
<thead>
<tr>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A^c</td>
<td>A^c</td>
</tr>
</tbody>
</table>

This means that Mon is unproductive, while Tuesday is productive and Wednesday is productive.

Thus \( P(\text{Mon only unproductive}) \)

\[ = P(\text{Mon is unproductive AND Tue productive AND Wed is productive}) \]

\[ = P(\text{Mon unproductive})P(\text{Tue productive})P(\text{Wed productive}) \]

\[ = P(A)P(A^c)P(A^c) \text{ by independence} \]

\[ = (0.4)(0.6)(0.6) = 0.144 \]

(ii) \( P(\text{only Tuesday and Wednesday are unproductive}) \)

\[ = P(\text{Monday productive and Tue unproductive and Wed unproductive}) \]

\[ = P(\text{Mon productive})P(\text{The unproductive})P(\text{Wed unproductive}) \]

\[ = P(A^c)P(A)P(A) \]

\[ = (0.6)(0.4)(0.4) = 0.096 \]

(iii) For the company to be unproductive on only one day there will be:

<table>
<thead>
<tr>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>corresponding probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>UP</td>
<td>P</td>
<td>P</td>
<td>(0.4)(0.6)(0.6) = 0.144</td>
</tr>
<tr>
<td>P</td>
<td>UP</td>
<td>P</td>
<td>(0.6)(0.4)(0.6) = 0.144</td>
</tr>
<tr>
<td>P</td>
<td>P</td>
<td>UP</td>
<td>(0.6)(0.6)(0.4) = 0.144</td>
</tr>
</tbody>
</table>

Thus the probability that the company will be unproductive for only one day

\[ = 3(0.4)(0.6)^2 = 0.432 \]
(iv) For the company to be unproductive for only 2 of the 3 days

<table>
<thead>
<tr>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>PROBABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>UP</td>
<td>UP</td>
<td>(0.6)(0.4)(0.4) = 0.096</td>
</tr>
<tr>
<td>UP</td>
<td>P</td>
<td>UP</td>
<td>(0.4)(0.6)(0.4) = 0.096</td>
</tr>
<tr>
<td>UP</td>
<td>UP</td>
<td>P</td>
<td>(0.4)(0.4)(0.6) = 0.096</td>
</tr>
</tbody>
</table>

P(company is unproductive for only two days) = 3(0.4)^2(0.6) = 0.288

(v) **Required:** P(Unproductive on all of the 3 days)

<table>
<thead>
<tr>
<th>MON</th>
<th>TUE</th>
<th>WED</th>
</tr>
</thead>
<tbody>
<tr>
<td>UP</td>
<td>UP</td>
<td>UP</td>
</tr>
</tbody>
</table>

With probability (0.4)(0.4)(0.4) = (0.4)^3 = 0.064

Hence P(Company is unproductive for all the 3 days is) = (0.4)^3 = 0.064

(vi) **Required:** unproductive on none of the three days

<table>
<thead>
<tr>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>P</td>
<td>P</td>
</tr>
</tbody>
</table>

With probability (0.6)(0.6)(0.6) = 0.216

Hence P (Company is unproductive in none of the 3 days or productive in all the 3 days) = (0.6)^3 = 0.216

(vii) Note that complement of ‘NONE’ is at least one, Hence the probability that the company will be unproductive for at least one of the 3 days

= 1 - P(unproductive in none of the three days)

= 1 - (0.6)^3 = 1 - 0.216 = 0.784

14.6 **SUMMARY AND CONCLUSIONS**

Elementary probability is treated using the basic concepts of a sample space (as a universal set), an event (as a subset) and the probability of an event. Examples are given to illustrate a probability as the ratio of the number of points in an event to the number of points in the sample space. The long run concept of probability using relative frequency approach is discussed. Addition and multiplication laws of probability are treated. The roles of conditional probability and independence are treated with examples.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
14.7 **REVISION QUESTIONS**

### 14.7.1 MULTIPLE CHOICE QUESTIONS

1. A random experiment is one that
   A. is performed randomly
   B. has finite number of outcomes
   C. has outcomes that cannot be predetermined
   D. has predictable outcomes
   E. is an experimental random trial

2. Two events are mutually exclusive when
   A. their union is the sample space
   B. they are disjoint
   C. they are independent
   D. one is a subset of the other
   E. they are both null events

3. If an unbiased die is rolled once, the probability of obtaining an odd number given that it is less than 5 is:
   A. \(\frac{1}{2}\)
   B. \(\frac{3}{4}\)
   C. \(\frac{2}{3}\)
   D. \(\frac{1}{6}\)
   E. \(\frac{1}{5}\)

### 14.7.2 SHORT ANSWERS QUESTIONS

**Use the following information to answer questions 1 – 3**

A man lives in a local government area where guinea worm prevalence is 60% while his father lives in another local government area with 30% guinea worm prevalence.

1. What is the chance that both father and son will be simultaneously infected with guinea worm?
2. What is the chance that neither will be infected?
3. What is the chance that at least one will be infected?

**Use the following information to answer questions 4 – 7**

Given a set of numbers \(S = \{3, 8, 11, 13, 15, 16, 18, 19, 21, 26\}\). If a number is picked at random, the probability that it is

4. A multiple of 3
5. Even
6. Even and less than 20
7. Odd given that it is less than 23

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
15.0 LEARNING OBJECTIVES

After studying this chapter, readers should be able to:

- Demonstrate a good understanding of a set, the universal set and subsets;
- Determine the complement, union and intersection of sets;
- Represent sets by Euler Venn diagram; and
- Solve simple business problems using the number of points in sets and Euler Venn diagrams.

15.1 INTRODUCTION

15.1.1 Definition and Concept of Sets

(a) **Definition**

A set is a collection of objects with some common characteristics. Examples include:

(i) The set of ICAN fellows
(ii) The set of shareholders of Mobil Plc
(iii) The set of all medium and small scale enterprises in Ikeja area of Lagos State
(iv) The set of all ministers (past and present) in the Yar’Adua Administration since 2007
(v) The set of all distressed banks in the country since 1990.

(b) **Set representation**

A set is usually represented by a capital letter. 
A is the set of all engineers in the Lagos State Ministry of Works (LSMW).
B is the set of mortgage finance institutions in Lagos State.

If, for example, Engineer Oluyemi is in LSMW, we write Oluyemi ∈ A to mean that Oluyemi is a member of the set A or Oluyemi belongs to A. If Nomand (Nig.) Ltd is a registered mortgage finance institution, we write: Normand ∈ B.

The set A above can also be referred to as
\[
\text{A} = \{\text{engineers who are in LSMW}\}
\]
(i) or \( \text{A} = \{e \text{ such that } e \in \text{LSMW}\} \)
(ii) or \( \text{A} = \{e \mid e \in \text{LSMW}\} \)

where in (i) e stands for an engineer, and in (ii) the vertical line “|” means “such that”. Thus, (ii) is read A is the set of all e’s such that e is in LSMW.

A set can also be described by listing out all elements of the set as follows:
\[
\text{C} = \{\text{Jan, Feb, Mar, Apr, May, Jun, July, Aug, Sept, Oct, Nov, Dec}\}
\]
Here, C is the set of all the months in any year.

For example, \( \text{Jan} \in \text{C}, \text{Apr} \in \text{C} \) while \( \text{Summer} \notin \text{C} \) is read as “Summer is not a member of the set C.”

15.2 TYPES OF SETS

(a) Concept of Universal set U

The universal set U consists of the totality of all the units under consideration in a given situation. Examples are:
(i) \( \text{U} = \text{the set of all Roman alphabets} \)
(ii) \( \text{U} = \text{the set of all integers from 0 up to and including 20} \)
(iii) \( \text{U} = \text{the set of all small and medium scale enterprises in Apapa area of Lagos State} \).

All other sets are, therefore, a part of the universal set.
(b) **Subsets**

A set \( R \) which is a part of another set \( S \) is said to be a subset of the bigger set \( S \). In other words, \( R \) is a subset of \( S \) if every member of \( R \) is also a member of \( S \). It is however possible to have some members of \( S \) which do not belong to \( R \). In set notation,

\[
R \subseteq S \quad \text{means that } R \text{ is a subset of } S
\]

or

\[
S \supseteq R \quad \text{means that } S \text{ contains } R
\]

As an illustration, let \( U = \{ \text{integer } x \mid 0 \leq x \leq 20 \} \). This can also be written as:

\[
U = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 \}
\]

Define \( R = \{ x \in U \mid 0 \leq x \leq 8 \} \) and \( S = \{ x \in U \mid 0 \leq x \leq 10 \} \)

In other words,

\( R = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8 \} \) and

\( S = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \} \)

Clearly, \( R \subseteq S \) (\( R \) is a subset of \( S \)) because every element of \( R \) is also an element of \( S \). However, \( 9 \notin S \), but \( 9 \in R \).

Notice that \( R \subseteq U \) and \( S \subseteq U \).

(c) **The Complement of a Set**

The complement of a set \( A \) is denoted by \( A^c \) or \( A^c \) and it consists of all points in \( U \) that are not in \( A \). In other words,

\[
A^c = \{ x \in U \mid x \notin A \}
\]

This is similar to the complement of an event discussed under probability in Chapter Fourteen with the sample space as the universal set.

This concept is now illustrated using the following example.
Illustration 15-1

There are ten friends whose weekly wages (₦'000) are 18, 10, 6, 15, 12, 21, 23, 8, 20, 25. The universal set of the wages of the friends (in thousand Naira) is 

\[ U = \{18, 10, 6, 15, 12, 21, 23, 8, 20, 25\} \]

Let \( A \) be the set of wages (₦'000) that are at most 19 thousand Naira. Then, 

\[ A = \{18, 10, 6, 15, 12, 8\} \]

Then \( A^c \) = set of wages in thousand Naira not in \( A \). In other words, 

\[ A^c = \{21, 23, 20, 25\} \]

(d) Finite Set

A set is finite if the number of elements in the set is finite. For example, in the example about the wages of 10 friends, the universal set \( U \) is finite since the number of elements in it is finite. Similarly, the sets \( A \) and \( A^c \) in the same example are both finite. A set which is not finite is said to be infinite.

(e) Equal Sets

Two sets \( A \) and \( B \) are said to be equal when the elements in \( A \) are exactly the same as the elements in \( B \), regardless of the order of listing of the sets. For example, the set of wages of the group of 10 that are at most 19 thousand can be written in thousands as: \( A = \{18, 10, 6, 15, 12, 8\} \) or \( B = \{6, 8, 10, 12, 15, 18\} \). The elements of \( A \) are merely rearranged to get \( B \). Hence, \( A = B \).

(f) Null Set

A null set is a set with no element. It is also called an empty set. It is usually denoted by \( \emptyset \), where \( \emptyset = \{\} \) which is the empty set. Recall that when we use \( \{\} \) to define a set, the members of the set are listed within the two curly brackets. However, for \( \emptyset = \{\} \), no element is listed between the two brackets, indicating that \( \emptyset \) has no element and hence, is empty. For example, the set of wages in (₦'000) of the group of 10 that are above 30 is empty because no member of the group is paid more than ₦30,000 in a week.
15.4 SET OPERATIONS

There are three (3) types of set operations. These are: (a) Complement, (b) Union and (c) Intersection

(a) The Complement of a Set
See discussion under types of sets.

(b) The Union of two sets
The union of two sets A and B is another set obtained by merely merging the elements in A with the elements in B without repetition. This union is written as \( A \cup B \), the elements of which are either in A or in B.

(c) The Intersection of two sets
The intersection of two sets A and B is another set obtained by selecting from A and B, those elements that belong to both sets. The intersection is denoted by \( A \cap B \). The following illustration is very informative.

ILLUSTRATION 15-2
Use the example of the wages of the group of 10 friends is used to illustrate the above concept.

(i) The universal set, \( U = \{6, 8, 10, 12, 15, 18, 20, 21, 23, 25\} \)
Let A be the set of wages in (N'000) that are at least 11 and at most 21 and B is the set of wages that are above 16, we then have \( A = \{12, 15, 18, 20, 21\} \)
\( B = \{18, 20, 21, 23, 25\} \)
Hence by definition,

(ii) \( A^c = \{6, 8, 10, 23, 25\} \)

(iii) \( A \cup B = \{12, 15, 18, 20, 21, 23, 25\} \)
Note that if we had ordinarily merged A and B, the elements 18, 20, 21 would have been duplicated.

(iv) \( A \cap B = \{18, 20, 21\} \)
These are the elements that are common to both sets.
The keyword for Union is OR while AND is for intersection. 
$A \cup B$ is described in words as $A$ or $B$, while $A \cap B$ is described as $A$ and $B$.

(v) **Set difference**
The set difference $A \mid B$ or $A - B$ consists of those points in $A$ which are not in $B$. Using the sets $A$ and $B$ defined in equations (1) and (2) above, leads to

$A - B = \{12, 15\}$

$B - A = \{23, 25\}$

Hence, $A - B \neq B - A$.

We notice in the above example that $A - B$ and $B - A$ are disjoint sets whose union consists of the elements of $A \cup B$. In other words,

$A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$ ………………………………..15.3.1

This is a very useful expression.

15.4 **AN EULER-VENN DIAGRAM**

An Euler Venn diagram is a diagrammatic representation of one or more sets in a single diagram. The concept is illustrated with the example of the wages of the group of 10 friends as obtained in example 15.2. The individual diagrams are as follows:

**Venn diagrams**

(i) Universal set $U = \{6, 8, 10, 12, 15, 18, 20, 21, 23, 25\}$

$$
\begin{align*}
A & : 12, 15, 20, 21 \\
B & : 23, 25
\end{align*}
$$
(ii) \[ A \cup B = \begin{array}{c c c c}
12, 15 & 20 & 23, 25 \\
21 & 18 &
\end{array} \]

(iii) \[ A \cap B = \begin{array}{c c c c}
18 & 20 & 21 
\end{array} \]

(iv) \[ A - B = \begin{array}{c c c c}
6, 8, 10 
\end{array} \]

(v) \[ B - A = \begin{array}{c c c c}
B - A & 23, 25 
\end{array} \]
All these can be put together in one diagram as:

(vi) All combined =

\[ \begin{array}{c}
6 \\
12, 15 \\
A \cap B \\
20, 18 \\
B-A \\
18 \\
21 \\
8 \\
10 \\
\end{array} \]

This combined diagram is the Euler-Venn diagram.

(i) The rectangle represents the universal set.

(ii) \( A \cup B \) has 7 elements: 12, 15, 18, 20, 21, 23, 25.

(iii) Three elements 6, 8, 10 are not in \( A \cup B \). Hence, they belong to \( (A \cup B)^c \). Thus \( (A \cup B)^c = \{6, 8, 10\} \). These are the elements in \( U \) which are not in \( A \cup B \). These elements correspond to the three elements that are within the rectangle but outside \( A \cup B \).

15.6 CARDINALITY OF A SET AND THE POWER OF A SET

15.5.1 Definitions

(a) The cardinality of a set is the number of all possible subsets of in the set. However, the power of a set consists of all possible subsets of the set including the empty set and every set under consideration. Among these are all singleton sets (one element set), all sets containing two elements and so on. For example, consider the set \( D = \{15, 20, 21\} \) in (v) of section 15.3.

We have \( D = \{15, 20, 21\} \)

Then all possible subsets of \( D \) are:

\[ \emptyset, \{15\}, \{20\}, \{21\}, \{15, 20\}, \{15, 21\}, \{20, 21\}, \{15, 20, 21\}. \]

There are 8 possible subsets of \( D \). Since \( D \) has only 3 points, the cardinality of \( D \) is 8, which is \( 2^3 \). In general, the cardinality of a finite set \( E \) containing \( n \) elements is \( 2^n \). Therefore, if \( E \) has 5 elements, the cardinality of \( E \) is \( 2^5 = 32 \).
(b) **The number of points in a set**

For a finite set \( E \), the number of points in \( E \) is obtained by counting all the elements of \( E \). This is denoted by \( n(E) \).

For example, the number of points in the subsets \( A, B, A \cup B \) and \( A \cap B \) of wages of a group of 10 friends defined are respectively given by:

\[
\begin{align*}
n(A) &= 5, \\
n(B) &= 5, \\
n(A \cup B) &= 7, \\
n(A \cap B) &= 3
\end{align*}
\]

Note that

\[
n(A) + n(B) - n(A \cap B) = 5 + 5 - 3 = 7 = n(A \cup B)
\]

In general,

\[
n(A \cup B) = n(A) + n(B) - n(A \cap B) \tag{15.5.1}
\]

or

\[
n(A \cup B) = n(A - B) + n(A \cap B) + n(A - B) \tag{15.5.2}
\]

For three sets, we have,

\[
n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C) \tag{15.5.3}
\]

### 15.6 APPLICATIONS

A few examples are presented below to illustrate the applications of set theory in solving business problems.

**ILLUSTRATION 15-3**

Each of the thirty members of a cooperative society of a large organisation can have no account or at least one type of account in FOLU Trust Bank. If 12 members have savings account only, 10 have current account only and 3 have neither current nor savings account in FOLU Bank,

(a) How many members have

(i) At least one account?

(ii) The two types of accounts?

(iii) Savings account?

(iv) Current account?

(b) Represent the pieces of information in (i) to (iv) in a Venn diagram.
Let A be the set of members who have savings account in FOLU Bank, and B the set who have current account in the same bank. Then, we have,

\[ n(U) = 30 \]

(i) Those who have at least one of the two accounts is \((A \cup B)\). However,

\[ (A \cup B) \cup (A \cap B)^c = U \]

\((A \cup B)\) and \((A \cap B)^c\) are disjoint sets. Therefore,

\[ n(U) = n(A \cup B) + n((A \cap B)^c) \quad \text{................. 15.5.4} \]

where \((A \cap B)^c\) is a set consisting of those who have neither of the two accounts in FOLU Bank. We are given that 

\[ n((A \cap B)^c) = 3. \]

Extracting from the information above, we have,

\[ 30 = n(U) = n(A \cup B) + 3 \]

\[ \Rightarrow n(A \cup B) = 30 - 3 = 27 \]

(ii) But \((A \cup B) = (A-B) + (A \cap B) + (B-A)\)

It is given that \(n(A-B) = 12\) and \(n(B-A) = 10\).

Hence, we have from 6.5.2

\[ 27 = 12 + n(A \cap B) + 10 \]

\[ = 22 + n(A \cap B) \Rightarrow n(A \cap B) = 27 - 22 = 5 \]

i.e., 5 members have both accounts.

(iii) \(n(A) = n(A-B) + n(A \cap B)\)......................... 15.5.5

\[ = 12 + 5 = 17 \]

i.e., 17 members have savings account.

(iv) \(n(B) = n(B-A) + n(A \cap B)\)......................... 15.5.6

\[ = 10 + 5 = 15 \]

i.e., 15 members have current account.

(b) The Euler-Venn Diagram is drawn as follows
There are 80 insurance agents who have sold insurance policies to their customers. Suppose 21 of the agents sold motor policy only, 15 home policy only, and 12 life policy only while 9 did not sell any of the 3 classes of insurance. Further, 11 agents sold both motor and home policies, 8 both home and life policies while 10 sold both motor and life policies. Use Euler-Venn diagram to determine the number of agents who sold

(i) All the 3 classes of policy.
(ii) Motor policy
(iii) Home policy
(iv) Life policy

**SUGGESTED SOLUTION 15-4**

Let

\[ M = \{ \text{Those who sold motor policies}\} \]
\[ H = \{ \text{Those who sold home policies}\} \text{ and } L = \{ \text{Those who sold life policies}\} \]

**Euler-Venn diagram for illustration 15-4**
No. of agents = 80
No. who sold none of the 3 policies = 9
Therefore, the number of agents who sold at least one of 3 policies is 71, i.e., \( n(M \cup H \cup L) = 71 \).

(i) Let \( x \) be the number that sold all the 3 policies, i.e., \( n(M \cap H \cap L) = x \). Using the additional information given, Venn diagram with the number of agents in each set is as shown in the following solutions. Hence,
\[
21 + (11-x) + x + (10-x) + (8-x) + 12 + 15 = 71 \\
(21 + 11 + 10 + 8 + 12 + 15 - 2x) = 71 \\
77 - 2x = 71 \\
2x = 77 - 71 = 6 \Rightarrow x = 3.
\]

(ii) No. of agents who sold motor policies
\[
n(M) = 21 + (11-x) + x + 10-x \\
= 21 + 11 + 10 - x \\
= 42 - x \\
= 42 - 3 with x = 3 \\
= 39
\]
\[
n(H) = 15 + (11-x) + x + (8-x) \\
= 15 + 11 + 8 - x \\
= 34 - x \\
= 34 - 3 with x = 3 \\
= 31
\]

(iii) \( n(L) = 12 + (8-x) + x + (10-x) \\
= 12 + 8 + 10 - x \\
= 30 - x \\
= 30 - 3 with x = 3 \\
= 27
\]

15.7 SUMMARY AND CONCLUSIONS

The treatment of elementary set theory begins with the introduction of the universal set, the null set as well as subsets. Other types of sets treated include finite sets, equal sets and the complement of a set. Some basic operations on sets such as the complement, union and intersection of sets in addition to the difference between two sets are treated with good examples. The diagrammatic representation of sets using Euler-Venn diagrams are treated with examples. The concepts of the cardinality of a set are applied to the solution of some business-like problems in set theory. These problems involve two or three sets.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
15.9 REVISION QUESTIONS

15.8.1 MULTIPLE CHOICE QUESTIONS

1. One of these is not a null set
   A. The set of professional accountants older than 200 years
   B. The set of all civil servants whose monthly basic salaries are over ₦10m
   C. The set of all lawyers who are also accountants
   D. The set of all male children under 10 years who have won a gold medal in the weight lifting event in the Olympic games
   E. The set of human beings with four legs.

2. Let A be the set of all ICAN Fellows and B the set of CIBN Fellows. Only one of these is true
   A. \( A \subseteq B \)
   B. \( B \subseteq A \)
   C. \( A = B \)
   D. \( A \cap B \subseteq A \)
   E. \( A \cup B \subseteq B \).

3. The complement of a set A is
   A. The set A
   B. The universal set
   C. The empty set
   D. The set of points in the universal set which are not in A
   E. The intersection of A and the universal set.

15.8.2 SHORT ANSWER QUESTIONS

Use the following information to answer questions 1-4
A social club has 50 members of whom 10 are accountants and 15 lawyers. If there are 28 members of the club who are neither accountants nor lawyers.
1. Find the number of club members who are either accountants or lawyers
2. How many club members are both accountants and lawyers?
3. How many club members are accountants only?
4. Find the number of club members that are lawyers only.
5. Write the set \( A \setminus B \) using both intersection and complement of a set.

Use the following information to answer questions 9 and 10
If \( A = \{1, 2, 3, 4, 5, 6, 7\} \) and \( B = \{5, 6, 7, 8, 9, 10, 11\} \)
6. Find \( A \cap B \)
7. Find the cardinality of \( A \cup B \).

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
16

INTRODUCTION TO MATRICES

16.0 LEARNING OBJECTIVES
After studying this chapter, readers should be able to:

- Write a set of numbers in matrix form;
- Recognise equal matrices, null matrices, identity matrix and triangular matrices;
- Add together two matrices that are conformable for addition;
- Find the difference (by way of subtraction) between two matrices that are conformable for subtraction;
- Multiply “a” matrix a by another matrix “b” in that order when the product is defined;
- Know that “a”. “b” may not necessarily be equal to “b . a”;
- Evaluate determinants and find the inverse of a square matrix; and
- Solve a system of linear equations.

16.1 INTRODUCTION

16.1.1 Basic Concepts
A matrix is an array (enclosed between brackets) of real numbers arranged in \(m\) rows and \(n\) columns. The matrix is then referred to as an \(m\) by \(n\) matrix or \((m \times n)\) matrix.
For example,

(a) $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ is a 2 by 3 matrix. Equivalently, we can write $A_{2,3} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

i.e. it has 2 rows and 3 columns

(b) $B = \begin{bmatrix} 8 & 1 \\ 4 & 3 \\ 1 & 0 \end{bmatrix}$ is a 3 by 2 matrix.

i.e. it has 3 rows and 2 columns

(c) $C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 5 \\ 0 & 4 & 3 \end{bmatrix}$ is a 3 by 3 matrix

i.e. it has 3 rows and 3 columns

Any $n$ by $n$ matrix is called a square matrix.

16.2 TYPES OF MATRICES

(a) Equal matrices

Two matrices $A$ and $B$ are said to be equal when

(i) The number of rows of $A = \text{the number of rows of } B$ and
(ii) The number of columns of $A = \text{the number of columns of } B$ and
(iii) The entries in corresponding positions are the same in both $A$ and $B$.

For example,

If

$\begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$ and $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 2 & 0 \end{bmatrix}$, then $A \neq B$ since $A$ has 2 columns while $B$ has 3 columns even though they have the same number of rows.
A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}, \text{ then } A \neq B \text{ since the entry in row 2 column 1 of } A \text{ which is 0, is not the same as the entry in row 2 column 1 of } B \text{ which is 1, even though other entries in corresponding positions in both matrices are equal.}

A = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \text{ then } A = B \text{ since entries in corresponding positions in both matrices are equal.}

(b) **Identity Matrix**

A square matrix A is said to be an identity matrix when each entry on the leading diagonal of A is unity and other entries off the leading diagonal are zero. For example, the following are identity matrices:

\[ I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

(c) **Diagonal Matrix**

A square matrix with zero entries off the leading diagonal is a diagonal matrix provided the entries on the leading diagonal are non-zero. For example,

\[ A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \]

are diagonal matrices.

(d) **Triangular Matrix**

A matrix whose entries above or below the leading diagonal are zero is referred to as a triangular matrix. It is said to be upper triangular where all entries below the leading diagonal are zero. It is lower triangular when all entries above the leading diagonal are zero. Following are two triangular matrices A and B.
A is an upper triangular matrix while B is a lower triangular matrix.

(e) The Transpose of a Matrix

A matrix B is the transpose of A if when the rows of B are turned into columns, the resulting matrix is the same as A. For example, let

\[
A = \begin{bmatrix}
1 & 5 & 3 \\
0 & 8 & 4 \\
0 & 0 & 7 \\
\end{bmatrix}, \quad B = \begin{bmatrix}
1 & 0 & 0 \\
5 & 8 & 0 \\
3 & 4 & 7 \\
\end{bmatrix}
\]

Then B is the transpose of A. This is written as

\[B = A^T\] or \[B = A^\top\]

(f) Symmetric Matrix

A matrix A is said to be symmetric if \(A = A^\top\). In other words, a matrix is symmetric if the matrix and its transpose are the same. For example,

\[
A = \begin{bmatrix}
5 & 1 & 4 \\
1 & 2 & 6 \\
4 & 6 & 3 \\
\end{bmatrix}
\]

is such that \(A = A^\top\). Hence A is symmetric.
(g) **Skew Symmetric Matrix**

A matrix \( A \) is said to be skew symmetric if \( A^T = -A \). It should be noted that this is only possible if all the leading diagonal entries of \( A \) are zero. For example,

\[
A = \begin{bmatrix}
0 & 1 & -4 \\
-1 & 0 & 6 \\
4 & -6 & 0
\end{bmatrix}, \quad A^T = \begin{bmatrix}
0 & -1 & 4 \\
1 & 0 & -6 \\
-4 & 6 & 0
\end{bmatrix}, \quad -A = \begin{bmatrix}
0 & 1 & 4 \\
1 & 0 & 6 \\
4 & 6 & 0
\end{bmatrix}
\]

Observe that \( A^T = -A \). Therefore, \( A \) is skew symmetric.

(h) **Null Matrix**

A null matrix is a matrix with all its entries zero. For example,

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

is a null matrix.

### 16.3 BASIC OPERATIONS ON MATRICES

(a) **Multiplication of a Matrix by a Constant**

Let \( c \) be a constant and \( A \) a matrix. The matrix \( B = cA \), is obtained from \( A \) by multiplying each entry of \( A \) by \( c \). For example, if

\[
A = \begin{bmatrix}
3 & 2 & 1 \\
5 & 6 & 4
\end{bmatrix}
\]

Then, \( cA = \begin{bmatrix}
3c & 2c & c \\
5c & 6c & 4c
\end{bmatrix} = \begin{bmatrix}
9 & 6 & 3 \\
15 & 18 & 12
\end{bmatrix} \) when \( c = 3 \).

(b) **Addition of Two Matrices**

Let \( A \) and \( B \) be two matrices. The sum \( (A + B) \) exists only when the number of rows of both matrices are the same, and the number of columns of both matrices are also the same. When \( A + B \) exists, then the sum is obtained by adding corresponding
entries of A and B and putting the sum in the corresponding positions of a new matrix A + B. For example, if

\[
A = \begin{bmatrix}
3 & 2 \\
5 & 8 \\
10 & 1 
\end{bmatrix}, \quad B = \begin{bmatrix}
6 & 5 \\
5 & 2 \\
3 & 4 
\end{bmatrix}
\]

Then, \( A + B = \begin{bmatrix}
3+6 & 2+5 \\
5+5 & 8+2 \\
10+3 & 1+4 
\end{bmatrix} = \begin{bmatrix}
9 & 7 \\
10 & 10 \\
13 & 5 
\end{bmatrix} \)

(c) **Difference of Two Matrices**

The difference of two matrices A and B, \( A - B \), exists when the number of rows of both matrices are equal and the number of columns of both matrices are also equal. Then, \( A - B = A + (-1)B \), where \((-1)B\) is a matrix whose entries are multiplied by \(-1\). \( A + (-B) \) is then obtained by addition of \((-1)B\) to A.

For example, if

If \( A = \begin{bmatrix}
5 & 2 & 1 \\
3 & 8 & 7 
\end{bmatrix} \), and \( B = \begin{bmatrix}
6 & 1 & 3 \\
2 & 7 & 5 
\end{bmatrix} \)

then, \( A - B = \begin{bmatrix}
5 & 2 & 1 \\
3 & 8 & 7 
\end{bmatrix} + (-1) \begin{bmatrix}
6 & 1 & 3 \\
2 & 7 & 5 
\end{bmatrix} \)

\[= \begin{bmatrix}
5 & 2 & 1 \\
3 & 8 & 7 
\end{bmatrix} + \begin{bmatrix}
-6 & -1 & -3 \\
-2 & -7 & -5 
\end{bmatrix} \]

\[= \begin{bmatrix}
-1 & 1 & -2 \\
1 & 1 & 2 
\end{bmatrix} \]

This may also be obtained as

\( A - B = \begin{bmatrix}
5-6 & 2-1 & 1-3 \\
3-2 & 8-7 & 7-5 
\end{bmatrix} = \begin{bmatrix}
-1 & 1 & -2 \\
1 & 1 & 2 
\end{bmatrix} \)

Note: \( A + B = B + A \), and \( B-A = -(A-B) \)

(d) **Product of Two Matrices**

Let A and B be two matrices. The condition that the product \( AB \) (in that order) exists is that the number of columns of A must be equal to the number of rows of B. Otherwise, \( AB \) does not exist.
When the product $AB$ exists, then the entry in row $r$ and column $s$ of $AB$ is obtained as the scalar product of the entries in row $r$ of $A$ and those in the column $s$ of $B$. For example, if

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

Let $AB = C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$,

then the matrices $A$ and $B$ are conformable for product

where

$$c_{11} = a_{11} \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = a_{11}b_{11} + a_{12}b_{21}$$

$$c_{12} = a_{12} \begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix} = a_{12}b_{12} + a_{22}b_{22}$$

$$c_{13} = a_{12} \begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} = a_{12}b_{12} + a_{23}b_{23}$$

$$c_{21} = a_{21} \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = a_{21}b_{11} + a_{22}b_{21}$$

$$c_{22} = a_{22} \begin{bmatrix} b_{12} \\ b_{22} \end{bmatrix} = a_{22}b_{12} + a_{22}b_{22}$$

$$c_{23} = a_{22} \begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} = a_{22}b_{13} + a_{23}b_{23}$$

Note: If $A$ is an $n \times r$ matrix and $B$ is an $r \times s$ matrix, then $AB$ exists since $A$ has $r$ columns and $B$ $r$ rows. The product $AB$ is an $n \times s$ matrix. For example, if

$$A = \begin{bmatrix} 6 & 1 & 4 \\ 5 & 3 & 2 \\ 7 & 0 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 4 \\ 8 & 6 \\ 1 & 7 \end{bmatrix}, \quad \text{then} \quad C = A \times B$$

$$3 \times 3 \quad 3 \times 2 \quad 3 \times 2 \quad 3 \times 3 \times 2$$
The product $\mathbf{C} = \mathbf{AB}$ is a $3 \times 2$ matrix and

$$
\mathbf{C} = \begin{bmatrix}
(6)(2) + (1)(8) + (4)(1) & (6)(4) + (1)(6) + (4)(7) \\
(5)(2) + (3)(8) + (2)(1) & (5)(4) + (3)(6) + (2)(7) \\
(7)(2) + (0)(8) + (5)(1) & (7)(4) + (0)(6) + (5)(7)
\end{bmatrix}
$$

$$
= \begin{bmatrix}
24 & 58 \\
36 & 52 \\
19 & 63
\end{bmatrix}
$$

Note: For matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{AB} \neq \mathbf{BA}$. In fact, one of these may not exist. For the example above,

$$
\begin{bmatrix} 3 \times 3 \end{bmatrix} \cdot \begin{bmatrix} 3 \times 2 \end{bmatrix} = \begin{bmatrix} 3 \times 2 \end{bmatrix}
$$

while

$$
\begin{bmatrix} 3 \times 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \times 3 \end{bmatrix}
$$

does not exist since $\mathbf{B}$ has 2 columns while $\mathbf{A}$ has 3 rows.

### 16.4 Determinants

An array of numbers arranged into $n$ rows and $n$ columns and placed between two vertical lines is called a determinant of order $n$. For example,

$$
\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}
$$

are determinants of orders 2 and 3 respectively.

**(a) Evaluating a $2 \times 2$ Determinant**

Let $\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

The value of this determinant is defined as:

$$
\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}
$$

For example,

$$
\begin{vmatrix} 5 & 6 \\ 2 & 1 \end{vmatrix} = 5(1) - 2(6) = -7
$$
(b) **Expansion of A 3×3 Determinant**

Consider the following 3×3 determinant.

\[
\begin{vmatrix}
 a_{11} & a_{12} & a_{13} \\
 a_{21} & a_{22} & a_{23} \\
 a_{31} & a_{32} & a_{33}
\end{vmatrix}
\]

This can be expanded by reducing it into a linear combination of 2×2 determinants using a row or a column of the determinant. The following 2×2 determinants are commonly used:

\[
A_{r,s} = \text{determinant obtained from } \Delta \text{ by deleting from } A \text{ its } r \text{ th row and } s \text{ th column. For example,}
\]

\[
A_{11} = \frac{1}{a_{22}} \begin{vmatrix}
 a_{11} & a_{12} \\
 a_{21} & a_{22}
\end{vmatrix}, \quad A_{12} = \frac{1}{a_{23}} \begin{vmatrix}
 a_{11} & a_{13} \\
 a_{21} & a_{23}
\end{vmatrix}, \quad A_{13} = \frac{1}{a_{32}} \begin{vmatrix}
 a_{11} & a_{12} \\
 a_{31} & a_{32}
\end{vmatrix}, \quad A_{21} = \frac{1}{a_{32}} \begin{vmatrix}
 a_{21} & a_{23} \\
 a_{31} & a_{33}
\end{vmatrix}, \quad A_{22} = \frac{1}{a_{31}} \begin{vmatrix}
 a_{11} & a_{13} \\
 a_{21} & a_{23}
\end{vmatrix}, \quad A_{23} = \frac{1}{a_{31}} \begin{vmatrix}
 a_{11} & a_{12} \\
 a_{21} & a_{22}
\end{vmatrix}, \quad A_{31} = \frac{1}{a_{22}} \begin{vmatrix}
 a_{11} & a_{13} \\
 a_{31} & a_{33}
\end{vmatrix}, \quad A_{32} = \frac{1}{a_{21}} \begin{vmatrix}
 a_{11} & a_{12} \\
 a_{31} & a_{32}
\end{vmatrix}, \quad A_{33} = \frac{1}{a_{21}} \begin{vmatrix}
 a_{11} & a_{12} \\
 a_{31} & a_{32}
\end{vmatrix}
\]

These can be put together as a matrix of **cofactors**:

\[
\text{i.e Matrix of cofactors } = \begin{bmatrix}
 A_{11} & A_{12} & A_{13} \\
 A_{21} & A_{22} & A_{23} \\
 A_{31} & A_{32} & A_{33}
\end{bmatrix}
\]

For example, to expand the determinant \( \Delta \) using (16.5.1) and the 1\(^{st} \) row, we have

\[
\Delta = (-)^{1+} a_{11} A_{11} + (-)^{1+} a_{12} A_{12} + (-)^{1+} a_{13} A_{13}
\]

To expand by the second column, we get,

\[
\Delta = (-)^{2+} a_{12} A_{12} + (-)^{2+} a_{22} A_{22} + (-)^{2+} a_{32} A_{32}
\]
ILLUSTRATION 16-1

Evaluate \[
\begin{vmatrix}
3 & 5 & 1 \\
8 & 0 & 2 \\
4 & 0 & 6
\end{vmatrix}
= \Delta
\]

SUGGESTED SOLUTION 16-1

To expand by the first row, we need:

\[
A_{11} = \begin{vmatrix} 2 \\ 6 \end{vmatrix}, \quad A_{12} = \begin{vmatrix} 2 \\ 6 \end{vmatrix} = 0, \quad A_{13} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}
\]

where, for example, \( A_{12} \) is the determinant obtained by deleting the 1st row and the 2nd column of \( \Delta \).

Hence, \( A_{11} = 0, A_{12} = 40, A_{13} = 0 \)

Expansion by the first row gives:

\[
\Delta = (-1)^{1+1} a_{11} A_{11} + (-1)^{1+2} a_{12} A_{12} + (-1)^{1+3} a_{13} A_{13} \quad \ldots \ldots \quad 16.4.2
\]

Substituting for \( a_{11}, A_{11}, a_{12}, A_{12}, a_{13}, A_{13} \), we get,

\[
= (1) (3) (0) + (-1) (5) (40) + (1) (1) (0)
\]

\[
= 0 - 200 + 0 = -200
\]

Expanding by the second column is the easiest since two of the entries in column 2 are zero. Expanding by column 2 gives

\[
\Delta = (-1)^{2+1} a_{21} A_{21} + (-1)^{2+2} a_{22} A_{22} + (-1)^{2+3} a_{23} A_{32} \quad \ldots \ldots \quad 16.4.3
\]

But \( a_{22} = a_{32} = 0 \) Hence, from 16.4.3, we have,

\[
\therefore \Delta = - a_{12} A_{12} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 16.4.4
\]

where \( a_{12} = 5 \) and \( A_{12} = \begin{vmatrix} 8 & 2 \\ 4 & 6 \end{vmatrix} = 8 - 2 = 0 \)

Substituting these in 16.4.3

\[
\therefore \Delta = - a_{12} A_{12} = (-1) (5) (40) = -200
\]
Again, expand by the 3rd row

\[ \Delta = (-)^{31} a_{31} A_{31} + (-)^{32} a_{32} A_{32} + (-)^{33} a_{33} A_{33} \quad \ldots \quad 16.4.5 \]

where \( a_{31} = 4, a_{32} = 0, a_{33} = 6 \) and

\[ A_{31} = \begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix} = 10, \quad A_{32} = \begin{bmatrix} 3 & 1 \\ 4 & 6 \end{bmatrix} = 14, \quad A_{33} = \begin{bmatrix} 3 & 5 \\ 8 & 0 \end{bmatrix} = -40 \]

So, the expansion of \( \Delta \) by the 2nd column gives

\[ \Delta = a_{31} A_{31} + (-) a_{32} A_{32} + (-) a_{33} A_{33} \quad \ldots \ldots \quad 16.4.6 \]

\[ \Delta = (4) (10) + (0) (14) + (6) (-40) = 40 - 240 = -200 \text{ as before.} \]

**ILLUSTRATION 16-2**

Evaluate

\[
\Delta = \begin{vmatrix} 4 & 1 \\ 7 & 5 \end{vmatrix} - \begin{vmatrix} 1 \\ 6 \end{vmatrix}
\]

**SUGGESTED SOLUTION 16-2**

Let us expand by the second column.

\[ \Delta = (-)^{12} (1) A_{12} + (-)^{22} (5) A_{22} + (-)^{32} (-) A_{32} \quad \ldots \quad 16.4.7 \]

where

\[ A_{12} = \begin{vmatrix} 1 & 6 \\ 7 & 1 \end{vmatrix} = -2 ; \quad A_{22} = \begin{vmatrix} 4 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 1 = 5 ; \text{ and} \]

\[ A_{12} = \begin{vmatrix} 4 & -1 \\ 1 & 6 \end{vmatrix} = 4 + 1 = 7 \]

we get.

\[ \therefore \Delta = (-)^{12} a_{12} A_{12} + (-)^{22} a_{22} A_{22} + (-)^{32} a_{32} A_{32} \]
INTRODUCTION TO MATRICES

\[
\begin{align*}
\Delta &= (-1)(1)(-41) + (1)(5)(25) + (-1)(-2)(27) \\
&= 41 + 125 + 54 = 220 \quad \ldots 16.4.8
\end{align*}
\]

Next, expand by the 3rd row to get
\[
\Delta = (-1)^{31}a_{31}A_{31} + (-1)^{32}a_{32}A_{32} + (-1)^{33}a_{33}A_{33} \quad \ldots \quad 16.4.9
\]

\[
A_{31} = \begin{vmatrix} 1 & -5 \\ 6 & 1 \end{vmatrix} = 27; \quad A_{32} = \begin{vmatrix} 4 & -1 \\ 1 & 6 \end{vmatrix} = 27; \quad A_{33} = \begin{vmatrix} 4 & 1 \\ 1 & 5 \end{vmatrix} = 19
\]

and \( a_{31} = 7, \quad a_{32} = -2, \quad a_{33} = 1. \)

\[
\therefore \Delta = (1)(7)(21) + (-1)(-2)(27) + (1)(1)(19)
\]

\[
= 147 + 54 + 19 = 220 \text{ as before}
\]

**Conclusion**

No matter what row or column is used for the expansion, the end result is the same and it is the value of the determinant. In practice, any row or any column can be used for the expansion.

### 16.6 MINOR, COFACTOR AND ADJOINT OF A MATRIX

**(a) Minor**

A minor of a matrix \( A \) is any square submatrix of \( A \). For example, if

\[
A = \begin{bmatrix} 3 & 4 & 1 \\ 8 & 3 & 5 \\ 1 & 7 & 2 \end{bmatrix}
\]

\[
\begin{bmatrix} 3 & 4 \\ 8 & 3 \end{bmatrix}, \quad \begin{bmatrix} 3 & 1 \\ 8 & 5 \end{bmatrix}, \quad \begin{bmatrix} 3 & 5 \\ 7 & 2 \end{bmatrix}, \quad \begin{bmatrix} 8 & 3 \\ 1 & 7 \end{bmatrix}
\]

are all submatrices of \( A \). Each of them is therefore a minor of \( A \).

Some particular types of minors of a matrix \( A \) are obtained by deleting from \( A \) its \( r \)th row and the \( s \)th column. The determinant of such a minor is denoted by \( A_{rs} \).
ILLUSTRATION 16-3

If \( \mathbf{A} = \begin{bmatrix} 3 & 4 & 1 \\ 8 & 3 & 5 \\ 1 & 7 & 2 \end{bmatrix} \),

Find the determinant of all 2\( \times \)2 minors of \( \mathbf{A} \).

SUGGESTED SOLUTION 16-3

The (1,1) minor is obtained by deleting from \( \mathbf{A} \) the first row and the first column. Its determinant is

\[
\begin{vmatrix} 3 & 5 \\ 2 & 2 \end{vmatrix} = 2
\]

\[
\begin{vmatrix} 8 & 5 \\ 7 & 2 \end{vmatrix} = 3
\]

\[
\begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix} = 1
\]

\[
\begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix} = 7
\]

\[
\begin{vmatrix} 2 & 4 \\ 4 & 7 \end{vmatrix} = 7
\]

\[
\begin{vmatrix} 3 & 1 \\ 5 & 5 \end{vmatrix} = 0
\]

\[
\begin{vmatrix} 3 & 4 \\ 8 & 3 \end{vmatrix} = -3
\]

(b) **Cofactor of a Matrix**

The \((r,s)\) cofactor of a matrix is the determinant of the matrix obtained by deleting the \(r\)th row and the \(s\)th column of \( \mathbf{A} \) and then multiplying the result by \((-1)^{r+s}\). Let \( C_{rs} \) denote the \((r,s)\) cofactor. Then,

\[
C_{rs} = (-1)^{r+s} A_{rs}
\]  

11.6.1.1
ILLUSTRATION 16-4

Using the matrix

\[
A = \begin{bmatrix}
3 & 4 & 1 \\
8 & 3 & 5 \\
1 & 7 & 2 \\
\end{bmatrix}
\]

Find the cofactors of A. Find also the matrix of cofactors of A.

SUGGESTED SOLUTION 16-4

This is the same matrix used in Illustration 16.3 Recall that

\[ C_{rs} = (-1)^{r+s} A_{rs} \]

Using 11.6.1.1, we have,

\[
\begin{align*}
C_{11} &= (-1)^{1+1} A_{11} = A_{11} = 9 \\
C_{12} &= (-1)^{1+2} A_{12} = -A_{12} = -1 \\
C_{13} &= (-1)^{1+3} A_{13} = A_{13} = 33 \\
C_{21} &= (-1)^{2+1} A_{21} = -A_{21} = - \quad \\
C_{22} &= (-1)^{2+2} A_{22} = A_{22} = 6 \\
C_{23} &= (-1)^{2+3} A_{23} = -A_{23} = -7 \\
C_{31} &= (-1)^{3+1} A_{31} = A_{31} = 7 \\
C_{32} &= (-1)^{3+2} A_{32} = -A_{32} = - \quad \\
C_{33} &= (-1)^{3+3} A_{33} = A_{33} = -3
\end{align*}
\]

The matrix of cofactors is, therefore, given as

\[
C = \begin{bmatrix}
-29 & -11 & 53 \\
-1 & 5 & -17 \\
17 & -7 & -23 \\
\end{bmatrix}
\]
(c) **The Adjoint of a Matrix**

The adjoint of a matrix A is the transpose of its matrix of cofactors.

Let A be a matrix whose matrix of cofactors is C. Then,

\[ \text{Adj} (A) = C^T \]

11.6.3.1

Thus, in the last example,

\[
A = \begin{bmatrix} 3 & 4 & 1 \\ 8 & 3 & 5 \\ 1 & 7 & 2 \end{bmatrix},
\]

\[
C = \begin{bmatrix} -29 & 17 & 53 \\ -1 & 5 & -17 \\ 17 & -7 & -23 \end{bmatrix}
\]

Then,

\[
\text{Adj} (A) = \begin{bmatrix} -29 & -1 & 17 \\ 17 & 5 & -7 \\ 53 & -17 & -23 \end{bmatrix}
\]

(d) **The Inverse \((A^{-1})\) of a matrix A**

Let A be an \(n \times n\) (i.e., a square matrix). Let \(I_n\) be the \(n \times n\) identity matrix. Then,

\[ AI = IA = A. \]

16.5.1

Further, let B be a square matrix, such that \(AB = BA = I\). Then, B is the inverse of B. It is denoted by \(A^{-1}\). Thus,

\[ AA^{-1} = A^{-1}A = I \]

16.5.2

**Warning:** Do not write \(A^{-1}\) as \(\frac{1}{A}\) since \(\frac{1}{A}\) has no meaning in the theory of matrices.

As an illustration, let

\[
A = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}
\]

and
\[ B = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} \]

\[ BA = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 4 - 3 & 1 - 1 \\ 12 - 12 & -3 + 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \]

Similarly, \( AB = I \).

Therefore, \( B \) is the inverse of \( A \). Hence, \( A^{-1} = B = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} \)

Let \( A \) be an \( n \times n \) matrix, such that \( A^{-1} \) exists. Then, the expression for the inverse of matrix \( A \) is given as:

\[
A^{-1} = \frac{1}{\det A} \text{Adj}(A) = \frac{1}{|A|} \text{Adj}(A) \quad \text{........... 8.6.3}
\]

The process of inverting a matrix is illustrated with the following examples:

**ILLUSTRATION 16-5**

If \( A = \begin{bmatrix} 8 & 4 \\ 3 & 1 \end{bmatrix} \), find \( A^{-3} \).

**SUGGESTED SOLUTION 16-5**

\[
|A| = \begin{vmatrix} 8 & 4 \\ 3 & 1 \end{vmatrix} = 8 - 12 = -4
\]

The cofactors are:

\[
C_{11} = (-1)^{1+1} A_{11} = A_{11} = 1
\]

\[
C_{12} = (-1)^{1+2} A_{12} = -A_{12} = -3
\]
COSTING AND QUANTITATIVE TECHNIQUES

\[
C_{21} = (-1)^{2+1} A_{21} = A_{21} = -4
\]
\[
C_{22} = (-1)^{2+2} A_{22} = A_{22} = -8
\]

Then, \( C = \begin{bmatrix} 1 & 3 \\ -4 & 8 \end{bmatrix} \)

\[
\text{Adj} (A) = \begin{bmatrix} 1 & -4 \\ -3 & 8 \end{bmatrix}
\]

Therefore, \( A^{-1} = \frac{1}{|A|} \text{Adj}(A) \)

\[
= \begin{bmatrix} 1 & 4 \\ -4 & -3 \end{bmatrix}
\]

To check the correctness of this, compute:

\[
AA^{-1} = \begin{bmatrix} 8 & 4 & 1 \\ 3 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -4 & -3 \\ -12 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} = I
\]

This confirms that \( A^{-1} = \begin{bmatrix} 1 & 1 & -4 \\ -4 & -3 & 8 \\ -12 & -8 & 1 \end{bmatrix} \)

Next is an example involving a \( 3 \times 3 \) matrix.

**ILLUSTRATION 16-6**

If \( A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \\ 3 & 1 & -1 \end{bmatrix} \), find \( A^{-1} \).
SUGGESTED SOLUTION 16-6

\[ |A| = (-)^{1+1}(2)A_{11} + (-)^{1+2}(1)A_{12} + (-)^{1+3}(0)A_{13} \]

\[ = 2A_{11} - A_{12} \]

\[ = 2 \begin{vmatrix} 3 & 2 \\ 1 & \end{vmatrix} - \begin{vmatrix} 2 & \\ 1 & \end{vmatrix} \]

\[ = 2(-3-2) - (-1-6) \]

\[ = -10 + 7 = -3 \]

\[ A_{11} = \begin{vmatrix} 3 & 2 \\ 1 & \end{vmatrix} = -5 \quad \Rightarrow \quad C_{11} = -5 \]

\[ A_{12} = \begin{vmatrix} 1 & 2 \\ 3 & \end{vmatrix} = -7 \quad \Rightarrow \quad C_{12} = 7 \]

\[ A_{13} = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = -8 \quad \Rightarrow \quad C_{13} = -8 \]

\[ A_{21} = \begin{vmatrix} 1 & 0 \\ 2 & \end{vmatrix} = -1 \quad \Rightarrow \quad C_{21} = 1 \]

\[ A_{22} = \begin{vmatrix} 2 & 0 \\ 3 & \end{vmatrix} = -2 \quad \Rightarrow \quad C_{22} = -2 \]

\[ A_{23} = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = -1 \quad \Rightarrow \quad C_{23} = 1 \]

\[ A_{31} = \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = 2 \quad \Rightarrow \quad C_{31} = 2 \]

\[ A_{32} = \begin{vmatrix} 2 & 0 \\ 1 & \end{vmatrix} = 4 \quad \Rightarrow \quad C_{32} = -4 \]

\[ A_{33} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 \quad \Rightarrow \quad C_{33} = 5 \]

Hence, the matrix of cofactors is:

\[
C = \begin{bmatrix}
-5 & 7 & -8 \\
1 & -2 & 1 \\
2 & -4 & 5
\end{bmatrix}, \text{ and}
\]
\[
\text{Adj (A)} = C^T = \begin{bmatrix}
-5 & 1 & 2 \\
7 & -2 & -4 \\
-8 & 1 & 5 \\
\end{bmatrix}
\]

\[
A^{-1} = \frac{1}{|A|} \text{Adj}(A) = \frac{1}{3} \begin{bmatrix}
-5 & 1 & 2 \\
7 & -2 & -4 \\
-8 & 1 & 5 \\
\end{bmatrix}
\]

\[
= \frac{1}{3} \begin{bmatrix}
5 & -1 & -2 \\
7 & 2 & 4 \\
8 & -1 & -5 \\
\end{bmatrix}
\]

Check:

\[
AA^{-1} = \begin{bmatrix}
2 & 1 & 0 \\
1 & 3 & 2 \\
3 & 1 & -1 \\
\end{bmatrix} \begin{bmatrix}
5 & -1 & -2 \\
1 & 7 & 2 \\
8 & -1 & -5 \\
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix} = I
\]

\textbf{Note:}

(i) There is shortcut method of finding the inverse of a \(2 \times 2\) matrix.

Let \(A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\), to get the inverse of \(A\), the following procedure is adopted:

- Interchange the leading diagonal entries.
- Take the negative of the off diagonal entries.
- Multiply the result by \(\frac{1}{|A|}\) to get.
INTRODUCTION TO MATRICES

\[
\begin{pmatrix}
\frac{1}{ad-bc} & d \\
-b & c \\
\end{pmatrix}
\]

**Warning:** This technique can be used for a 2 x 2 matrix ONLY.

(iii) When \(|A| = 0\), \(A^{-1}\) does not exist. For example, if \(A = \begin{pmatrix} 8 & 4 \\ 2 & 1 \end{pmatrix}\)

\(|A| = 8 - 8 = 0\). Hence, \(A^{-1}\) does not exist. If \(|A| \neq 0\), \(A\) is said to be non-singular while \(A\) is singular if \(|A| = 0\). Hence the inverse of a singular matrix does not exist.

16.6 SYSTEMS OF LINEAR EQUATIONS

A system of linear equations is of the form:

\[
\begin{align*}
a_1x_1 + a_2x_2 &= c_1 \\
b_1x_1 + b_2x_2 &= c_2
\end{align*}
\]

16.6.1

where \(a_1, a_2, b_1, b_2, c_1\) and \(c_2\) are constants. It is usually required to find the values of \(x_1\) and \(x_2\) which satisfy the system of equations.

Another system of linear equations is:

\[
\begin{align*}
a_1x_1 + a_2x_2 + a_3x_3 &= d_1 \\
b_1x_1 + b_2x_2 + b_3x_3 &= d_2 \\
c_1x_1 + c_2x_2 + c_3x_3 &= d_3
\end{align*}
\]

16.6.2

Three methods of solution will be treated in this pack.

(a) Solution by direct algebraic method.

(b) Solution using the matrix inverse.

(c) Solution using Cramer's rule.
(a) **Direct Method of Solution**

(i) **2 Variables**

\[ x_1 + 2x_2 = 22 \] \hspace{1cm} (i)

\[ x_1 + x_2 = 7 \] \hspace{1cm} (ii)

Eq (i) - Eq (ii) gives,

\[ 3x_2 = 5 \]

\[ x_2 = \frac{5}{3} \]

Substituting this in equation (1), we get,

\[ x_1 + 20 = 22 \]

\[ x_1 = 2 \]

In other words, the solution is \( x_1 = 2; \ x_2 = \frac{5}{3} \), which can be written as

\[
\begin{bmatrix}
  x_1 \\
  x_2 
\end{bmatrix}
= 
\begin{bmatrix}
  2 \\
  \frac{5}{3} 
\end{bmatrix}
\]

(ii) **3 variables**

\[ 3x_1 - x_2 + x_3 = 2 \] \hspace{1cm} (i)

\[ x_1 + 2x_2 + x_3 = 9 \] \hspace{1cm} (ii)

\[ 5x_1 + x_2 - x_3 = 4 \] \hspace{1cm} (iii)

Equation (ii) \( \times 3 \) gives \( 3x_1 + x_2 + 2x_3 = 37 \) \hspace{1cm} (iv)

Eq (iv) - Eq (i) gives \( 11x_2 + x_3 = 35 \)

\[ \Rightarrow \ x_2 + x_3 = 3 \] \hspace{1cm} (v)

Eq (ii) \( \times 5 \) gives \( 5x_1 + 5x_2 + 10x_3 = 35 \) \hspace{1cm} (vi)

Eq (iii) is \( 5x_1 + x_2 - x_3 = 4 \)

Eq (vi) - Eq (ii) gives \( 11x_2 + 13x_3 = 91 \) \hspace{1cm} (vii)

From (v) and (vii),

\[ x_2 + x_3 = 3 \] \hspace{1cm} (v)

\[ 11x_2 + 13x_3 = 91 \] \hspace{1cm} (viii)

Eq (v) \( \times 11 \) gives \( 11x_2 + 13x_3 = 35 \) \hspace{1cm} (ix)
Eq (viii) – eq (ix) gives
\((23 - 11)x_3 = 36\)
\[12x_3 = 36\]
\[x_3 = 3\]

Using \(x_3 = 3\) in (v), we get,
\[x_2 + 2 = 3\]
\[x_2 = 1\]

Putting \(x_2 = 1\) in (i), we get,
\[3x_1 - 4 + 3 = 3\]
\[3x_1 - 1 = 2\]
\[3x_1 = 3\]
\[x_1 = 1\]

In other words,
\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix} = \begin{bmatrix}
  1 \\
  2 \\
  3
\end{bmatrix}
\]
meaning \(x_1 = 1, x_2 = 2, x_3 = 3\)

(b) Solution using Inverse Matrix

(i) A 2 \times 2 Matrix
Refer to the equations
\(x_1 + 4x_2 = 22\)
\(x_1 + x_2 = 7\)

solved by direct algebraic method (by elimination). This system of equations can be written in matrix form as
\[
\begin{bmatrix}
  1 & 4 \\
  1 & 1
\end{bmatrix} \begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} = \begin{bmatrix}
  22 \\
  7
\end{bmatrix}
\]

The solution of which is
\[
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix} = \begin{bmatrix}
  1 & 4 \\
  1 & 1
\end{bmatrix}^{-1} \begin{bmatrix}
  22 \\
  7
\end{bmatrix}
\]
\[
\begin{align*}
\begin{bmatrix}
1 & -4 \\ -3 & 1 
\end{bmatrix} &= 22 \\
\begin{bmatrix}
-6 \\ -15 
\end{bmatrix} &= \frac{6}{3} = 2 \\
\frac{15}{3} &= \frac{5}{1}
\end{align*}
\]

which is the result obtained by the elimination method.

In general, the solution to a system of linear equations,

\[
A X = b \quad \text{………………………….. 8.7.3}
\]

is given by

\[
X = A^{-1} b \quad \text{……………………… 8.7.4}
\]

provided \( |A| \neq 0 \) i.e the matrix is non-singular.

(ii) A 3 \times 3 Matrix

Consider the system of linear equations

\[
\begin{align*}
3x_1 - 2x_2 + x_3 &= 2 \\
x_1 + 3x_2 + 4x_3 &= 9 \\
5x_1 + 4x_2 - 3x_3 &= 4
\end{align*}
\]

which can be written as

\[
\begin{bmatrix}
3 & -2 & 1 \\
1 & 3 & 4 \\
5 & 4 & -3 
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
2 \\
9 \\
4
\end{bmatrix}
\]

or

\[
A X = b
\]
Hence,

$$X = A^{-1}b$$ or

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ 1 & 3 & 4 \\ 5 & 4 & -3 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 19 \\ 4 \end{bmatrix}$$

thus $$A^{-1}$$ is needed

$$A = \begin{bmatrix} 3 & -2 & 1 \\ -1 & 3 & 4 \\ 5 & 4 & -3 \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} 3 & 4 \\ 4 & -1 \end{vmatrix} = -25 \Rightarrow C_{11} = -25$$

$$A_{12} = \begin{vmatrix} 1 & 4 \\ 5 & -1 \end{vmatrix} = -23 \Rightarrow C_{12} = 23$$

$$A_{13} = \begin{vmatrix} 1 & 3 \\ 5 & 4 \end{vmatrix} = -11 \Rightarrow C_{13} = -11$$

$$A_{21} = \begin{vmatrix} -1 & 1 \\ 4 & -4 \end{vmatrix} = 2 \Rightarrow C_{21} = -2$$

$$A_{22} = \begin{vmatrix} 3 & 1 \\ 5 & -4 \end{vmatrix} = -14 \Rightarrow C_{22} = -14$$

$$A_{23} = \begin{vmatrix} 3 & -1 \\ 5 & 4 \end{vmatrix} = 22 \Rightarrow C_{23} = -22$$

$$A_{31} = \begin{vmatrix} -1 & 1 \\ 3 & 4 \end{vmatrix} = -11 \Rightarrow C_{31} = -11$$

$$A_{32} = \begin{vmatrix} 3 & 1 \\ 1 & 4 \end{vmatrix} = 11 \Rightarrow C_{32} = -11$$

$$A_{33} = \begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} = 11 \Rightarrow C_{33} = 11$$
Therefore, the matrix of co-factors is
\[
C = \begin{bmatrix}
-22 & -23 & -11 \\
-2 & -14 & -22 \\
-11 & -11 & 11
\end{bmatrix}
\]

\[
\text{Adj} (A) = C^T = \begin{bmatrix}
-25 & -2 & -11 \\
-23 & -14 & -11 \\
-11 & -22 & 11
\end{bmatrix}
\]

Now, \(|A| = 3A_{11} \cdot (-2)A_{12} + (1)A_{13} = 3A_{11} + 2A_{12} + A_{13} = (3)(-25) + 2(-23) + (-11) = -75 - 46 - 11 = -132\)

Since \(|A| = -132\), then
\[
A^{-1} = \frac{1}{|A|} \text{Adj}(A) = -\frac{1}{132} \begin{bmatrix}
-25 & -2 & -11 \\
-23 & -14 & -11 \\
-11 & -22 & 11
\end{bmatrix}
\]

\[
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A^{-1} \begin{bmatrix} 2 \\ 19 \\ 4 \end{bmatrix}
\]

\[
= \frac{1}{132} \begin{bmatrix}
25 & 2 & 11 \\
-23 & 14 & 11 \\
11 & 22 & -11
\end{bmatrix} \begin{bmatrix} 2 \\ 19 \\ 4 \end{bmatrix}
\]

\[
= \frac{1}{132} \begin{bmatrix}
50 + 38 + 44 \\
-46 + 266 + 44 \\
22 + 418 - 44
\end{bmatrix} = \frac{1}{32} \begin{bmatrix} 132 \\ 264 \\ 392 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}
\]
(c) Solution by Cramer’s rule

(i) The \((2 \times 2)\) matrix case.

\[ x_1 + 4x_2 = 22 \]
\[ x_1 + x_2 = 7 \]

or

\[
\begin{bmatrix}
1 & 4 \\
1 & 1 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix}
=
\begin{bmatrix}
22 \\
7 \\
\end{bmatrix}
\]

\[
AX = b
\]

By Cramer’s rule,

\[
x_1 = \frac{\begin{bmatrix} 22 & 4 \\ 7 & 1 \end{bmatrix}}{\Delta},
\quad
x_2 = \frac{\begin{bmatrix} 1 & 22 \\ 1 & 7 \end{bmatrix}}{\Delta}
\]

\[ x_1 = \frac{\Delta_1}{\Delta}, \quad x_2 = \frac{\Delta_2}{\Delta} \]

where \(\Delta : A|, \Delta\) is the determinant obtained by replacing the 1st column of \(A\) by the matrix \(\begin{bmatrix} 22 \\ 7 \end{bmatrix}\) and \(\Delta\) is the determinant obtained by replacing the 2nd column of \(A\) by the matrix \(\begin{bmatrix} 22 \\ 7 \end{bmatrix}\).

Thus,

\[ \Delta = \begin{vmatrix} 22 & 4 \\ 7 & 1 \end{vmatrix} = 22 - 4 \cdot 7 = 2 \]

\[ \Delta = \begin{vmatrix} 1 & 22 \\ 1 & 7 \end{vmatrix} = 1 \cdot 7 - 22 \cdot 1 = -15 \]

\[ \Delta = \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = -3 \]

Hence,

\[ x_1 = \frac{\Delta_1}{\Delta} = \frac{2}{2 - 15} = \frac{2}{-13} \]
\[ x_2 = \frac{x_2}{\Delta} = -\frac{5}{32} = : \]

Therefore, \[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix} = \begin{bmatrix}
  2 \\
  19 \\
  4
\end{bmatrix}
\]

(ii) The \(3 \times 3\) matrix case

\[3x_1 - x_2 + x_3 = 2\]
\[x_1 + x_2 + 4x_3 = 9\]
\[5x_1 + x_2 - x_3 = 1\]

\[
\begin{bmatrix}
  3 & -2 & 1 \\
  1 & 3 & 4 \\
  5 & 4 & -3
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix} = \begin{bmatrix}
  2 \\
  19 \\
  4
\end{bmatrix}
\]

\[\Delta = \begin{vmatrix}
  3 & -2 & 1 \\
  1 & 3 & 4 \\
  5 & 4 & -3
\end{vmatrix} = -32 \text{ from previous calculation}
\]

\[
\Delta_1 = \begin{vmatrix}
  2 & -2 & 1 \\
  19 & 3 & 4 \\
  4 & 4 & -3
\end{vmatrix} = 2(-9-16) + 2(-57-16) + 1(76-12) = 132
\]

\[
\Delta_2 = \begin{vmatrix}
  3 & 2 & 1 \\
  1 & 19 & 4 \\
  5 & 4 & -3
\end{vmatrix} = 3(-57-16) - 2(-3-20) + 1(95-95) = -264
\]

\[
\Delta_3 = \begin{vmatrix}
  3 & -2 & 2 \\
  1 & 3 & 19 \\
  5 & 4 & 4
\end{vmatrix} = 3(12-76) + 2(4-95) + 2(4-15) = -396
\]

Hence,

\[ x_1 = \frac{x_1}{\Delta} = -\frac{32}{32} = 1 \]

\[ x_2 = \frac{x_2}{\Delta} = -\frac{64}{32} = : \]
Recall that $\Delta$ is the determinant obtained by replacing the 1\textsuperscript{st} column of $A$ by $\begin{bmatrix} 2 \\ 19 \\ 4 \end{bmatrix}$ and so on.

\textbf{16.7 BUSINESS APPLICATION}

The following is a typical example of the use of matrices in business.

\textbf{ILLUSTRATION 16-7}

There are 30 secondary schools and 60 primary schools in a Local Government Area (LGA) in Ekiti State. Each of the secondary schools and the primary schools has 1 messenger (ME), 5 clerks (CL) and 1 cashier (CA). Each secondary school in addition has 1 accountant (AC) and 1 head clerk (HC). The monthly salary (₦'000) of each of them is as follows:

- Messenger – ₦20,
- Clerk – ₦40,
- Cashier – ₦35,
- Accountant – ₦50,
- Head clerk – ₦60.

Use matrices to find

(i) The total number of posts of each kind of primary and secondary schools taken together.

(ii) The total monthly salary of each primary school and each secondary school separately.

(iii) The total monthly salary bill of each primary school and each secondary school separately.

(iv) The total monthly salary bill of all secondary and primary schools.

\textbf{SUGGESTED SOLUTION 16-7}

Consider the row matrix of the two types of schools:

$$A = (30, 60)$$

Representing the number of secondary schools and primary schools respectively.
Let  \[ \begin{bmatrix} ME & CL & CA & AC & HC \\ 1 & 5 & 1 & 1 & 1 \\ 1 & 5 & 1 & 0 & 0 \end{bmatrix} = B \]

This is the matrix of the number of persons in each position for each type of school.

Consider the product matrix

\[ AB = \begin{bmatrix} 30 & 60 \\ 1 & 5 & 1 & 1 & 1 \\ 1 & 5 & 1 & 0 & 0 \end{bmatrix} \]

\[ = \begin{bmatrix} 90 & 450 & 90 & 30 & 30 \end{bmatrix} \]

Then, there are in the two types of schools,

90 messengers
450 clerks
90 cashiers
30 accountants
30 head clerks

(ii) The monthly salaries (₦'000) can be put as a column matrix

\[ C = \begin{bmatrix} 20 \\ 40 \\ 35 \\ 50 \\ 60 \end{bmatrix} \]

Total monthly bill of each kind of school is

\[ BC = \begin{bmatrix} 1 & 5 & 1 & 1 & 1 \\ 1 & 5 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 20 \\ 40 \\ 35 \\ 50 \\ 60 \end{bmatrix} \]

\[ = \begin{bmatrix} 20 + 200 + 35 + 50 + 60 \\ 20 + 200 + 35 + 0 + 0 \end{bmatrix} \]
Total monthly salary bill for secondary schools = ₦365,000 while total monthly salary for primary schools = ₦255,000.

(iii) Now, A (BC) gives the total monthly salary bill for all secondary and primary schools.

\[
A \ (B) = \begin{bmatrix}
30 & 60 \\
6 & 270
\end{bmatrix}
\]

\[
= 7950 + 15300
\]

\[
= ₦23,250 (\text{'}000)
\]

\[
= ₦23,250,000
\]

**16.8 SUMMARY AND CONCLUSIONS**

A matrix is introduced as a 2 – dimensional array of numbers in rows and columns. Few commonly used matrices like the null matrix, the identity matrix and a square are introduced. Others include Triangular matrix, symmetric and skew-symmetric matrices. Some basic matrix operations such as transpose of a matrix, addition of two matrices and multiplication of a matrix by a scalar are presented.

The technique of multiplying one matrix on the left by another one on the right (when the product can be performed in that order) is outlined. It is emphasized that the product AB is not necessarily the same as BA. The evaluation of 2 by 2 and 3 by 3 determinants is treated with some examples. Condition for the inverse of a square matrix is discussed and the evaluation technique of such an inverse (when it exists) is done using the adjoint method for both 2 by 2 and 3 by 3 matrices. The techniques of finding the solution of a system of linear equations in not more than 3 independent variables are presented by means of illustrative examples. The methods of solution include (i) direct elimination by substitution, (ii) matrix inversion and (iii) Cramer’s rule. The treatment of matrices is concluded with the applications of the theory to solving problems in economics and business.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
16.10 REVISION QUESTIONS

16.9.1 MULTIPLE CHOICE QUESTIONS

1. The matrix \[
\begin{pmatrix}
4 & 2 \\
3 & 1 \\
8 & 11
\end{pmatrix}
\]
is

A. A 3 by 2 matrix  
B. A 3 by 3 matrix  
C. A 2 by 3 matrix  
D. An identity matrix  
E. A 2 by 2 matrix.

2. The matrix \[
\begin{pmatrix}
0 & 5 \\
-5 & 0
\end{pmatrix}
\]
is

A. A diagonal matrix  
B. A triangular matrix  
C. A systematic matrix  
D. A skew-symmetric matrix  
E. A Null matrix

3. Evaluate \[
\begin{pmatrix}
5 & 3 \\
-5 & 3
\end{pmatrix}
\]

A. 9  
B. -9  
C. 21  
D. -11  
E. 1

16.9.2 SHORT ANSWER QUESTIONS

Use the following information to answer questions 1 - 4

\[
A = \begin{pmatrix}
2 & 4 & 1 \\
3 & -5 & 0
\end{pmatrix}
\]
and

\[
B = \begin{pmatrix}
6 & 7 & 1 \\
0 & 0 & 2
\end{pmatrix}
\]

1. find \(A + B\)
2. find \(AB\)
3. find \(A^TB\)
4. find \(|AB^T|\)

Use the following information to answer questions 5 – 7

\[
\begin{pmatrix}
2 & 3 \\
4 & 5
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} = \begin{pmatrix}
25 \\
43
\end{pmatrix}
\]
is written as \(AX=b\)
5. Write down $A$ and $b$
6. Find $A^{-1}$
7. Find the matrix $X$.

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
BASIC CONCEPTS OF DIFFERENTIATION

17.0 LEARNING OBJECTIVES

After studying this chapter, readers should be able to:

◆ Understand the concept of derivatives;
◆ Differentiate various functions such as polynomial, product, quotient, function;
Of function, implicit function, exponential and logarithmic functions;
◆ Obtain the second-order derivatives;
◆ Apply the principle of differentiation to economic and business problems such as finding marginals, elasticity, maximum and minimum values; and
◆ Understand the concept of simple partial differentiation of first order only.

17.1 INTRODUCTION

(a) Concept and Meaning of Differentiation

The concept of differentiation is important as it has a lot of applications in many areas of human endeavours such as the fields of Engineering, Science, Economics, Business just to mention a few.

Differentiation is a mathematical concept dealing with the rate of change. This rate of change is generally termed the derivative, slope, gradient or marginal measure in various areas of usage.

Formally, differentiation (the differential calculus) is the process of finding the derivative of a function.
The question is how to find a derivative?

To answer this question, there is need to apply the Limit's theory to the incremental principle of the variables involved in the function. For instance, let us consider the function \( y = f(x) \) (i.e. \( y \) is a function of \( x \)). A small increment in \( x \) (represented by \( \Delta x \)) will cause a small increment in \( y \) (also represented by \( \Delta y \)).

Then, for
\[
y = f(x) \quad \text{................................. (i)}
\]
we have

\[
y + \Delta y = f(x + \Delta x) \quad \text{(as there are small increments in both} \ x \ \text{and} \ y \ \text{variables) ......................... (ii)}
\]

Equation (ii) - equation (i) gives

\[
(y + \Delta y) - y = f(x + \Delta x) - f(x)
\]
\[
\Delta y = f(x + \Delta x) - f(x)
\]
\[
\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad \text{.................. (iii)}
\]

As \( \Delta x \to 0 \) Equation (iii) becomes

\[
\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left\{ \frac{f(x + \Delta x) - f(x)}{\Delta x} \right\} \quad \text{.......... (iv)}
\]

By the Limits theory, (iv) becomes

\[
\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad \text{............... (v)}
\]

Note that \( \frac{dy}{dx} \) is also represented by \( f'(x) \), which is called the derivative of \( f(x) \). Other notations that are used for derivative are:

\[
\frac{d}{dx} (f) \quad \text{(read as dee } \ x \text{ of } f)
\]
\[
\frac{df}{dx} \quad \text{(read as } f \text{ dee } x)
\]
\[
y' \quad \text{(read as prime of } y \text{ or } y \text{ prime)}
\]
If the left hand side of equations (v) exists, then \( \frac{dy}{dx} \) is called the differential coefficient of \( y \) with respect to \( x \).

A collection of equations (i) to (v) expressed above in finding the derivative of a function from the consideration of limiting value, is called “differentiation from first principles”.

Let us further demonstrate the principle by obtaining the derivative of the following function from the first principle.

\[ y = ax^n, \] (where \( a \) is a constant)

The derivative from the first principles is as follows:

Given \( y = ax^n \)  ..................................................  (i)

Let \( y + \Delta y = a(x+\Delta x)^n \)  ....................................  (ii)

Then \( (y+\Delta y) - y = a(x+\Delta x)^n - ax^n \)

\[
\frac{\Delta y}{\Delta x} = ax^n \left\{ \left[ 1 + n \frac{(\Delta x)}{x} + \frac{n(n-1)}{2!} \left( \frac{(\Delta x)}{x} \right)^2 + \ldots \right] - 1 \right\} 
\]

\[
\frac{\Delta y}{\Delta x} = a(nx^{n-1}\Delta x + \frac{n(n-1)}{2!} x^{n-2} (\Delta^2 x + \ldots})
\]

\[
\therefore \frac{\Delta y}{\Delta x} = \text{an}x^{n-1} + \text{terms containing higher powers of } \Delta x
\]

Limit \( \frac{\Delta y}{\Delta x} \) = Limit \( a(nx^{n-1}) + \text{terms containing powers of } \Delta x \)

\[
\Delta x \to 0 \quad \Delta x \to 0
\]

\[ \therefore \frac{dy}{dx} = a(nx^{n-1}) \]

Therefore, \( \frac{dy}{dx} = \text{an}x^{n-1} \)  ..................................  (iii)

Equation (iii) is the derivative of \( y = ax^n \) and it shall be applied as a rule.
ILLUSTRATION 17-1

Obtain the derivative of the following:

(i) \( x^3 \)
(ii) \( 2x^5 \)

SUGGESTED SOLUTION 17-1

(i) \( y = x^3 \)
Here, \( a = 1, \ n = 3 \)
\[ \frac{dy}{dx} = 3(X^{3-1}) = 3x^2 \]

(ii) \( y = 2x^5 \)
Also, \( a = 2, \ n = 5 \)
\[ \frac{dy}{dx} = 2(5)(X^{5-1}) = 10x^4 \]

ILLUSTRATION 17-2

Obtain the value of the derivative for the following at \( x = 2 \):

\( y = 3X^3 \)
(ii) \( 2X^5 \)

SUGGESTED SOLUTION 17-2

(i) \( y = 3X^3 \)
\[ \frac{dy}{dx} = 3(3)(X^{3-1}) = 9X^2 \]
At \( x = 2, \) \[ \frac{dy}{dx} = 9(2)^2 = 36 \]

(ii) \( y = 2X^5 \)
\[ \frac{dy}{dx} = 2(5)(X^{5-1}) = 10X^4 \]
At \( x = 2, \) \[ \frac{dy}{dx} = 10 \times (2)^4 = 160 \]

(b) Rules for Differentiation: The following rules shall be applied to various functions:

Rule 1. If a Polynomial \( y = u + v + w + \ldots \ldots \ldots \) where \( u, v, w \) are functions of \( x \). then
\[ \frac{dy}{dx} = \frac{d}{dx} (u + v + w + \ldots \ldots \ldots) \]
\[
\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} + \ldots
\]

Hence, the derivative of a sum is the sum of the derivates.

ILLUSTRATION 17-3

Find the derivative of each of the following:

(i) \(2x^4 - 3x^3 + 5x^2\);  
(ii) \(2x^3 + 3x^2 - \frac{2}{x}\)

SUGGESTED SOLUTION 17-3

(i) \(y = 2x^4 - 3x^3 + 5x^2\)

\[
\frac{dy}{dx} = 2(4)(x^{4-1}) - 3(3)(x^{3-1}) + 5(2)(x^{2-1})
\]

\[
\frac{dy}{dx} = 8x^3 - 9x^2 + 10x
\]

(ii) \(y = 2x^3 + 3x^2 - \frac{2}{x}\)

\[
y = 2x^3 + 3x^2 - 2x^{-1}
\]

\[
\frac{dy}{dx} = 2(3)(x^{3-1}) + 3(2)(x^{2-1}) - 2(-1)(x^{-1-1})
\]

\[
= 6x^2 + 6x + 2x^2
\]

\[
= 6x^2 + 6x + \frac{2}{x^2}
\]

Rule 2. If \(y = Cu\), where \(C\) is a constant and \(u\) a function of \(x\), then

\[
\frac{d}{dx} Cu = C \frac{du}{dx}
\]
ILLUSTRATION 17-4
Find the derivative of the following:

\[ y = 3(x^2 + 2x) \]

SUGGESTED SOLUTION 17-4

\[
\frac{dy}{dx} = 3 \left( \frac{d}{dx} (x^2 + 2x) \right)
\]

\[
= 3(2x + 2)
\]

Rule 3. If \( y = C \), where \( C \) is a constant, then

\[
\frac{dy}{dx} = 0
\]

Rule 4. If a product \( y = uv \), where \( u \) and \( v \) are functions of \( x \), then

\[
\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}
\]

By extension, if \( y =uvw \), where \( u, v \) and \( W \) are functions of \( x \), then

\[
\frac{dy}{dx} = vw \frac{du}{dx} + uw \frac{dv}{dx} + uv \frac{dw}{dx}
\]

The above is the product rule.

ILLUSTRATION 17-5
Find the derivative for the following:

(i) \( y = (2x + 1) (3x^2 + 2x) \)

SUGGESTED SOLUTION 17-5

Here, let \( u = 2x + 1 \), \( \frac{du}{dx} = \)

\( v = 3x^2 + 2x \), \( \frac{dv}{dx} = 6x + 2 \)
But for \( y = uv \), the derivative is

\[
\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}
\]

\[
= (2x +1) (6x +2) + (3x^2 + 2x)2
\]

\[
= 12x^2 + 6x + 4x + 2 + 6x^2 + 4x
\]

\[
\frac{dy}{dx} = 18x^2 + 14x + 2
\]

(ii) \( y = (2x+4) (x^2 +3x) (2x^3 + 5x) \)

Let \( u = 2x + 4, \) \( \frac{du}{dx} = 2 \)

\( v = x^2 + 3x, \) \( \frac{dv}{dx} = 2x + 3 \)

\( w = 2x^3 + 5x, \frac{dw}{dx} = 6x^2 + 5 \)

But

\[
\frac{dy}{dx} = vw \frac{du}{dx} + uw \frac{dv}{dx} + vu \frac{dw}{dx}
\]

\[
= (x^2 + 3x) (2x^3 + 5x)2 + (2x + 4) (2x^3 + 5x) (2x + 3)
\]

\[
+ (x^2 +3x) (2x + 4) (6x^2 + 5)
\]

**Rule 5.** If quotient \( y = \frac{u}{v} \), where \( u \) and \( v \) are functions of \( x \), then

\[
\frac{dy}{dx} = \frac{v \frac{du}{dx} + u \frac{dv}{dx}}{v^2}
\]

**ILLUSTRATION 17-6**

Differentiate the following with respect to \( x \):

(i) \( y = \frac{1 + x^2}{3x} \); (ii) \( \frac{2 + 3x}{x^2 + 3x + 5} \)
SUGGESTED SOLUTION 17-6

(i) \( y = \frac{1 + x^2}{3x} \)

Let \( u = 1 + x^2 \), \( \frac{du}{dx} = 2x \)

\( v = 3x \), \( \frac{dv}{dx} = 3 \)

But \( \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \)

\( = \frac{3x(2x) - (1 + x^2)3}{(3x)^2} \)

\( = \frac{6x^2 - 3 - 3x^2}{(3x)^2} = \frac{3x^2 - 3}{(3x)^2} \)

\( = 1 - \frac{1}{x} \)

(ii) \( y = \frac{2 + 3x}{x^2 + 3x + 5} \)

Let \( u = 2 + 3x \), \( \frac{du}{dx} = 3 \)

\( v = x^2 + 3x + 5 \), \( \frac{dv}{dx} = 2x + 3 \)

But \( \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \)

\( \therefore \) substituting gives

\( \frac{dy}{dx} = \frac{(x^2 + 3x + 5)3 - (2 + 3x)(2x + 3)}{(x^2 + 3x + 5)^2} \)

\( = \frac{3x^2 + 9x + 15 - 4x - 6x^2 - 6 - 9x}{(x^2 + 3x + 5)^2} \)

\( = \frac{-3x^2 - 4x + 9}{(x^2 + 3x + 5)^2} \)
Rule 6. If a function of function \( y = (ax + b)^n \), where \( a \) and \( b \) are constants and \( n \) is the power of index, then

\[
\frac{dy}{dx} = \frac{d}{dx} (ax + b)^n
\]

\[
= n(ax + b)^{n-1} \left( \frac{d}{dx} (ax + b) \right)
\]

This rule is also called the \textbf{composite or chain rule}. In another way, this can be expressed as:

\[
\frac{dy}{dx} = \left( \frac{dy}{du} \right) \left( \frac{du}{dx} \right)
\]

where \( y \) is a function of \( u \) and \( u \) is a function of \( x \). It is also called \textbf{function of a function}.

**ILLUSTRATION 17-7**

Differentiate the following:

(i) \( y = (x + 2)^3 \) ;

(ii) \( y = (3x^2 - 3)^4 \)

**SUGGESTED SOLUTION 17-7**

(i) \( y = (x + 2)^3 \)

Let \( u = (x + 2) \), \( \frac{du}{dx} = 1 \)

then, \( y = u^3 \), \( \frac{dy}{du} = 3u^2 \)

\[
\therefore \quad \frac{dy}{dx} = \left( \frac{dy}{du} \right) \left( \frac{du}{dx} \right) = 3u^2 (1) = 3u^2 = 3(x + 2)^2
\]

(ii) \( y = (3x^2 - 4)^4 \)

Let \( u = 3x^2 - 4 \), \( \frac{du}{dx} = 6x \)

\[
y = u^4 \quad , \quad \frac{dy}{du} = 4u^3
\]

\[
\therefore \quad \frac{dy}{dx} = \left( \frac{dy}{du} \right) \left( \frac{du}{dx} \right) = 4u^3 (6x) = 24xu^3 = 24x(3x^2 - 4)^3
\]
**Rule 7.** For the implicit, the relationship between \( y \) and \( x \) variables may not be expressed explicitly (e.g. \( x^2 + y^2 = 4 \)). In this case, treat \( y \) as if it is a function of \( x \) and the rules of differentiation discussed above are applied as appropriate.

The following examples are used for explanation:

**ILLUSTRATION 17-8**

Differentiate the following implicitly:

(i) \( x^2 + y^3 = 7 \) ;
(ii) \( 5y^2 - 3x^2y^3 + 2y = 0 \)

**SUGGESTED SOLUTION 17-8**

(i) \( x^2 + y^3 = 7 \)

Differentiate the function term by term with respect to variable \( x \), then:

\[
2x \frac{dx}{dx} + 3y^2 \frac{dy}{dx} = 0
\]

\[
2x + 3y^2 \frac{dy}{dx} = 0
\]

\[
3y^2 \frac{dy}{dx} = -2x
\]

\[
\frac{dy}{dx} = \frac{x}{3y^2} \text{ (by making } \frac{dy}{dx} \text{ the subject)}
\]

(ii) \( 5y^2x - 3x^3 + 2y = 0 \)

\[
5(y^2 \frac{dx}{dx} + 2yx \frac{dy}{dx}) - 3(2x \frac{dx}{dx} y^3 + 3x^2y^2 \frac{dy}{dx}) + 2 \frac{dy}{dx} = 0
\]

\[
5(y^2 + 2xy \frac{dy}{dx}) - 3(2xy^3 + x^3y^2 \frac{dy}{dx}) + 2 \frac{dy}{dx} = 0 \text{ .......}
\]

*In this expression, 1\textsuperscript{st} and 2\textsuperscript{nd} terms are handled by the product rule. To simplify the expression,

\[
5y^2 + 10xy \frac{dy}{dx} - 9x^2y^2 \frac{dy}{dx} - 6xy^3 + 2 \frac{dy}{dx} = 0
\]

\[
\Rightarrow 10xy \frac{dy}{dx} - 9x^2y^2 \frac{dy}{dx} + 2 \frac{dy}{dx} = -5y^2 + 6xy^3
\]

\[
\Rightarrow \frac{dy}{dx} \left[10xy - 9x^2y^2 + 2\right] = -5y^2 + 6xy^3
\]
\[ \frac{dy}{dx} = \frac{-y^2 + x^3}{[10xy - 9x^2y^2 + y]} \]

**Rule 8.** If the exponential function \( y = e^{ax+b} \), where \( a \) and \( b \) are constants, then
\[
\frac{dy}{dx} = \frac{d}{dx}(e^{ax+b})
\]
\[ = \frac{d}{dx}(ax+b)e^{ax+b} \]
\[ = ae^{ax+b} \]

**ILLUSTRATION 17-9**

Differentiate the following with respect to \( X \):

(i) \( y = 3e^{2x^2+x} \) ;

(ii) \( y = X^3e^{x^3} \)

**SUGGESTED SOLUTION 17-9**

(i) \( y = 3e^{2x^2+x} \)
\[
\frac{dy}{dx} = \frac{d}{dx}(2x^2 + x) \cdot 3e^{2x^2+x}
\]
\[ = (4x + 1) \cdot 3e^{2x^2+x} \]
\[ = 3(4x + 1)e^{2x^2+x} \]

(ii) \( y = x^4e^{x^3} \)

The product rule will be applied here.

Let \( u = x^4 \), \( \frac{du}{dx} = 4x^3 \)
\( v = e^{x^3} \), \( \frac{dv}{dx} = 3x^2 e^{x^3} \)

But \[ \frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx} \]
\[ = e^{x^3} \cdot (4x^3) + x^4(3x^2 e^{x^3}) \]
\[ = e^{x^3}[4x^3 + 3x^6] \]
Rule 9. If the logarithmic function $y = \log_e (ax+b) = \ln(ax+b)$, where $a$ and $b$ are constant, then

$$\frac{dy}{dx} = \frac{1}{ax+b} \cdot \frac{d}{dx} (ax+b) = \frac{a}{ax+b}.$$ 

ILLUSTRATION 17-10

Differentiate the following with respect to $X$:

(i) $y = \log_e (3X+2)$

(ii) $y = \log_e (4X-1)^2$

SUGGESTED SOLUTION 17-10

(i) $y = \log_e (3x+2)$

$$\frac{dy}{dx} = \frac{1}{3x+2} \cdot \frac{d}{dx} (3x+2) = 3$$

(ii) $y = \log_e (4x-1)^2$

$$\frac{dy}{dx} = \frac{1}{(4x-1)^2} \cdot \frac{d}{dx} (4x-1)^2$$

$$= \left\{ \frac{1}{(4x-1)^2} \right\} (4)(2)(4x-1)$$

$$= \frac{8}{(4x-1)}$$

17.2 SECOND – ORDER DERIVATIVE

Second – order derivative is one of higher derivatives where successive differentiations are carried out. The process of differentiating a function more than once is called Successive differentiation.

If $y = f(x)$, $\frac{dy}{dx}$ is also a function of $x$.

The derivative of $\frac{dy}{dx}$ with respect to $x$ is $\frac{d}{dx} \left( \frac{dy}{dx} \right)$. The expression $\frac{d}{dx} \left( \frac{dy}{dx} \right)$ is called the second – order derivative of $y$ with respect to $x$, and it is denoted by $\frac{d^2y}{dx^2}$ or $y''$ or $f''(x)$. It is useful when determining the turning point of a function.
ILLUSTRATION 17-11
Find the first and second derivatives of the following: (i) \( y = 3x^5 \); (ii) \( y = 2x^5 - 3x^4 + 2x^2 - 6 \)

SUGGESTED SOLUTION 17-11
(i) \( y = 3x^5 \), \( \frac{dy}{dx} = (3)(5)x^{5-1} = 15x^4 \)
for the second derivative,
\[
\frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} (15x^4) = (15)(4)x^{4-1} = 60x^3
\]
(ii) \( y = 2x^5 - 3x^4 + 2x^2 - 6 \)
First derivative, \( \frac{dy}{dx} = (2)(5)x^{5-1} - (3)(4)x^{4-1} + (2)(2)x^{2-1} \)
\[
= 10x^4 - 12x^3 + 4x
\]
Second derivative, \( \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d^2y}{dx^2} \)
\[
= 10(4)x^{4-1} - 12(3)x^{3-1} + 4
= 40x^3 - 36x^2 + 4
\]

17.3 APPLICATION OF DIFFERENTIATION
As earlier stated, differentiation has a lot of applications in various fields. However, in this study-pack the focus is on the business areas of application such as marginal cost and revenue, elasticity of demand and, maximum and minimum values of a function.

(a) Applications to Maximum and Minimum values
The concept of maximum and minimum values is termed the zero slope analysis. The focus here is to find a point where the slope of the function to be optimized is zero.

It should be noted that the concept is not restricted to plotting of curves alone, but more importantly to determine the maximum and minimum values of a function such as maximizing profit or minimizing cost, etc.

We have the following procedure for the determination of maximum and minimum values of a function \( y = f(x) \) if it exists:
Step 1: Obtain the first derivative and set it equal to zero, i.e.
\[ \frac{dy}{dx} = 0 \] This occurs at the turning point

Step 2: From the \[ \frac{dy}{dx} = 0 \], determine the stationary point.

Step 3: Compute \[ \frac{d^2y}{dx^2} \] at these stationary points.

If \( \frac{d^2y}{dx^2} < 0 \), the stationary point is a maximum point, and if \( \frac{d^2y}{dx^2} > 0 \), the stationary point is a minimum point.

If \( \frac{d^2y}{dx^2} = 0 \), higher derivative (than second) can be used to decide. This is called point of inflexion.

**ILLUSTRATION 17-12**

A Company planning to have a new product in the market came up with a total sales function \( S = -2000P^2 + 20000P \) and the total cost function \( C = -4000P + 50000 \), where \( P \) and \( C \) are respectively the price and cost (in Naira) of the new product.

Determine the optimal price for the new product.

**SUGGESTED SOLUTION 17-12**

The profit \( \pi \) is obtained by the difference in total sales and total cost.

\[ \therefore \pi = S - C \]
\[ = -2000P^2 + 20000P - (-4000P + 50000) \]
\[ = -2000P^2 + 24000P - 50000 \]

i.e. \( \frac{d\pi}{dP} = -4000P + 24000 \)
But at the turning point, \( \frac{d\pi}{dP} = 0 \), then
\[
0 = -4000P + 24000
\]
\[
4000P = 24000
\]
\[
P = \frac{24000}{4000} = 6
\]

But \( \frac{d^2\pi}{dP^2} = -4000 \) (which is less than zero)

Hence, \( P = 6 \), gives a maximum point.

Therefore, the optimal price is ₦6.

(a) Applications to Marginal Cost and Revenue

This section deals with the applications of differentiation to the:

(i) Relationship between Average and Marginal Costs.

(ii) Relationship between Average and Marginal revenues.

For the cost analysis, i.e. case (i):

Let \( T = \) Total Cost
\( q = \) quantity demand

Then, the Average Cost (AC) = \( \frac{T}{q} \) and,

the Marginal Cost (MC) = \( \frac{dT}{dq} \)

Also, the slope of average cost is obtained by:
\[
\frac{d}{dq} \left( \frac{T}{q} \right) = \frac{\frac{dT}{dq} q - T \frac{dq}{dq}}{q^2} = \frac{1}{q} \left( \frac{dT}{dq} - \frac{T}{q} \right) = \frac{1}{q} (MC - AC).
It is established that if the cost curve is of U-shape and the AC has a sloping downward curve, then
\[
\frac{d}{dq} \frac{T}{q} < 0
\]

It means that \( MC < AC \).

When \( AC \) decreases, \( MC < AC \) and on the lowest point of \( AC \) curve, the tangent will be horizontal. Therefore
\[
\left( \frac{d}{dq} \right) \frac{T}{q} = 0,
\]
i.e. \( MC = AC \).

Also, when \( AC \) is increasing, \( MC > AC \) and
\[
\frac{d}{dq} \left( \frac{T}{q} \right) > 0,
\]

In case (ii) above

Let Total Revenue = \( R \) and we know that \( R = Price \times quantity \), i.e. \( R = Pq \), where \( P = Price \) and \( q = quantity \)

This price is also called Demand

Then, Average Revenue (AR) = \( \frac{R}{q} \) and the

Marginal Revenue (MR) = \( \frac{d}{dq} (R) \)

\[
= P + \frac{dP}{dq}
\]

In terms of relationship, it has been established that the rate of fall of MR is twice of AR.

**ILLUSTRATION 17-13**

Determine the minimum average cost if the cost function is given by:

\( T = 72q - 20q^2 + 4q^3 \). Also obtain the marginal cost at the point of minimum average cost.
**SUGGESTED SOLUTION 17-13**

Given  \( T = 72q - 20q^2 + 4q^3 \)

\[ \text{AC} = \frac{T}{q} = 72 - 20q + 4q^2 \]

For maximum or minimum, \( \frac{dT}{dq} \) = 0

i.e. \( -20 + 8q = 0 \)

\[ \Rightarrow q = \frac{20}{8} = \frac{5}{2} \]

Further, \( \frac{d^2T}{dq^2} \) = 8, which is greater than zero.

Hence, \( q = \frac{5}{2} \) is at minimum point.

\[ \text{The Minimum Average Cost} = 72 - 20\left(\frac{5}{2}\right) + 4\left(\frac{5}{2}\right)^2 \]

\[ = 47 \]

For the MC, we have

\[ \text{MC} = \frac{dT}{dq} = 72 - 40q + 12q^2 \]

MC at \( q = \frac{5}{2} \) is equal to \( 72 - 40\left(\frac{5}{2}\right) + 12\left(\frac{5}{2}\right)^2 \)

i.e. \( \text{MC} = 72 - 100 - 75 = 47 \)

**ILLUSTRATION 17-14**

Determine the maximum profit of a company with revenue function

\( R = 200q - 2q^2 \) and the cost function \( T = 2q^3 - 57q^2 \).

**SUGGESTED SOLUTION 17-14**

\[ \text{Profit (} \pi \text{)} = R - T \]

\[ = 200q - 2q^2 - 2q^3 + 57q^2 \]

i.e. \( \pi = 200q + 55q^2 - 2q^3 \)
At the turning point, \( \frac{d\pi}{dq} = 0 \)

\[
\frac{d\pi}{dq} = 200 + 110q - 6q^2 = 0
\]

\[
\Rightarrow 100 + 55q - 3q^2 = 0
\]

\[
\Rightarrow 3q^2 - 55q - 100 = 0
\]

\[
\Rightarrow 3q^2 - 60q + 5q - 100 = 0
\]

\[
\Rightarrow 3q(q - 20) + 5(q - 20) = 0
\]

\[
\Rightarrow (3q + 5)(q - 20) = 0
\]

\[
\Rightarrow q = 20 \text{ or } -5/3
\]

But \( \frac{d^2\pi}{dq^2} = 110 - 12q \)

\[
\therefore \text{ at } q = 20, \frac{d^2\pi}{dq^2} = 110 - 240 = -130 < 0
\]

Hence, at \( q = 20 \), we have the maximum profit.

\[
\therefore \text{ The maximum profit } = 200(20) + 55(20)^2 - 2(20)^3
\]

\[
= 4000 + 22000 - 16000
\]

\[
= 10,000
\]

(c) **Application of differentiation to the elasticity of demand**

The price elasticity of demand is the rate of change in response to quantity demanded to the change in price. Therefore, the price elasticity of demand is given by \( e_d \)

\[
e_d = \frac{p}{x} \cdot \frac{1}{\frac{dP}{dx}} \text{ or } \left[ \frac{p}{x} \right] \left[ \frac{dp}{dx} \right]
\]

where \( P = \text{Price} \) and \( x = \text{quantity demanded} \)

When \( e_d > 1 \), the demand is elastic and,

When \( e_d < 1 \), the demand is inelastic.

Also, we have income elasticity of demand. This is an elasticity of quantity demanded in response to change in income. It is
denoted by \( e_y \) and defined by \( e_y = \frac{y}{x} \cdot \frac{1}{\frac{dy}{dx}} \) or \( \frac{d}{dx} \right) \),

where \( x \) is the quantity demanded and \( y \) is the income.

**ILLUSTRATION 17-15**

Given a demand function \( P = 50 - x - x^2 \), determine its elasticity of demand.

**SUGGESTED SOLUTION 17-15**

\[
\begin{align*}
P &= 50 - x - x^2 \\
\frac{dp}{dx} &= \frac{P}{x} + \frac{dp}{dx} \\
\text{But } e_d &= \frac{P}{x} + \frac{dp}{dx} \\
&= \left( \frac{P}{x} \right) \left( \frac{1}{\frac{dp}{dx}} \right) \\
&= \frac{50 - x - x^2}{x} \cdot \frac{1}{x} \\
&= \frac{50 - x - x^2}{x(1 + x)} = \frac{-x^2 + x - 0}{x(1 + x)} \\
&= \frac{x^2 + x - 0}{x(1 + x)}
\end{align*}
\]

**17.4 PARTIAL DIFFERENTIATION OF FIRST ORDER**

Partial derivative is applicable when there are more than one variable in a function and the focus is to obtain the derivative with respect to one of these variables.
For instance, let \( Z = f(x, y) \) be a function of two variables \( x \) and \( y \); and each variable can take any value independently of each other. If we assume variable \( x \) to be fixed (as a constant) and allow variable \( y \) to vary, then the derivative with respect to \( y \) can be obtained in the usual way of differentiation. This process is called the partial derivative of \( Z \) with respect to \( y \) and it is denoted by \( \frac{\partial}{\partial y} \).

**ILLUSTRATION 17-16**

Given \( Z = \frac{x^2}{x - y + 1} \), determine (i) \( \frac{\partial}{\partial x} \) and; (ii) \( \frac{\partial}{\partial y} \).

**SUGGESTED SOLUTION 17-16**

\[
Z = \frac{x^2}{x - y + 1}
\]

(i) \( \frac{\partial}{\partial x} \) = \[
2x - x \ \frac{\partial}{\partial x} (x - y + 1)
\]

\[
= \frac{2x^2 - x(y + 1) - x^2}{x - y + 1}
\]

\[
= \frac{x^2 - y + 1}{x - y + 1}
\]

(ii) \( \frac{\partial}{\partial y} \) = \[
\frac{\partial}{\partial y} \left( \frac{x^2}{x - y + 1} \right)
\]

\[
= \frac{x^2}{(x - y + 1)^2}
\]

**17.5 SUMMARY AND CONCLUSIONS**

The concept of differentiation has a lot of applications in many areas of human endeavours such as Engineering, Science, Business and Economics.

The basic rules of differentiation are given as well as how they can be applied to problems on marginal cost and revenue, elasticity of demand and maximum and minimum values of a function.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
17.6 REVISION QUESTIONS

17.6.1 MULTIPLE CHOICE QUESTIONS

1. Which of the following is not a differentiation?
   a. rate of change
   b. derivative
   c. slope
   d. interception
   e. gradient.

2. Differentiate \( y = ax^n \) with respect to \( x \),
   a. \( ax^{n-1} \)
   b. \( nax^{n-1} \)
   c. \( nax^n \)
   d. \( \frac{ax^n}{n} \)
   e. \( \frac{ax^n}{n-1} \)

3. Which of the following is the derivative of \( \frac{U}{V} \) with respect to \( x \)?
   \[
   \left( \frac{\frac{d}{dx} U}{\frac{d}{dx} V} + \frac{\frac{d}{dx} V}{\frac{d}{dx} U} \right) / U^2
   \]
   a. \[
   \left( \frac{\frac{d}{dx} U}{\frac{d}{dx} V} - \frac{\frac{d}{dx} V}{\frac{d}{dx} U} \right) / V^2
   \]
   b. \[
   \left( \frac{\frac{d}{dx} U}{\frac{d}{dx} V} + \frac{\frac{d}{dx} V}{\frac{d}{dx} U} \right) / V^2
   \]
   c. \[
   \left( \frac{\frac{d}{dx} U}{\frac{d}{dx} V} - \frac{\frac{d}{dx} V}{\frac{d}{dx} U} \right) / V^2
   \]
   d. \[
   \left( \frac{\frac{d}{dx} U}{\frac{d}{dx} V} + \frac{\frac{d}{dx} V}{\frac{d}{dx} U} \right) / U^2
   \]
   e. \[
   \left( \frac{\frac{d}{dx} U}{\frac{d}{dx} V} - \frac{\frac{d}{dx} V}{\frac{d}{dx} U} \right) / U^2
   \]

17.6.2 SHORT ANSWER QUESTIONS

1. In differentiation, the function of function is also known as ………… function

2. Second derivative is used to determine the type of ………………………

3. Given the demand function \( P = 150 - 3x - x^2 \), the elasticity of demand at \( x =5 \) is ……………………………

Use the following information to answer questions 4, 5, and 6.  
The Total Cost Function is given by \( TC = 18q - 5q^2 + q^3 \)
4. The quantity at turning point of the average cost is ……………………………

5. The minimum average cost is ………………………………………………………

6. The marginal cost at the point of minimum average cost is ………………..

7. The value of the derivative of \( y = 3(x^2 + 5x) \) with respect to \( x \) at \( x = 5 \) is …………………

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
18

BASIC LINEAR PROGRAMMING

18.0 LEARNING OBJECTIVES

After studying this chapter, readers should be able to:

- Understand the concept and meaning of linear programming;
- Know the underlying basic assumptions;
- Formulate the linear programming problem;
- Solve linear programming problem using graphical and Simplex methods; and make appropriate and correct interpretations; and
- Understand the concepts of duality and shadow cost in linear programming.

18.1 INTRODUCTION

This is an Operations Research technique that is popular and frequently used in industry, business and other areas of human endeavour. The major focus of Linear Programming (L.P) technique, in decision-making, is to optimize the use of limited available resources. That is, it is an economic allocation of scarce resources by means of mathematical modeling. The history of Operations Research tells us that George B. Dantzing developed Linear Programming technique during Second World War. His primary aim of developing the technique then was to solve some military logistics problems. But now, it is being used extensively in wide areas of human endeavours.

18.2 CONCEPT AND MEANING OF LINEAR PROGRAMMING.

The term “Linear Programming” consists of two words Linear and Programming. The word “Linear” implies linear relationship among the variables in a model while the word “Programming” implies modeling and solving a problem mathematically. By the combination of these two words, it is obvious that the Linear Programming technique
involves modeling and solving problems (stated in functional forms) mathematically. Hence, the process of converting verbal description and numerical data into mathematical expressions is essential to the problem solving in Linear Programming.

In summary, Linear Programming is a mathematical technique used in optimizing the value of a linear objective function subject to the linear relations or constraints. A general statement of a Linear Programming Model consists of three basic components or elements. These elements are the Decision variables, Objective function and constraints.

(a) Decision Variables

These are unknown variables that we seek to determine. It is essential to properly define the decision variables as a first step towards the development of Linear Programming Model. Alphabetical letters (e.g. x, y, z) or letters with subscripts (e.g. x₁, x₂,….xₙ) are used to represent the decision variables. All decision variables in Linear Programming are controllable, continuous and non-negative (i.e. xⱼ ≥ 0)

(b) Objective Function

This is a function that states the goal that needs to be optimized. By optimisation, the major aim is either to minimize cost or maximise profit.

Furthermore, the objective function is expressed in terms of the stated decision variables. For instance, if x₁, x₂,…xₙ are decision variables and cⱼ are associated costs /profits, then the objective function can be stated as :

Minimise (Maximise) \[ Z = \sum_{j=1}^{N} cⱼ xⱼ \] .................................. 18.2 .1

where j = 1,2…..n, and C is real number.

(c) Constraints

The limit of achievement for an objective function is controlled by the available resources. Therefore, the constraints of the linear programming model depend on the resources such as man/ labour, machine, raw materials, space etc. These
constraints are expressed as linear functions in terms of the decision variables.

A set of linear constraints which form the equations or inequalities can be expressed as:

\[
\begin{align*}
\begin{cases}
    a_{11} X_1 + \cdots + a_{1n} X_n \leq (\geq \text{ or } =) b_1 \\
    a_{21} X_1 + \cdots + a_{2n} X_n \leq (\geq \text{ or } =) b_2 \\
    \quad \quad \quad \quad \quad \quad \quad \vdots \\
    a_{m1} X_1 + \cdots + a_{mn} X_n \leq (\geq \text{ or } =) b_m \\
\end{cases}
\end{align*}
\]

where coefficients \( a_{ij} \), \((i = 1, \ldots, m \text{ and } j = 1, 2, \ldots, n)\) are unrestricted in magnitude and signs. The coefficients \( a_{ij} \) are also called the input-output or the technological coefficients. They express the rate at which a given resource is being used up.

By combining the above components of linear programming, the general structure of a linear programming problem can be stated as:

Minimise (maximise ) \( Z = \sum C_j X_j \)

Subject to:
\[
\begin{align*}
    a_{11} X_1 + \cdots + a_{1n} X_n &\leq (\geq \text{ or } =) b_1 \\
    a_{m1} X_1 + \cdots + a_{mn} X_n &\leq (\geq \text{ or } =) b_m \\
    X_1, X_2, \ldots, X_n &\geq 0
\end{align*}
\]

For example, a particular problem can be stated as

Minimise \( Z = 2X_1 + 3X_2 \)

Subject to:
\[
\begin{align*}
    3X_1 + X_2 &\leq 10 \\
    X_1 + 2X_2 & = 15 \\
    2X_1 + 3X_2 &\geq 20 \\
    X_1, X_2 &\geq 0
\end{align*}
\]

18.3 BASIC ASSUMPTIONS OF LINEAR PROGRAMMING

The underlying basic assumptions of a linear programming model are

(a) Non-negativity of the decision variables.

(b) Linearity of the objective function and the constraints.
(c) Certainty: This assumes that all relationships are exact and the modeling linear programming is deterministic.

(d) Additive of the resources; i.e. resources can be added proportionally.

(e) Proportionality requires the contribution of each variable to be directly proportional to the value of variable.

(f) Divisibility of Decision Variables; i.e. the decision variables can be divided into small and more convenient units or components.

(g) Independence of decision variables; i.e., the effects of the contribution of each decision variables are independent of each other. Therefore, there is no interaction between the decision variables.

(h) Existence of optimal solution corresponding to the optimal value of the objective function.

18.6 PROBLEM FORMULATION IN LINEAR PROGRAMMING

In most cases, the linear Programming problems are usually stated in words. It is therefore necessary to express the given problem in a mathematical form in order to facilitate easy solving of the problem. The basic steps in formulating the linear programming problem are:

(a) Identify the decision variables (the unknown variables). These are the variables to be determined; and are usually represented by the algebraic symbols.

(b) Identify all constraints and/or restrictions in the problem and express them as linear function of decision variables. The resources limitation dictates the nature of constraint’s statement.

(c) Identify the objective function of the problem and state it as linear function of the decision variables.

Here, it is necessary to optimise the function by minimizing the cost or maximizing the profits subject to the given constraints.

The following examples shall be used to explain all the above steps:
ILLUSTRATION 18-1 (Problem on Production)

A food processing company specialises in the production of “Gari” and “Lafun”. As these two products are derived from the same source (cassava), the time in hours to produce a unit (kg) of each product, and the weekly capacity of operations and cost per unit of product are given in the table below:

<table>
<thead>
<tr>
<th>Operations</th>
<th>Time per Unit (in hrs)</th>
<th>Operation capacity (in Hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (Piling of Cassava)</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2. (Granding, soaking, fermention, etc)</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>3. (Frying/ Drying)</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Cost per unit (₦)</td>
<td>40</td>
<td>30</td>
</tr>
</tbody>
</table>

Formulate a mathematical model for the linear programming problem.

SUGGESTED SOLUTION 18-1

Step 1: The decision variables are Gari and Lafun and can be represented by $X_1$ and $X_2$ respectively. (Note that $X_1, X_2 \geq 0$).

Step 2: The constraints of the problem are

\[
\begin{align*}
4X_1 + 3X_2 & \leq 45 \quad \text{(for operation 1)} \\
3X_1 + 10X_2 & \leq 180 \quad \text{(for operation 2)} \\
6X_1 + 5X_2 & \leq 150 \quad \text{(for operation 3)}
\end{align*}
\]

Step 3: The objective function is Minimize \( Z = 40X_1 + 30X_2 \)

Therefore, the combination of the above steps gives the summary of the problem as:

\[
\begin{align*}
\text{Minimize } Z &= 40X_1 + 30X_2 \\
\text{Subject to: } 4X_1 + 3X_2 & \leq 45 \\
& 3X_1 + 10X_2 \leq 180 \\
& 6X_1 + 5X_2 \leq 150 \\
& X_1, X_2 \geq 0
\end{align*}
\]

ILLUSTRATION 18-2 (Problem on Advertising Media Selection)

A Company plans to advertise its products in three different media in order to reach a large number of her potential customers. The Company’s choices of media are magazines, Television and Radio. A market survey was earlier carried out and the following result was obtained:
If the company does not want to spend more N200,000 on advertising and further requires that:
(a) at least 2.5 million exposures to take place among women;
(b) advertising on television is restricted to N700,000;
(c) the number of advertising units on radio and television should each be between 4 and 12; formulate the L.P. model for the problem.

**SUGGESTED SOLUTION 18-2**

**Step 1:** The decision variables are Magazine, Television and Radio and they can be represented by $x_1$, $x_2$, $x_3$ respectively.

**Step 2:** The constraints of the problem are formulated as follows:
- Number of female reached
  \[ 100,000x_1 + 150,000x_2 + 300,000x_3 \geq 2,500,000 \]
- Budget constraint
  \[ 20,000x_1 + 15,000x_2 + 10,000x_3 \leq 200,000 \]
- Constraint on Television
  \[ 15,000x_2 \leq 700,000 \]
- Constraints on Radio and Television
  \[ 4 \leq x_2 \leq 12 \text{ for Television} \]
  \[ 4 \leq x_3 \leq 12 \text{ for Radio} \]

**Step 3:** The objective function is focused on the potential customers. Hence, it is required to maximize $Z = 250,000x_1 + 200,000x_2 + 500,000x_3$
Therefore, the combination of the above steps gives:
Max. $Z = 250,000x_1 + 200,000x_2 + 500,000x_3$
Subject to $100,000x_1 + 150,000x_2 + 300,000x_3 \geq 2,500,000$
\[ 20,000x_1 + 15,000x_2 + 10,000x_3 \leq 200,000 \]
\[ 15,000x_2 \leq 700,000 \]
\[ 4 \leq x_2 \leq 12 \]
\[ 4 \leq x_3 \leq 12 \]
\[ x_1, x_2, x_3 \geq 0 \]
18.7 METHODS OF SOLVING LINEAR PROGRAMMING PROBLEMS

There are two major methods of solving an L.P. problem. The methods are:

(a) Graphical method; and

(b) Simplex Method.

Each of these methods will be considered in the subsequent sections.

(a) Graphical Method of Solving L.P Problem

This is a method that demands the use of geometric representation of a linear optimization model. It is an easy and common method usually used for a L.P problem with two decision variables.

The procedure of graphical method is summarized in the following steps:

Step 1: Obtain the mathematical model of the LP problem. That is, formulate the problem as discussed in section 17.4

Step 2: On a graph paper, draw the graphs of the constraints (i.e. the inequalities) and the non-negativity restrictions.

Step 3: Identify the feasible region. This is the region common to all the graphs drawn in step 2. It is also called the “solution space” in the sense that all the points within the region satisfy the constraints simultaneously.

Step 4: Identify and determine the coordinates of corners of the feasible solution space. Then evaluate the values of the objective function at each extreme point. Pick the extreme point of the feasible region that gives the optimum (best) value. The concept of optimum can be determined by picking the highest or smallest of these values respectively for maximization or minimization problem. Note that there is a theory that backs-up the issue of extreme point which always gives the minimum or maximum value of Z (objective function).
ILLUSTRATION 18-3 (Maximization Problem)

Solve the following L.P. problem by graphical method.

Maximize \[ Z = 3X_1 + 2X_2 \]

Subject to

\[ 2X_1 + 3X_2 \leq 18 \quad \cdots \cdots \quad I \]
\[ 3X_1 + X_2 \leq 9 \quad \cdots \cdots \quad II \]
\[ X_1 + 5X_2 \leq 10 \quad \cdots \cdots \quad III \]
\[ X_1, X_2 \geq 0 \]

SUGGESTED SOLUTION 18.3

Step 1: The problem is already in mathematical form

Step 2: Variables \( X_1 \) and \( X_2 \) are represented respectively on horizontal and vertical axes.

Consider the constraints and treat as equations in order to draw the graphs. Since the equations are of straight lines, two points will fix each line.

Consider:

**Constraint I \((2X_1 + 3X_2 = 18)\)**

\( X_1 = 0, \ X_2 = 6 \) corresponds to the point \((0, 6)\)
\( X_2 = 0, \ X_1 = 9 \) corresponds to the point \((9, 0)\)

**Constraint II \((3X_1 + X_2 = 9)\)**

\( X_1 = 0, \ X_2 = 9 \) corresponds to the point \((0, 9)\)
\( X_2 = 0, \ X_1 = 3 \) corresponds to the point \((3, 0)\)

**Constraint III \((X_1 + 5X_2 = 10)\)**

\( X_1 = 0, \ X_2 = 2 \). Then we have point \((0, 2)\)
\( X_2 = 0, \ X_1 = 10 \). Then we have point \((10, 0)\)

With the above points, the graphs of the constraints are drawn as follows:
Step 3: From the graph, the feasible region (i.e. the region common to all the constraints) is ABCO.

Step 4: The extreme points of the feasible region are: A(0,2), B(2.5,1.7), C(3,0) and O(0,0).
The next line of action is to use the extreme points to determine the optimal value thus:

<table>
<thead>
<tr>
<th>Points</th>
<th>Max ( Z = 3X_1 + 2X_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(0,2)</td>
<td>( Z = 4 )</td>
</tr>
<tr>
<td>B(2.5,1.7)</td>
<td>( Z = 7.5 + 3.5 = 11 )</td>
</tr>
<tr>
<td>C(3,0)</td>
<td>( Z = 9 )</td>
</tr>
<tr>
<td>O(0,0)</td>
<td>( Z = 0 )</td>
</tr>
</tbody>
</table>

**Decision:** Since the problem is a maximization one, the highest value of \( Z \) is 11. Hence the optimal value of \( Z = 11 \) is obtained at \( X_1 = 7.5 \) and \( X_2 = 1.75 \).

**ILLUSTRATION 18-4 (Minimization Problem)**

Use the graphical method to solve the following L.P. problem

Minimize \( Z = 3X_1 + 4X_2 \)
Subject to:

\[ \begin{align*}
X_1 + 3X_2 & \geq 9 \\
2X_1 + 5X_2 & \geq 20 \\
3X_1 + 2X_2 & \geq 12 \\
X_1, X_2 & \geq 0
\end{align*} \]

SUGGESTED SOLUTION 18-4

Step 1: The problem is already in mathematical form

Step 2:

**Constraint I** \((X_1 + 3X_2 = 9)\)

If \(X_1 = 0, X_2 = 3\). corresponds to the point \((0,3)\)
If \(X_2 = 0, X_1 = 9\). corresponds to the point \((9,0)\)

**Constraint II** \((2X_1 + 5X_2 = 20)\)

If \(X_1 = 0, X_2 = 4\). corresponds to the point \((0,4)\)
If \(X_2 = 0, X_1 = 10\). corresponds to the point \((10,0)\)

**Constraint III** \((3X_1 + 2X_2 = 12)\)

If \(X_1 = 0, X_2 = 6\). corresponds to the point \((0,6)\)
If \(X_2 = 0, X_1 = 4\). corresponds to the point \((4,0)\)

Step 3: The use of the above points, will create the following graph:
Step 4: The extreme points of the feasible region are: A(0,6), B(2,3.5), and C(10,0). Therefore, the following table gives the objective function:

<table>
<thead>
<tr>
<th>Points</th>
<th>Min $Z = 3X_1 + 4X_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(0,6)</td>
<td>$Z = 24$</td>
</tr>
<tr>
<td>B(2,3.5)</td>
<td>$Z = 6 + 14 = 20$</td>
</tr>
<tr>
<td>C(10,0)</td>
<td>$Z = 30$</td>
</tr>
</tbody>
</table>

Decision: Since the value of $Z$ at B(2,3.5) is the smallest (i.e. $Z = 20$), the optimal value is 20 with $X_1 = 2$ and $X_2 = 3.5$.

(b) Definition of Some Useful Terms in Simplex Method for solving LP Problem
The following terms or concepts are needed in the Simplex Method. Hence, they are discussed as preambles to the method.

(i) Standard Form
The standard form of a L.P. problem is the conversion of the original given problem of L.P. to the form that is amenable to algebraic computations and manipulations.

The main characteristics of the standard form are:

- The objective function is either of a Minimisation or Maximisation type.

- All inequalities’ constraints are converted to equations by adding slack or surplus variables. Slack variable(s) is added to the inequality of type $\leq$ (less than or equal to); while Surplus variable(s) is/are subtracted from the inequality of type $\geq$ (greater than or equal to). A slack variable represents unused resources while the surplus variable represents the amount by which solution values exceed a resource. Surplus variable is also called the negative slack variable.

- All variables are restricted to be non-negative.
The right-hand side of each constraint must be non-negative. If it is not, just multiply both sides of the constraint by −1.

In a compact form, the standard form can be expressed in the following matrix notation:

Optimize (Max or Min) \( Z = CX + OS \)

Subject to: \( AX + S = b \)

\( X, S \geq 0 \)

Where \( C = (C_1, C_2, \ldots, C_n) \) is the row vector, \( X = (X_1, X_2, \ldots, X_n) \)

\( b = (b_1, b_2, \ldots, b_m) \) and

\( S = (S_1, S_2, \ldots, S_m) \) are column vectors and

\[
A = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\]

is the \( m \times n \) matrix and the coefficients of variables \( x_1, x_2, \ldots, x_n \) in the constraints.

**ILLUSTRATION 18-5**

The L.P. problem

\[
\begin{align*}
\text{Min} \ Z &= 2x_1 + 3x_2 + x_3 \\
\text{Subject to:} \quad &x_1 + 3x_2 + 2x_3 \leq 10 \\
&2x_1 + x_2 + 3x_3 \geq 20 \\
&3x_1 + x_2 + 2x_3 = 15 \\
&x_1, x_2, x_3 \geq 0
\end{align*}
\]

can be expressed in standard form as:

\[
\begin{align*}
\text{Min} \ Z &= 2x_1 + 3x_2 + x_3 + OS_1 + OS_2 \\
\text{Subject to:} \quad &x_1 + 3x_2 + 2x_3 + S_1 + OS_2 = 10 \\
&2x_1 + x_2 + 3x_3 + OS_1 - S_2 = 20 \\
&3x_1 + x_2 + 2x_3 + OS_1 + OS_2 = 15 \\
&x_1, x_2, x_3, S_1, S_2 \geq 0
\end{align*}
\]

N.B:

\( \blacklozenge \) Instead of \( S_1 \) and \( S_2, x_4 \) and \( x_5 \) can be used as slack and surplus variables.

\( \blacklozenge \) s.t. means subject to.
(ii) **Canonical Form**

A system of $m$ equations and $n$ variables with $m \leq n$ is in canonical form with a distinguished set of $m$ basic variables if each basic variable has coefficient 1 in one equation and zero in the others; and also each equation has exactly one basic variable with coefficient 1.

For example, variables $x_1$ and $x_2$ are distinguished Basic Variables in the following problem:

Maximize \[ Z = x_1 - 2x_2 + 3x_3 \]

Subject to:
\[
\begin{align*}
  x_1 + 2x_3 &= 10 \\
  x_2 + 3x_3 &= 15 \\
  x_1, x_2, x_3 &\geq 0
\end{align*}
\]

(iii) **Basic Solution**

A solution set which has at most $m$ zero values in a L.P. problem that has $n$ variables and $m$ constraints ($m < n$) is a basic solution. If it is otherwise ($m > n$), the solution degenerates.

(iv) **Standard L.P. Problem in Canonical Form**

This is the required form for a L.P. problem before a simplex method or procedure can be applied. Here, the L.P problem must be in both standard and canonical forms.

A standard L.P. problem is in canonical form if it satisfies the following conditions:

(i) The constraints are in canonical form with respect to a distinguished set of basic variables

(ii) Associated basic solution is feasible.

(iii) Objective function is expressed in terms of non-basic variables only.

For instance, $x_1$ and $x_2$ are basic variables in the example under canonical form (Equation 17.5.2.1), while $x_3$ is the non-basic variable.

(v) **Artificial Variable**

This is a variable represented by a letter to make a standard form of L.P. problem which is not in canonical form. A standard
canonical form is usually applied to the constraints of types $\geq$ and $=$ (equal to) in order to make them canonical.

(c) The Simplex Algorithm

The Simplex Algorithm (for both Minimization and Maximization) of a L.P. problem can be stated in the following procedural steps:

**Step 1:** Start by stating the problem in a standard canonical form.

**Step 2:** Set up the initial simplex table or tableau from the data in step 1.

A general form of initial simplex table is given below:

<table>
<thead>
<tr>
<th>Cost/Profit of Basic Variables $C_B$</th>
<th>Variables in Basis $x_B$</th>
<th>Variables $(x_j)$</th>
<th>Value of Basic Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{B1}$</td>
<td>$x_1$</td>
<td>$a_{11} a_{12} \ldots a_{1n}$</td>
<td>$b_1$</td>
</tr>
<tr>
<td>$C_{B2}$</td>
<td>$x_2$</td>
<td>$a_{21} a_{22} \ldots a_{2n}$</td>
<td>$b_2$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$C_{Bm}$</td>
<td>$x_m$</td>
<td>$a_{m1} a_{m2} \ldots a_{mn}$</td>
<td>$b_m$</td>
</tr>
</tbody>
</table>

\[ Z_j = \sum C_{Bk} x_k \]

\[ Z = C_j - Z_j \]

**Step 3:** Compute $Z_j$ and $^*Z_j$ for each variable $x_j$ as stated in step 2.

**Step 4:** Carry-out the optimality test.

For a Minimization problem, find out if there is any negative value of $^*Z_j$. If there is none, the problem is optimal and then stop but if not go to step 5. Also for a Maximization problem, find out if there is any positive value of $^*Z_j$. If there is none, the problem is optimal and then stop but if otherwise, go to step 5.

**Step 5:** Determine the variable to enter into the basis:

For a Minimization problem, select key column with largest negative value of $^*Z_j$ in order to go into the basis.
While for the Maximization problem, select the key column with largest positive value of $Z_j$ in order to go into the basis.

**Step 6:** Determine the variable to leave the basis.
Here, select key row with Min $(X_{bi}/a_{ij}, a_{ij}>0)$ to leave the basis. If all $a_{ij} \leq 0$, then the current solution is unbounded and stop the procedure. Also, if minimum of these ratios $(b_i/a_{ij})$ is attained in several rows, a simple rule of thumb such as choosing that row with the smallest index can be used to select the pivot row.

**Step 7:** Identify the key element at the intersection of key row and key column. Then update the entries in the simplex table by:
(i) first obtaining the key row values, and then
(ii) pivoting or applying elementary row operations to form a new simplex table. That is, the new table is formed by:
(a) stating the key element. If it is 1, the new row remains the same as old. But if it is not 1, then divide each element in the key row by this key element in order to obtain a new row;
(b) obtaining the new values for the elements in the remaining rows (other than the key row) by the elementary row operations as follows:

\[
\text{New value for the element of remaining row;} = \left( \text{element value in the old row;} \right) \pm \left( \text{Value of element in row i which is either above or below key element} \right) \times \left( \text{New corresponding value of element in the key row} \right)
\]

**Step 8:** Go to step 3 in order to compute $Z_j$ and $\hat{Z}_j$ for the new simplex table and continue along the other sequence steps.
In the same vein, the above algorithm (Simplex Algorithm) can be depicted in the following flow chart:
Start

Convert the L.P. problem into standard Canonical form

Setup the initial simplex table in order to obtain the initial solution

Compute $Z_j$ and $\overset{*}{Z}_j$ where $Z_j = \sum c_j x_j$ and $\overset{*}{Z}_j = c_j \cdot Z_j$

(Minimization)

Is it a Minimization problem?

Yes

Does negative values of $\overset{*}{Z}_j$ exist?

Yes

Select the key column with negative value of $\overset{*}{Z}_j$

No

(Maximization)

Is it a Maximization problem?

No

Does positive values of $\overset{*}{Z}_j$ exist?

No

Select the key column with largest positive value of $\overset{*}{Z}_j$

Yes

Is all $a_{ij} \leq 0$?

Yes

The current problem is unbounded

No

Select the key row with M{\text{in}} \{b_i / a_{ij}; a_{ij} > 0\}

Identify the key element at the intersection of key row and key column.

Update the entries in the simplex table by

(a) obtaining key row values; and
(b) pivot or apply elementary row operations.

The solution set is optimal

Stop

Fig. 18.1. Flow chart of Simplex Algorithm
ILLUSTRATION 18-6
Use the simplex method to solve the following LP problem:
Max  \( Z = 2x_1 + 3x_2 + 3x_3 \)
Subject to:
\[
\begin{align*}
3x_1 + 2x_2 & \leq 60 \\
-x_1 + x_2 + 4x_3 & \leq 10 \\
2x_1 - 2x_2 + 5x_3 & \leq 50 \\
x_1, x_2, x_3 & \geq 0
\end{align*}
\]

SUGGESTED SOLUTION 18-6

Step 1: Convert the given problem to standard canonical form by the use of slack variables \( S_1, S_2 \) and \( S_3 \):
Max  \( Z = 2x_1 + 3x_2 + 3x_3 + OS_1 + OS_2 + OS_3 \)
S. t.  \[
\begin{align*}
3x_1 + 2x_2 + 0x_3 + S_1 + OS_2 + OS_3 &= 60 \\
-x_1 + x_2 + 4x_3 + S_2 + OS_3 &= 10 \\
2x_1 - 2x_2 + 5x_3 + OS_1 + OS_2 + S_3 &= 50 \\
x_1, x_2, x_3, S_1, S_2, S_3 & \geq 0
\end{align*}
\]

Step 2: Form the initial simplex table from step 1:

<table>
<thead>
<tr>
<th>( C_j )</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_B )</td>
<td>( X_B )</td>
<td>( X_1 )</td>
<td>( X_2 )</td>
<td>( X_3 )</td>
<td>( S_1 )</td>
<td>( S_2 )</td>
</tr>
<tr>
<td>0</td>
<td>( S_1 )</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>( S_2 )</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>( S_3 )</td>
<td>2</td>
<td>-2</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Z_i )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \dot{Z} = C_j - Z )</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The ratio \( (b_i/a_{ij}) \)

Step 3: Computations of \( Z \) and \( \dot{Z} \) are made in step 2 as follows:

\[
Z = \sum C_B X_j
\]
i.e.

\[
Z = \sum C_B X_j = (0 \times 3) + (0 \times -1) + (0 \times 2) = 0
\]

and

\[
\dot{Z} = C_j - Z = 2 - 0 = 2
\]

Also for \( Z = \sum C_B X_2 \)

\[
Z = \sum C_B X_2 = (0 \times 2) + (0 \times 1) + (0 \times -2) = 0
\]

\[
\dot{Z} = C_j - Z = 3 - 0 = 3
\]
Step 4: By the optimality test, the solution is not optimal yet because series of positive values $Z$ are present.

Step 5: The largest of the positive value of $Z$ is 3 from the table in step 2. Then, take the one with variable $X_2$ in order to go into the basis.

Step 6: From the ratio obtained in the above tableau (Table 1), the minimum ratio is 10 and therefore take variable $S_2$ as the one leaving the basis.

Step 7: The key element at the intersection of key row and key column is 1. It is circled in Table 1.

Next is to carry-out pivot operation on table 1 in order to have a new table as Table 2.

<table>
<thead>
<tr>
<th></th>
<th>$C_j$</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_B$</td>
<td>$X_B$</td>
<td>$X_1$</td>
<td>$X_2$</td>
<td>$X_3$</td>
<td>$S_1$</td>
<td>$S_2$</td>
<td>$S_3$</td>
<td>$b_i$</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0</td>
<td>$S_1$</td>
<td>(5)</td>
<td>0</td>
<td>-8</td>
<td>1</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>$R_2$</td>
<td>3</td>
<td>$X_2$</td>
<td>-1</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$R_3$</td>
<td>0</td>
<td>$S_3$</td>
<td>0</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$Z$</td>
<td>-3</td>
<td>3</td>
<td>12</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>Z= 30</td>
</tr>
<tr>
<td>$\dagger Z$</td>
<td>5</td>
<td>0</td>
<td>-9</td>
<td>0</td>
<td>-3</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Pivot Operations

New $R_1 = \text{Old } R_1 + (-2)R_2$
New $R_2 = \text{Old } R_2$
New $R_3 = \text{Old } R_3 + 2R_2$

$Z$ for $\sum_{i} C_{Bi}X_i = 0(5) + (-1)(3) + 0(0) = -3$

$Z = C_j - Z_j = 2 - (-3) = 5$

$Z$ for $\sum_{i} C_{Bi}X_i = 0(0) + 3(1)(3) + 0(0) = 3$

$Z = C_j - Z_j = 3 - 3 = 0$

$Z$ for $\sum_{i} C_{Bi}X_i = 0(-8) + 3(4) + 0(13) = 12$

$Z = C_j - Z_j = 3 - 12 = -9$
Table 3

<table>
<thead>
<tr>
<th>C_j</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_1</td>
<td>1</td>
<td>0</td>
<td>-8/5</td>
<td>1/5</td>
<td>-2/5</td>
<td>0</td>
</tr>
<tr>
<td>X_2</td>
<td>0</td>
<td>1</td>
<td>2^{2/5}</td>
<td>1/5</td>
<td>3/5</td>
<td>0</td>
</tr>
<tr>
<td>X_3</td>
<td>0</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Z</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

Elementary Operations

New R_1 = old R_1 + 5
New R_2 = old R_2 + New R_1
New R_3 = old R_3

Z for \( \sum C_{b_1} X_1 = 2(1) + 3(0) + 0(0) \) = 2
\( Z = C_j - Z_j = 2 - 2 = 0 \)

Z for \( \sum C_{b_2} X_2 = 2(0) + 3(1) + 0(0) \) = 3
\( Z = C_j - Z_j = 3 - 3 = 0 \)

Z for \( \sum C_{b_3} X_3 = 2(-8/5) + 3(12/5) + 0(13) \) = 4
\( Z = C_j - Z_j = 3 - 4 = -1 \)

Since there is no positive value of \( Z \) in table 3, the solution is optimal with
\( X_1 = 8, \; X_2 = 18 \) and \( S_3 = 70 \)

Therefore, The Optimal Value \( Z = 2(18) + 3(18) + 3(0) = 70 \).

ILLUSTRATION 18-7

By simplex method, solve the following LP problem:

Min. \( Z = x_1 + x_2 - 4x_3 \)
Subject to:
\( x_1 + x_2 + 2x_3 \leq 9 \)
\( x_1 + x_2 - x_3 \leq 2 \)
\( -x_1 + x_2 + x_3 \leq 4 \)
\( x_1, x_2, x_3 \geq 0 \)

SUGGESTED SOLUTION 18-7

The problem in standard canonical form:

Min. \( Z = x_1 + x_2 - 4x_3 + OS_1 + OS_2 + OS_3 \)
Subject to:
\( x_1 + x_2 + 2x_3 + S_1 + OS_2 + OS_3 = 9 \)
\( x_1 + x_2 - x_3 + S_1 + OS_2 + OS_3 = 2 \)
\( -x_1 + x_2 + x_3 + OS_1 + OS_2 + S_3 = 4 \)
\( x_1, x_2, x_3, S_1, S_2, S_3 \geq 0 \)
Presentation in Table 1 below:

<table>
<thead>
<tr>
<th></th>
<th>$C_j$</th>
<th>1</th>
<th>1</th>
<th>-4</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>$b_i$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_B$</td>
<td>$X_B$</td>
<td>$X_1$</td>
<td>$X_2$</td>
<td>$X_3$</td>
<td>$S_1$</td>
<td>$S_2$</td>
<td>$S_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$S_1$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>$9/2 = 4.5$</td>
</tr>
<tr>
<td>0</td>
<td>$S_2$</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>Not valid</td>
</tr>
<tr>
<td>0</td>
<td>$S_3$</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>$4/1 = 4$</td>
</tr>
<tr>
<td>$Z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$Z = 0$</td>
<td></td>
</tr>
<tr>
<td>$Z = C_j - Z_j$</td>
<td>1</td>
<td>1</td>
<td>-4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

By optimality test, the solution is not yet optimal. Therefore, variable $X_3$ goes into basis because it is not the most negative value of $Z$; while $S_3$ with the minimum ratio leaves the basis.

Table 2

<table>
<thead>
<tr>
<th>$C_j$</th>
<th>1</th>
<th>1</th>
<th>-4</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>$b_i$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_B$</td>
<td>$X_B$</td>
<td>$X_1$</td>
<td>$X_2$</td>
<td>$X_3$</td>
<td>$S_1$</td>
<td>$S_2$</td>
<td>$S_3$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$S_1$</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>$S_2$</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>$S_3$</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$Z$</td>
<td>4</td>
<td>-4</td>
<td>-4</td>
<td>0</td>
<td>0</td>
<td>-4</td>
<td>Z = 16</td>
<td></td>
</tr>
<tr>
<td>$Z = C_j - Z_j$</td>
<td>-3</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

New $R_1 = \text{old } R_1 - 2(R_3)$
New $R_2 = \text{old } R_2 + R_4$
New $R_3 = \text{old } R_3$

$Z$ for $\sum C_{B_1}X_1 = 3(0) + 0(0) + (-4)(-1) = 4$

$Z = C_j - Z_j = 4 - 4 = -3$

$Z$ for $\sum C_{B_1}X_2 = 0(-1) + 0(2) + (-4)(1) = -4$

$Z = C_j - Z_j = -4 - (-4) = 0$ and so on.

Since $X_3$ is having negative value of $Z$, $X_3$ moves to the basis while $S_1$ (with ratio $1/3$) leaves the basis.
**Table 3**

<table>
<thead>
<tr>
<th>C</th>
<th>C_B</th>
<th>X_1</th>
<th>X_2</th>
<th>X_3</th>
<th>S_1</th>
<th>S_2</th>
<th>S_3</th>
<th>b_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X_1</td>
<td>1</td>
<td>-1/3</td>
<td>0</td>
<td>1/3</td>
<td>0</td>
<td>-2/3</td>
<td>1/3</td>
</tr>
<tr>
<td>0</td>
<td>S_2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>-4</td>
<td>X_3</td>
<td>0</td>
<td>2/3</td>
<td>1</td>
<td>1/3</td>
<td>0</td>
<td>1/3</td>
<td>4^(3/3)</td>
</tr>
<tr>
<td>Z</td>
<td>1</td>
<td>-3</td>
<td>-4</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Pivot Operations

New R_1 = \( \frac{1}{3} \) (old R_1)

New R_2 = old R_2

New R_3 = old R_3 + New R_3

**Z** for \( \sum C_{B1} X_1 \) = \( 1 \) \( (1) + 0(0) + (-4)(0) \) = 1

\( Z \) = 1 - 1 = 0

**Z** for \( \sum C_{B1} X_2 \) = \( 1 \) \( (-1/3) + 0(2) + (-4)(2/3) \) = -3

\( Z \) = 1 - (-3) = 4

**Z** for \( \sum C_{B3} X_3 \) = 0 \( (1) + 0(0) + (-4)(1) \) = -4

\( Z \) = -4 - (-4) = 0

Since no more negative values of \( Z \), the solution is optimal with

\( X_1 = \frac{1}{3}, X_3 = 4^{3/3} \) and \( X_2 = 0 \).

The optimal value is \( Z = 1 \) \( (1/3) + 0(1) + (-4)(4^{3/3}) \)

\( = \frac{1}{3} + 0 - 17^{3/3} \)

\( = -17 \).

**Remarks:** It is essential to know that:

(vi) The above example can be handled by the Maximization approach.

The following relationship will make it possible:

Max \( Z = \text{Min.} \ (-Z) \); or vice versa.

(vii) The unrestricted variable can be handled by re-defining the variable with new ones.
ILLUSTRATION 18-8

Suppose it is required to maximize.
\[
Z = 2x_1 + 3x_2
\]
Subject to:
\[
\begin{align*}
2x_1 + x_2 & \leq 10 \\
x_1 + 3x_2 & \leq 15 \\
x_1 & \geq 0, \ x_2 \text{ unrestricted.}
\end{align*}
\]

The unrestricted variable \( x_2 \) is re-defined by \( x_2 = x_3 - x_4 \), where \( x_3, x_4 \geq 0 \).

Therefore, the above problem can be re-written as:
\[
\text{Max } Z = 2x_1 + 3(x_3 - x_4)
\]
\[
\text{s.t.} \quad \begin{align*}
2x_1 + x_3 - x_4 & \leq 10 \\
x_1 + 3x_3 - 3x_4 & \leq 15 \\
x_1, x_3, x_4 & \geq 0.
\end{align*}
\]

(viii) Some authors prefer using \( Z^* = Z_j - C_j \). In this case, take decision in opposite way to \( Z^* = C_j - Z_j \).

(ix) In economic and business terms, the various variables stated in step 3 of section 17.5.3 can be interpreted as follows:
- \( Z \) represents return
- \( C_j \) represents return per unit of Variable \( x_j \)
- \( x_j \) represents number of units of Variable \( j \)
- \( a_{ij} \) represents units of resource, \( i \) consumed (required) per unit of Variable \( j \)
- \( b_j \) represents units of resource, \( i \) available.

18.8 USE OF ARTIFICIAL VARIABLE AND METHODS OF HANDLING IT IN L.P. PROBLEM

It is a common practice to use the artificial variable when it is not possible to have a canonical form for a given L.P. problem. This usually happens with the constraints of types \( \geq \) and \( = \). By using the artificial variable (A), the problem that is not in standard canonical form will be made canonical.

When artificial variable is used, the usual methods of obtaining the Initial Basic Feasible Solution are:
(a) The Penalty or Big – M Method
(b) The Two-Phase Technique.

(a) **Penalty or Big-M Method**

In this method, the artificial variable that is added to the problem is assigned very large value per unit or cost $M$ in the objective function.

This large value per unit cost $M$ is negative for the Maximization problem and positive for the Minimization problem. The penalty $M$ will make the basic solution infeasible.

The operation of simplex method will replace these artificial variables with decision variables in the basis with more realistic unit cost.

Succinctly, the summary of the procedural steps of this method is given below:

**Step 1:** Express the given L.P. problem in standard form.

**Step 2:** Add artificial variable ($A_i$, $i = 1, 2, \ldots$) to the left hand side of all the constraints of type $=$ or $\geq$. Assign large penalty of negative $M$ to the objective function if it is Maximization problem; while positive $M$ for Minimization problem.

**Step 3:** Arrange the result of step 2 in tableau.

**Step 4:** Apply the regular steps of simplex method to what you have in step 3. The simplex iteration will drive out the artificial variables from the basis and allow for real decision variables to enter the basis. With further iteration, the optimal solution will be obtained.

**ILLUSTRATION 18-9**

Minimize $Z = 3x_1 + 2x_2$
Subject to:
$\begin{align*}
x_1 + x_2 & \geq 5 \\
-2x_1 - x_2 & \leq -6 \\
x_1, x_2 & \geq 0
\end{align*}$
**SUGGESTED SOLUTION 18-9**

The given problem can be re-written as

Min. \( Z = 3x_1 + 2x_2 \)

Subject to:

\[
\begin{align*}
  x_1 + x_2 & \geq 5 \\
  2x_1 + x_2 & \geq 6 \\
  x_1, x_2 & \geq 0
\end{align*}
\]

**Step 1:** To express the problem in standard form as follows:

Min. \( Z = 3x_1 + 2x_2 + OS_1 + OS_2 \)

Subject to:

\[
\begin{align*}
  x_1 + x_2 - S_1 & \geq 5 \\
  2x_1 + x_2 - S_2 & \geq 6 \\
  x_1, x_2, S_1, S_2 & \geq 0
\end{align*}
\]

**Step 2:** Artificial variables are added as follows:

Min. \( Z = 3x_1 + 2x_2 + OS_1 + OS_2 + MA_1 + MA_2 \)

Subject to:

\[
\begin{align*}
  x_1 + x_2 - S_1 + OS_2 + A_1 + OA_2 & = 5 \\
  2x_1 + x_2 + OS_1 - S_2 + OA_1 + A_2 & = 6 \\
  x_1, x_2, S_1, S_2, A_1, A_2 & \geq 0
\end{align*}
\]

**Step 3:** Table Formulation

### Table 1

<table>
<thead>
<tr>
<th>( C_B )</th>
<th>( C )</th>
<th>( 3 )</th>
<th>( 2 )</th>
<th>( 0 )</th>
<th>( 0 )</th>
<th>( M )</th>
<th>( M )</th>
<th>( b_i )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>( A_1 )</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( M )</td>
<td>( A_2 )</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>3 ( \leftarrow ) To leave the basis</td>
</tr>
<tr>
<td>( Z )</td>
<td>( 3 )</td>
<td>( 3 )</td>
<td>( 2 )</td>
<td>( -M )</td>
<td>( -M )</td>
<td>( M )</td>
<td>( M )</td>
<td>( Z = 11M )</td>
<td></td>
</tr>
<tr>
<td>( Z )</td>
<td>( = C_j - Z_i )</td>
<td>( 3 - 3M )</td>
<td>( 2 - 2M )</td>
<td>( M )</td>
<td>( M )</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Most negative value of \( Z \) allows \( x_1 \) to enter into basis.

Based on the above ratios, variable \( A_2 \) will leave the basis.

### Table 2

<table>
<thead>
<tr>
<th>( C_B )</th>
<th>( C )</th>
<th>( 3 )</th>
<th>( 2 )</th>
<th>( 0 )</th>
<th>( 0 )</th>
<th>( M )</th>
<th>( M )</th>
<th>( b_i )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>( 3 )</td>
<td>( A_1 )</td>
<td>0</td>
<td>0.5</td>
<td>-1</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>2 ( \frac{2}{0.5} = 4 ) ( \leftarrow ) To leave the basis</td>
</tr>
<tr>
<td>( M )</td>
<td>( 3 )</td>
<td>( A_2 )</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>-0.5</td>
<td>0</td>
<td>0.5</td>
<td>3 ( \frac{3}{0.5} = 6 )</td>
</tr>
<tr>
<td>( Z )</td>
<td>( 3 )</td>
<td>( (1.5 + 0.5M) )</td>
<td>( -M )</td>
<td>( (0.5M - 1.5) )</td>
<td>( M )</td>
<td>( (1.5 - 0.5M) )</td>
<td>( Z = 9 + 2M )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Z )</td>
<td>( = C_j - Z_i )</td>
<td>( 0 )</td>
<td>( (0.5 - 0.5M) )</td>
<td>( M )</td>
<td>( (1.5 - 0.5M) )</td>
<td>0</td>
<td>( (0.5M - 1.5) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Pivot Operations

\[ \text{New } R_2 = \frac{1}{2} \text{(old } R_2) \]
\[ \text{New } R_1 = \text{old } R_1 - \text{New } R \]

Table 3

<table>
<thead>
<tr>
<th>( C_j )</th>
<th>2</th>
<th>2</th>
<th>0</th>
<th>0</th>
<th>M</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_B )</td>
<td>( X_B )</td>
<td>( X_1 )</td>
<td>( X_2 )</td>
<td>( S_1 )</td>
<td>( S_2 )</td>
<td>( A_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( X_2 )</td>
<td>0</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>( X_3 )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>( Z_j )</td>
<td>3</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( Z = C_j - Z )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>M-1</td>
<td>M-1</td>
</tr>
</tbody>
</table>

Pivot Operations

\[ \text{New } R_2 = 2 \text{(old } R_1) \]
\[ \text{New } R_2 = \text{old } R_2 - \frac{1}{2} \text{(New } R_1) \]

Since all \( Z \) values are positive, the problem is now optimal.

Thus we assign \( X_1 = 6 \), \( X_2 = 5 \) and the optimal value is

\[ Z = 3(6) + 2(5) = 28. \]

ILLUSTRATION 18-10

Minimize \( Z = 5y_1 + y_2 \)
Subject to:
\[ \begin{align*}
2y_2 & \leq 10 \\
2y_1 & \geq 6 \\
10y_1 + 4y_2 & \leq 40 \\
y_1, y_2 & \geq 0
\end{align*} \]

SUGGESTED SOLUTION 18-10

Step 1: The problem in standard form.
Minimize \( Z = 5y_1 + y_2 \) \( OS_1 + OS_2 + OS_3 \)
Subject to:
\[ \begin{align*}
0y_1 + 2y_2 + S_1 & = 10 \\
2y_1 + 0y_2 - S_2 & = 6 \\
10y_1 + 4y_2 + S_3 & = 40 \\
y_1, y_2, S_1, S_2, S_3 & \geq 0
\end{align*} \]

Step 2: Artificial variable added to the result in step 1.
Minimize \( Z = 5y_1 + y_2 \) \( OS_1 + OS_2 + OS_3 \) \( MA_1 \)
Subject to:
\[ \begin{align*}
0y_1 + 2y_2 + S_1 + OS_2 + OS_3 + OA_1 & = 10 \\
2y_1 + 0y_2 + OS_1 - S_2 + OS_3 + A_1 & = 6 \\
10y_1 + 4y_2 + OS_1 + OS_2 + S_3 + OA_3 & = 40 \\
y_1, y_2, S_1, S_2, S_3, A_1 & \geq 0
\end{align*} \]
Step 3: Table formulation.

**Table 1**

<table>
<thead>
<tr>
<th>C</th>
<th>5</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>X</td>
<td>y1</td>
<td>y2</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
</tr>
<tr>
<td>0</td>
<td>S1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-M</td>
<td>A1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>S3</td>
<td>10</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ Z = -2M \quad 0 \quad 0 \quad M \quad 0 \quad -M \]

\[ Z = 5 + 2M \quad 1 \quad 0 \quad -M \quad 0 \quad 0 \]

The largest \( Z \) is 5 + 2M. Therefore \( y_1 \) goes into the basis, while \( A_1 \) (with ratio 3) leaves the basis.

**Table 2**

<table>
<thead>
<tr>
<th>C</th>
<th>5</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>X</td>
<td>y1</td>
<td>y2</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
</tr>
<tr>
<td>0</td>
<td>S1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>y1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>S3</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ Z = 5 \quad 0 \quad 0 \quad -2.5 \quad 0 \quad 2.5 \]

\[ Z = 15 \]

\[ Z = C_j - Z_j \]

Pivot Operations:

- New Row 2 = \( \frac{1}{2} \) (old R2) \( (R = \text{Row}) \)
- New Row 1 = old R1
- New Row 3 = \( \frac{1}{10} \) (old R3) – New R3

\( S_3 \) goes into the basis while \( S_3 \) leaves the basis.

**Table 3**

<table>
<thead>
<tr>
<th>C</th>
<th>5</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>-M</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>X</td>
<td>y1</td>
<td>y2</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
</tr>
<tr>
<td>0</td>
<td>S1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>y1</td>
<td>1</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>0</td>
<td>S2</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ Z = 5 \quad 1 \quad 0 \quad 0 \quad 0.25 \quad 0 \quad Z = 25 \]

\[ Z = C_j - Z_j \]

\[ Z = 0 \quad 0 \quad 0 \quad 0 \quad -0.25 \quad -M \]
Pivot Operations

\[
\begin{align*}
\text{New } R_1 &= \text{old } R_1 \\
\text{New } R_2 &= \text{old } R_2 + \frac{1}{2} (\text{New } R_3)
\end{align*}
\]

Since all values of \( Z \leq 0 \), the problem is optimal. Therefore, \( y_1 = 5, \ y_2 = 0 \) with optimal value of \( Z = 5(5) + 0 = 25 \)

(b) **The Two-Phase Method**

The method involved two phases of simplex optimization procedure. In the first phase, the sum of the artificial variables is minimized subject to the constraint of the given L.P. problem. This will help to obtain the initial basic feasible solution, which can be used in the next phase (second phase).

The second phase optimizes the original objective function starting with the basic feasible solution obtained at the end of the first phase.

The summary of procedural steps of the method is as follows:

In phase 1, these are the following steps:

**Step 1:** Express the given L.P. problem in standard form. Also ensure that all \( b_i \) constant terms on the right hand side of constraints are non-negative and the constraints of type \( \geq \) or \( = \) are assigned necessary artificial variables.

**Step 2:** Obtain a new objective function \( (w) \), which consists of the sum of artificial variables and the decision variables with assigned zero coefficients.

**Step 3:** Form the initial simplex table.

**Step 4:** Use the simplex method to minimize the function \( w \) subject to original constraints (as obtained in step 1) in order to have the optimal basic feasible solution.

If optimal basic feasible solution gives \( \text{Min. } w = 0 \), then continue with phase II of solving. Otherwise, if \( \text{Min } w \neq 0 \), you stop because feasible solution does not exist.

In phase II, use the optimum basic feasible solution of the phase I as the starting solution for the original L.P problem. Here, make further iterations by the simplex method until the optimal basic feasible solution is obtained.
ILLUSTRATION 18-11

Use the two-phase method to solve the following:

Minimize \( Z = -3x_1 + x_2 + x_3 \)
Subject to:
\[
\begin{align*}
  x_1 - 2x_2 + x_3 & \leq 11 \\
  -4x_1 + x_2 + 2x_3 & \geq 3 \\
  -2x_1 + x_3 & = 1 \\
  x_1, x_2, x_3 & \geq 0
\end{align*}
\]

SUGGESTED SOLUTION 18-11

Step 1: To convert to standard form and add necessary artificial variables.

Minimize \( Z = -3x_1 + x_2 + x_3 \)
Subject to:
\[
\begin{align*}
  x_1 - 2x_2 + x_3 + S_1 + 0S_2 + 0S_3 & = 11 \\
  -4x_1 + x_2 + 2x_3 + 0S_1 - S_2 + A_1 & = 3 \\
  -2x_1 + 0x_2 + x_3 + 0S_1 + 0S_2 + A_2 & = 1 \\
  x_1, x_2, x_3, S_1, S_2, A_1, A_2 & \geq 0
\end{align*}
\]

Step 2: Objective function consisting of artificial variables alone:

Min. \( w = A_1 + A_2 \)
Subject to:
\[
\begin{align*}
  x_1 - 2x_2 + x_3 + S_1 + 0S_2 + 0S_3 & = 11 \\
  -4x_1 + x_2 + 2x_3 + 0S_1 - S_2 + A_1 & = 3 \\
  -2x_1 + 0x_2 + x_3 + 0S_1 + 0S_2 + A_2 & = 1 \\
  x_1, x_2, x_3, S_1, S_2, A_1, A_2 & \geq 0
\end{align*}
\]

Step 3: Formulation of simplex table.

<table>
<thead>
<tr>
<th>( C )</th>
<th>( 0 )</th>
<th>( 0 )</th>
<th>( 0 )</th>
<th>( 0 )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 1 )</th>
<th>( b_i )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_8 )</td>
<td>( X_8 )</td>
<td>( X_1 )</td>
<td>( X_2 )</td>
<td>( X_3 )</td>
<td>( S_1 )</td>
<td>( S_2 )</td>
<td>( A_1 )</td>
<td>( A_2 )</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-4</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>3</td>
<td>1.5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
  Z & = -6 + 1 + 3 - 1 + 1 + 1 = 4 \\
  w & = 4 \\
  Z' = C_iZ & = 6 - 1 - 3 + 1 + 0 + 0
\end{align*}
\]
Table 2

<table>
<thead>
<tr>
<th></th>
<th>( C_j )</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_B )</td>
<td>( X_B )</td>
<td>( X_1 )</td>
<td>( X_2 )</td>
<td>( X_3 )</td>
<td>( S_1 )</td>
<td>( S_2 )</td>
<td>( A_1 )</td>
<td>( A_2 )</td>
</tr>
<tr>
<td>0</td>
<td>( S_1 )</td>
<td>3</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>( A_1 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>( X_3 )</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( Z )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>( Z = C_j - Z_i )</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Table 3:

<table>
<thead>
<tr>
<th></th>
<th>( C_j )</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_B )</td>
<td>( X_B )</td>
<td>( X_1 )</td>
<td>( X_2 )</td>
<td>( X_3 )</td>
<td>( S_1 )</td>
<td>( S_2 )</td>
<td>( A_1 )</td>
<td>( A_2 )</td>
</tr>
<tr>
<td>0</td>
<td>( S_1 )</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-2</td>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>( X_2 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>( X_3 )</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( Z )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Since \( w = 0 \), we move to phase II.
Phase II with objective function
Min. \( Z = -3x_1 + x_2 + x_3 \)
Subject to the constraints in table 3.

Table 4

<table>
<thead>
<tr>
<th></th>
<th>( C_j )</th>
<th>-3</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_B )</td>
<td>( X_B )</td>
<td>( X_1 )</td>
<td>( X_2 )</td>
<td>( X_3 )</td>
<td>( S_1 )</td>
<td>( S_2 )</td>
<td>( A_1 )</td>
<td>( A_2 )</td>
</tr>
<tr>
<td>0</td>
<td>( S_1 )</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-2</td>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>( X_2 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>( X_3 )</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Z )</td>
<td>-2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>( Z = C_j - Z_i )</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

We should not allow the artificial variable to go into Basis again. Hence, variable \( X_1 \) enters the basis while \( S_1 \) leaves the basis.
Table 5

<table>
<thead>
<tr>
<th>C</th>
<th>-3</th>
<th>1</th>
<th>1</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>b</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.33</td>
<td>-0.67</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.67</td>
<td>-1.33</td>
</tr>
<tr>
<td>Z</td>
<td>-3</td>
<td>1</td>
<td>1</td>
<td>-0.33</td>
<td>-0.33</td>
</tr>
<tr>
<td>Z = C - Z</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.33</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The problem is optimal at X₁ = 4 , X₂ = 1 , X₃ = 9 with the optimal value Z = -3(4) + 1(1) + 1(9) = -12 + 1 + 9 = -2

18.7 DUALITY CONCEPT IN L.P. PROBLEM

Duality concept can be stated as follows:

For any L.P. problem, there exists another associated L.P. problem called the dual of the given problem. The given problem is called Primal Problem. A solution to the primal L.P. problem also gives the same solution to the dual.

(a) Some Rules for Transforming Primal to a Dual L.P. Problem are listed as follows:

(ii) Define one non-negative dual variable for each primal constraint.

(iii) Make the cost vector of the primal the right hand side (R.H.S.) constraints of the dual.

(iv) Make the R.H.S. vector of the primal the cost vector of the dual.

(v) The transpose of co-efficient of the primal becomes the constraints matrix of the dual.

(vi) Reverse the direction of constraint inequalities.

(vii) Reverse the optimization direction i.e. change Max. to Min and vice versa.
ILLUSTRATION 18-12

Max. Z = \( x_1 + 2x_2 - 3x_3 + 4x_4 \)
Subject to:
\[ x_1 + 2x_2 + 2x_3 - 3x_4 \leq 25 \]
\[ 2x_1 + x_2 - 3x_3 + 2x_4 \leq 15 \]
\[ x_i \geq 0. \quad \forall i = 1, 2, 3, 4 \]

Dual
Min. D = \( 25y_1 + 15y_2 \)
Subject to:
\[ y_1 + 2y_2 \geq 1 \]
\[ 2y_1 + y_2 \geq 2 \]
\[ 2y_1 - 3y_2 \geq -3 \]
\[ -3y_1 + 2y_2 \geq 4 \]
\[ y_1, y_2 \geq 0 \]

So \( x_j; \ j = 1, 2, ..., 4 \) are called the primal variables and \( y_i; \ i = 1, 2 \) are called the dual variables.

Note that the dual problem can be formed when the primal problem is symmetric.

(b) **Definition of Symmetric Form**

A L.P. problem is said to be in the symmetric form if all the variables are restricted to be non-negative and all the constraints are inequality of the same type, otherwise, it is said to be asymmetric i.e. if any of the decision variables is unrestricted in sign and the inequality constraints are of the mixed type.

There is a way of handling asymmetric form. It is illustrated by the following example:

Consider a primal problem in asymmetric form as follows:

Max. \( Z = 4x_1 + 5x_2 \)
Subject to:
\[ 3x_1 + 2x_2 \leq 20 \quad \ .......... \quad 1 \]
\[ 4x_1 - 3x_2 \geq 10 \quad \ .......... \quad 2 \]
\[ x_1 + x_2 = 5 \quad \ .......... \quad 3 \]
\[ x_1 \geq 0, \ x_2 \text{ unrestricted in sign} \]

Convert the above problem to symmetric form. This means that all the constraints must be \( \leq \) type inequalities (since the primal is a maximisation problem), and all the variables are non-negative. This can be accomplished as follows:

(i) Inequality 2 is multiplied by \(-1\)
Equation 3 is replaced by a pair of inequalities i.e. \( x_1 + x_2 \leq 5 \) and \( x_1 + x_2 \geq 5 \).

The unrestricted variable \( x_4 \) is replaced by the difference of two non-negative variables, \( x_3 \) and \( x_4 \).

Thus, the symmetric form of primal problem and its Dual are:

<table>
<thead>
<tr>
<th>Primal</th>
<th>Dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. ( Z = 4x_1 + 5x_4 - 5x_4 )</td>
<td>Min. ( D = 20y_1 - 10y_2 + 5y_3 - 5y_4 )</td>
</tr>
<tr>
<td>Subject to: ( 3x_1 + 2x_3 - 2x_4 \leq 20 )</td>
<td>Subject to: ( 3y_1 - 4y_2 + y_3 - y_4 \geq 4 )</td>
</tr>
<tr>
<td>(-4x_1 + 3x_3 - 3x_4 \leq -10 )</td>
<td>( 2y_1 + 3y_2 + y_3 - y_4 \geq 5 )</td>
</tr>
<tr>
<td>( x_1 + x_3 - x_4 \leq 5 )</td>
<td>(-2y_1 - 3y_2 - y_3 + y_4 \geq 5 )</td>
</tr>
<tr>
<td>(-x_1 + x_3 + x_4 \leq -5 )</td>
<td></td>
</tr>
<tr>
<td>( x_1, x_3, x_4 \geq 0 )</td>
<td>( y_1, y_2, y_3, y_4 \geq 0 )</td>
</tr>
</tbody>
</table>

Summary of the Primal/Dual Relationship (Symmetric Case).

<table>
<thead>
<tr>
<th>Primal</th>
<th>Dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Constraints coefficient matrix</td>
</tr>
<tr>
<td>b</td>
<td>R.H.S. Constant</td>
</tr>
<tr>
<td>c</td>
<td>Cost (price) vector</td>
</tr>
<tr>
<td>d</td>
<td>Objective function = Max. ( Z )</td>
</tr>
<tr>
<td>e</td>
<td>( = C )X</td>
</tr>
<tr>
<td>f</td>
<td>Constraints inequalities ( AX \leq b )</td>
</tr>
<tr>
<td>Decision variables ( X \geq 0 )</td>
<td>Transpose of the coefficient (constant) matrix</td>
</tr>
<tr>
<td></td>
<td>Cost (price) vector</td>
</tr>
<tr>
<td></td>
<td>R.H.S. constraints</td>
</tr>
<tr>
<td></td>
<td>Objective function Min. ( D = b'Y )</td>
</tr>
<tr>
<td></td>
<td>( A'Y \geq C' )</td>
</tr>
<tr>
<td></td>
<td>( Y \geq 0 )</td>
</tr>
</tbody>
</table>

In matrix notations; we have:

Primal

Max. \( Z = CX \)

Subject to: \( AX \leq b \)

\( X \geq 0 \)

Dual

Min. \( D = b'Y \)

Subject to: \( A'Y \geq C' \)

\( Y \geq 0 \)

Suppose the primal is

Max. \( Z = 20x_1 + 30x_2 \)

Subject to:

\( 3x_1 + 2x_2 \leq 210 \)

\( x_1 + 2x_2 \leq 100 \)

\( x_1, x_2 \geq 0 \)

Min. \( D = 210y_1 + 100y_2 \)

Subject to:

\( 3y_1 + y_2 \geq 20 \)

\( 2y_1 + 2y_2 \geq 30 \)

\( y_1, y_2 \geq 0 \)
Note that Dual problem can also be solved by graphical or simplex method as done for primal problem. However, the following difference in terms of economic evaluation is made about primal and dual problems.

By using primal L.P. problem, attempt is made to optimize resource allocation in a way that the quantities produced will maximize profit. While on the other hand, the dual L.P. problem attempts to optimize resource allocation in a way that quantity produced will make the marginal opportunity cost equals its marginal return. Therefore, the major focus of dual problem is to find for each resource its best Marginal Value or Shadow Price. This value is an indicator of the level of resource scarcity.

Formally, shadow price can be defined as rate of change in the optimal objective function value with respect to the unit change in the availability of a resource. i.e.

\[
\text{Shadow Price} = \frac{\text{Change in Optimal Objective function value}}{\text{Unit change in the availability of resource}}.
\]

18.8 SUMMAR Y AND CONCLUSIONS

This is a mathematical technique used in optimizing the value of a linear objective function subject to the linear relations or constraints. The model consists of three basic components or elements. These are: Decision Variables, Objective Functions and Constraints.

The graphical and simplex methods of solving L.P. problems are considered. The ideas of Duality and shadow price are considered in the text.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
18.9 REVISION QUESTIONS

18.9.1 MULTIPLE CHOICE QUESTIONS
1. The role of surplus variables in the simplex method of solving L.P. problem is to
   (a) start phases of simplex method
   (b) have inequalities of type $<\text{ in equation form}$
   (c) aid in finding dual problem
   (d) have inequalities of type $\geq\text{ in equation form}$
   (e) make the L.P. problem feasible.

2. In a minimization problem, the objective function coefficient for an artificial variable by the penalty method is
   (a) zero
   (b) $M$
   (c) $-M$
   (d) $M+1$
   (e) $-(M+1)$

3. For formulation of solution to the simplex method, there is need to add artificial variable to:
   (a) inequalities of “less than” constraints
   (b) inequalities of “greater than” constraints
   (c) equality constraints
   (d) both (a) and (b)
   (e) both (b) and (c).

18.9.2 SHORT ANSWER QUESTIONS

1. A L.P problem in which all the variables are non-negative and all the constraints are inequalities of the same type is said to be __________

2. Penalty or Two-phase method of solving L.P. problem is used when _______ variable is introduced.

3. The Canonical form of equations in L.P. problem enables us to obtain _______ solution.

4. The region that satisfies all the constraints of a L.P. problem simultaneously is known as __________

5. The increase in the objective function value by providing one additional unit of resource is known as __________

6. The value of a dual variable represents _______ of each additional unit of resource.
7. If the primal has unbounded solution, the dual has no __________

Use the following information to answer questions 8 and 9

Given the following L.P. problems;
Min. \( Z = 3x_1 + 2x_2 \)
s.t. \( x_1 + 3x_2 \leq 10 \)
\( 2x_1 + x_2 \geq 0 \)
\( 3x_1 - x_2 \geq 4 \)
\( x_1, x_2 \geq 0 \)

8. the symmetric form is __________

9. the dual problem is __________

Refer to Suggested Solutions to Revision Question in Appendix 1, page 483.
19.0 LEARNING OBJECTIVES

After studying this chapter, readers should be able to understand:

- The concept and meaning of Network analysis;
- Analyze how to draw a network diagram based on:
  - Activity-on-arrow
  - Activity-on-note
  - Concepts;
- The two types of precedence network models;
- The concept of some important terms such as floats, slack, earliest time and latest time; and
- The two popular methods of handling the network analysis: the Critical Path Method (CPM) and Programme Evaluation and Review Technique (PERT).

19.2 INTRODUCTION: CONCEPT AND MEANING OF NETWORK ANALYSIS

Network Analysis (N.A.) is an important concept that deals with project management. A project is a well-defined set of jobs, tasks, or activities, which must be completed in a specified time and sequence order. The jobs, tasks or activities are interrelated and require resources such as money, material, personnel and other facilities.

Therefore, the major focus of project management is to schedule jobs, tasks or activities in an efficient way so as to reduce or have minimum cost of its completion within the specified period.

Network analysis, sometimes called network planning and scheduling techniques, is a set of operations research techniques needed and useful for planning, scheduling and controlling large and complex...
network analysis

443

projects. The diagrammatical representation of these techniques, which consists of arrows (activities) and nodes (events) is known as network diagram.

Possible areas of application of network analysis include:

(a) Construction projects such as building highways, house, bridge, e.t.c;
(b) Maintenance or planning of oil refinery, ship repair and other large operations;
(c) Preparation of bids and proposals for large projects;
(d) Development of new weapon/system or manufacturing system;
(e) Manufacturing and assemblage of large items such as airplanes, ships and computers; and
(f) Simple projects such as home remodeling, moving to a new house, house cleaning and painting.

19.2 SOME BASIC TERMS IN NETWORK

Some basic terms that are important as components in the network include:

(a) **Activities:** An activity represents an action, project operation or task which is time and resource required effort to complete a particular path of an overall project. It incurs cost and has a point of time where it begins and a point where it ends. Hence, an arrow symbol (→) is usually used to represent an activity.

(b) **Dummy activity:** This is an activity which carries zero duration because it does not consume either time or resource. The symbol commonly used for this is the arrow with dotted line (••••••). The two main reasons for using dummy activity are to present (i) the logical sequence of activities and, (ii) the unique activity numbering system.

(c) **Event:** An event represents the start or completion of an activity or task and as such does not consume time and resource. Hence, it can be said to be scientific instant of time that denotes the beginning or end of an activity. It is usually represented by a circle (O) as node in the network diagram.

Event can be further categorized into **burst or merge type.** When an event is the starting event of two or more activities, it is
called **burst event**. While an event which is the ending event of two or more activities is called **merge event**. They are both depicted below:

![Diagram showing burst and merge events]

**Fig 19.1 (Burst and Merge Events)**

### 19.3 NETWORK DIAGRAM

As earlier stated, the pictorial representation of the interrelation of the various activities and events concerning a project is known as network diagram.

Network diagram has two types of precedence network models. Each depicts the precedence requirements of the various activities. The two precedence models are:

(a) *Activity-on-Arrow Network (Arrow Diagram)*

(b) *Activity-on-Node Network.*

**Activity-on-Arrow (AOA) Network Diagram:**

This is a popular method of network analysis. In arrow diagram, the network is drawn by putting the various activities of the project on arrows which are arranged in logical sequence. Every event is assigned a number which is placed inside the circle (node). The numbering sequence allows the flow of network.

The basic rules of numbering events are:

(i) The initial event is numbered 1. It is an event which has all outgoing arrows with no incoming arrow.
(ii) The events after 1 are numbered 2, 3, 4, - - - - in the order of the logical sequence of the activities. They have arrows going out and coming in.

(iii) The terminal or final event has all arrows coming in with no arrow going out. It has the last number.

In summary, the arrow diagram has a single starting node from which all activities (with no predecessors) may begin and then work its way left to right by ending with single node.

ILLUSTRATION 19-1
Consider six jobs or tasks A, B, C, D, E, F, of a project with the following job sequence:
Job A precedes B and C
Job B precedes D
Job C and D precedes E
Job E precedes F
Draw an arrow diagram for this problem.

SUGGESTED SOLUTION 19-1

(b) Activity-on-node Network (AON diagram)

In this network, the name of activity and its duration are usually placed within the nodes. The arrows are used to indicate the sequencing order requirements of the tasks or jobs.

This approach eliminates the problem of using dummy activities. It has no particular starting and ending events (nodes) for the entire project.
ILLUSTRATION 19-2
Use the data in example 19-1, draw the AON diagram for the problem.

SUGGESTED SOLUTION 19 - 2

![AON Diagram]

Start → A → C → D → E → F → Finish

19.3.1 Comparison of AOA and AON Diagrams

Arrow diagrams are more popular than AON diagrams but both are extensively used in practice. However, there is no computational proof to support that one is better than the other. However, each has its advantages and disadvantages.

(a) Advantages of Arrow Diagram

(i) It has a better adaptation to PERT [program evaluation review technique].
(ii) Arrow diagrams are used in many computer programs.
(iii) It gives a better flow of time sequence for the entire project.
(iv) It can be drawn by superimposing a time scale with an arrow to indicate the correct length of time duration.

(b) Advantages of AON diagram

(i) It is widely used and adapted to construction industries.
(ii) It is simpler and easier to explain and understand.
(iii) It is also easier to revise and update.
(iv) It does not use dummy activity.

The disadvantages of Arrow diagram are those activities that AON diagram can handle but which the Arrow diagram cannot. While the AON diagram disadvantages are those activities the Arrow diagram can handle but the AON can not.
For example, the following shall be used to demonstrate the issue of dummy activity.

**ILLUSTRATION 19-3**

Given five jobs J,K,L,M,N, with durations of 4,8,7,5 and 11 days respectively, draw network diagram using both arrow and AON diagrams. The predecessor to activities K,L and M is the activity J; while predecessors to activity N are activities K,L,M.

**SUGGESTED SOLUTION 19-3**

From the above diagrams, we can see that dummy activity is used in arrow diagram in order to have the needed logical sequence; while we do not need it in AON diagram.

**19.6 METHODS OF NETWORK ANALYSIS**

The two well known methods of network analysis are:

(a) Critical path method (CPM); and

(b) Program evaluation review technique (PERT).

Both methods (CPM and PERT) focus their procedures on identifying the critical path. The critical path takes the longest or maximum time duration of the project; while activities on the critical path are called the critical activities.
Some major features of both methods are discussed as follows:

(a) **CPM**

(i) CPM is used for activities which are deterministic;
(ii) It is an activity-oriented technique;
(iii) It is good in establishing a trade-off for optimum balancing between scheduled time and cost of the project; and
(iv) CPM was developed mostly for maintenance and construction projects.

(b) **PERT**

(i) It is an event-oriented technique;
(ii) It handles probabilistic type of data; and
(iii) It has activities of non-receptive nature and therefore used for one-time project.

19.4.1 CPM procedure

The procedure of identifying the critical path in the CPM involves three phases. These phases are:

(a) Forward pass computations;
(b) Backward pass computations; and
(c) Float (slack) computations.

(a) **Forward pass computation**

This is also known as computation of earliest time. The cumulative durations of jobs from the start and to the end of a project are obtained by this computation.

In order to carry out the necessary computations, we need to know that an event (node) is a point in time at which activities are terminated and others are started. Also, computations start at node 1 and advance recursively to the end node. By this, we have the following steps for forward pass:

**Step 1:** set $ET_1 = 0$ to indicate that the project starts at time 0.
Step 2: obtain the next ET\textsubscript{i} by the general formula:
\[ ET_{i} = \text{Maximum of all } [ET_{i} + t_{ij}] \text{ for all } i,j \text{ leading to the events} \]
Where \( ET_{i} \) is Earliest expected time at time zero, \( ET_{j} \) is Earliest expected time of the successor event \( j \)
\( ET_{i} \) is Earliest expected time of the predecessor event \( i \), and \( t_{ij} \) is the expected Time of activity \( ij \) (i.e. \( i \rightarrow j \)).
\[ i = 1, 2, 3, \ldots, n - 1 \]
\[ j = 2, 3, 4, \ldots, n. \]

(b) Backward Pass Computations

This is known as latest time computation. The computations begin from the last event (n) and precede in descending order of the node till the initial event (1). The basic steps of backward pass computation are summarized below:

Step 1: set \( LT_{n} = ET_{n} \)

Step 2: Obtain the next LT\textsubscript{i} by the general formular:
\[ LT_{i} = \text{Minimum of all } [LT_{j} - t_{ij}] \text{ for all } i,j \text{ emerging from } i \]
Where \( LT_{n} \) is the latest time for event \( n \)
\( LT_{i} \) is the latest allowable time for event \( i \)
\( LT_{j} \) is the latest allowable time for event \( j \), and \( t_{ij} \) is the expected time of activity \( ij \).

(c) Float (slack) Computations

The term “float” or “slack” refers to the period of time by which a particular activity or event can be delayed without the time schedule of the network affected. The term “float” is referred to CPM when activities are considered, while PERT goes with slack when events are involved.

For the computation, float or slack can be obtained by finding the difference between the latest allowable time and earliest expected time for event \( j \) or activity \( j \).
\[ \text{i.e. Float (slack)} = LT_{j} - ET_{j} \]
The events (activities) with zero float (slack) will give the expected critical path.

### ILLUSTRATION 19-4

An electrification project in SAO village has to pass through six tasks. The duration and the precedence order of these tasks are given below:

<table>
<thead>
<tr>
<th>TASKS (ACTIVITY)</th>
<th>PREDECESSOR ACTIVITY</th>
<th>DURATION IN DAYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>14</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>16</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>18</td>
</tr>
<tr>
<td>D</td>
<td>B</td>
<td>15</td>
</tr>
<tr>
<td>E</td>
<td>C</td>
<td>17</td>
</tr>
<tr>
<td>F</td>
<td>D, E</td>
<td>15</td>
</tr>
</tbody>
</table>

(a) Construct the network diagram for the project.

(b) Determine:

(i) the forward passes
(ii) the backward passes
(iii) the float; and hence the critical path.

### SUGGESTED SOLUTION 19-4

(b) (i) The forward passes can be obtained by formula:

\[
ET_j = \text{Max}[Et_i + t_{ij}]
\]

\[
ET_i = 0
\]

\[
ET_2 = \text{Max}[ET_1 = 0, ET_j + t_{12}]
\]

\[
= \text{Max}(0,14) = 14 \text{ days}
\]
\[ ET_3 = \text{Max} [ET_2 = 14, ET_2 + t_{23}] \]
\[ \text{Max} (14, 32) = 32 \text{ days} \]
\[ ET_4 = \text{Max} [ET_2 = 14, ET_2 + t_{24}] \]
\[ \text{Max}(14, 30) = 30 \text{ days} \]
\[ ET_5 = \text{Max} [ET_3 = 32, ET_3 + t_{35}, ET_4 = 30, ET_4 + t_{45}] \]
\[ \text{Max}(32, 49, 45) = 49 \text{ days} \]
\[ ET_6 = \text{Max} [ET_5 = 49, ET_5 + t_{56}] \]
\[ \text{Max}(49, 64) = 64 \text{ days} \]

(ii) The backward passes are also obtained by formula:
\[ LT_i = \text{Min} [LT_j - t_{ij}] \]
\[ LT_6 = ET_6 = 64 \text{ days}. \]
\[ LT_5 = \text{Min} \{LT_6 = 64, LT_6 - 15 \} = 49 \text{ days}. \]
\[ LT_4 = \text{Min} \{LT_5 = 49, LT_5 - 15 \} = 34 \text{ days}. \]
\[ LT_3 = \text{Min} \{LT_4 = 49, LT_4 - 17 \} = 32 \text{ days}. \]
\[ LT_2 = \text{Min} \{LT_3 = 34, LT_3 - 16, LT_4 = 32, LT_4 - 18 \} = 14 \text{ days}. \]
\[ LT_1 = \text{Min} \{LT_2 14, LT_2 - 14 \} = 0 \text{ day}. \]

(iii) The float = \( LT_i - ET_i \)

<table>
<thead>
<tr>
<th>Event</th>
<th>( LT_i )</th>
<th>( ET_i )</th>
<th>Float(Slack)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>34</td>
<td>32</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>49</td>
<td>49</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
<td>64</td>
<td>0</td>
</tr>
</tbody>
</table>

The critical path is \( 1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6 \) with activities \( A \rightarrow C \rightarrow E \rightarrow F \)
The critical length is \( 14 + 18 + 17 + 15 = 64 \text{ days} \).

19.4.2 Alternative Methods of obtaining Critical Path

It is possible to obtain the critical path without using the float (slack) approach. All that is needed is to draw first the network diagram and then list out all possible routes in the network. Compute the total days for each route and then pick the route with the highest total as the critical path. For example, using the data in illustration 19.4, we have the following routes:

(a) ABDF (With total duration of \( 14 + 16 + 15 + 15 = 60 \text{ days} \))
(b) ACEF (With total duration of 14 + 18 + 17 + 15 = 64 days)  
The higher of the two routes is ACEF with total of 64 days.  
This gives the same result as the float (slack) approach.

19.4.3 PERT PROCEDURE

In CPM, it is assumed and known that the duration of activities is  
deterministic and of a single time estimate. But for PERT, it is  
probabilistic and activity is based on three time estimates of  
performance. The three time estimates that are normally  
employed in PERT are:

(a) The most optimistic time (t_o)  
(b) The most pessimistic time (t_p)  
(c) The most likely time (t_i)

Where t_o is the shortest time, possibly assuming everything goes  
perfectly well  
t_p is the longest possible time including time for usual delay  
t_i is the best estimate of what would normally occur.

By the assumption of Beta distribution and application of  
necessary statistical theory, the expected activity time (t_e) is  
obtained by:

\[ t_e = \frac{t_o + 4t_i + t_p}{6} \]  \hspace{1cm} 16.4.3.1

which provides 50 – 50 chance of activity trial. The variance (\(\delta^2\))  
and standard deviation \(\delta\) of time required to complete an  
activity are respectively given by:

\[ \delta^2 = \frac{1}{6} (t_p - t_o)^2 \]
\[ \delta = \frac{1}{6} (t_p - t_o) \]

while the variance and standard deviation of time required to  
complete the entire project are respectively given by:

\[ \delta^2 = \left( \frac{t_{p1} - t_{j1}}{6} \right)^2 + \left( \frac{t_{p2} - t_{j2}}{6} \right)^2 + \ldots \ldots + \left( \frac{t_{pn} - t_{jn}}{6} \right)^2 \]  \hspace{1cm} 16.4.3.2

\[ \delta = \sqrt{\left( \frac{t_{p1} - t_{j1}}{6} \right)^2 + \left( \frac{t_{p2} - t_{j2}}{6} \right)^2 + \ldots \ldots + \left( \frac{t_{pn} - t_{jn}}{6} \right)^2} \]  \hspace{1cm} 16.4.3.3
After obtaining the expected activity time \( t_e \) for each activity, we then apply the same procedure of obtaining forward pass and backward pass as done in CPM in order to have the critical path. Hence, project completion time can be obtained. Also by assumptions of normality and central unit theorem, the probability of completing the project in some given time can be obtained by:

\[
\text{Prob. } \left( Z = \frac{T_s - T_e}{\delta} \right)
\]

Where \( T_s \) is the expected project scheduled time
\( T_e \) is the expected completion time of project
\( \delta \) is the expected standard deviation to complete the project.
\( Z \) is the number of standard deviations of the scheduled time or target date.

Note further that \( T_e \) can be obtained by adding the expected times of all the activities lying on the critical path.

**ILLUSTRATION 19-5**

A small project consists of 7 activities whose three time estimates are given below:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Activity’s Name</th>
<th>Time Estimates (days)</th>
<th>( t_o )</th>
<th>( t_i )</th>
<th>( t_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 2</td>
<td>H</td>
<td>3 3 12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 – 3</td>
<td>I</td>
<td>3 6 9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 – 4</td>
<td>J</td>
<td>6 6 15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 – 5</td>
<td>K</td>
<td>3 3 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 – 5</td>
<td>L</td>
<td>6 15 24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 – 6</td>
<td>M</td>
<td>6 15 21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 – 6</td>
<td>N</td>
<td>9 15 27</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Find the expected duration and variance for each activity.

(b) Draw a network diagram.

(c) What is the expected project length (duration)?

(d) Calculate the variance and standard deviation of the project length.

(e) What is the probability that the project will be completed at least 5 days earlier?
SUGGESTED SOLUTION 19-5

(A) The \( t_e \) and \( \delta^2 \) are computed in the following table:

<table>
<thead>
<tr>
<th>Activity's</th>
<th>Time estimates (days)</th>
<th>( t_e )</th>
<th>( \delta^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t_0 )</td>
<td>( t_i )</td>
<td>( t_p )</td>
</tr>
<tr>
<td>H</td>
<td>3</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>I</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>J</td>
<td>6</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>K</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>L</td>
<td>6</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>M</td>
<td>6</td>
<td>15</td>
<td>21</td>
</tr>
<tr>
<td>N</td>
<td>9</td>
<td>15</td>
<td>27</td>
</tr>
</tbody>
</table>

(b) The network diagram of the problem

(c) The critical path
The possible routes are
1. \( 1 - 2 - 5 - 6 \) with total duration \( 4.5 + 3 + 16 = 23.5 \) days
2. \( 1 - 3 - 5 - 6 \) with total duration \( 6 + 15 + 16 = 37 \) days
3. \( 1 - 4 - 6 \) with total duration \( 7.5 + 14.5 = 22 \) days

The highest total duration is 37 days which gives the critical path \( 1 - 3 - 5 - 6 \) \((I,L,N)\). Therefore, the expected project length is 37 days.

(Note that forward pass and backward pass can also be used to obtain the critical path).

(d) The variance of the project length is
\[ \text{var. of } I + \text{var. of } L + \text{var. of } N = 1 + 9 + 9 = 19 \text{ days} \]
NETWORK ANALYSIS

\[
\text{Standard Deviation Project Length } = \sqrt{19} = 4.36 \text{ days}
\]

(e) The Probability of project to be completed at least 5 days earlier (i.e. \( 37 - 5 = 32 \) days) is

\[
\text{Prob. } Z = 32 - 37 = \text{Prob. } Z = \frac{1.1468}{4.36} = 0.1251
\]

\[.\] The probability of the project completed at least 5 days earlier is \( 1 - 0.1251 = 0.8749 \)

19.7 SUMMARY AND CONCLUSIONS

Network analysis is an important concept that deals with project management and it helps in having an efficient way of reducing or having low cost of its completion time.

The two well known methods of network analysis are CPM and PERT. CPM and PERT are respectively used for activities, which are deterministic and probabilistic in nature.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

19.6 REVISION QUESTIONS

19.6.1 MULTIPLE CHOICE QUESTIONS

1. Which of the following is the main focus or objective of the network analysis?
   A. To minimize total project cost
   B. To minimize production delays
   C. To minimize total project duration
   D. To minimize interruption and conflicts
   E. (b) and (d).

2. Which of the following is not an advantage of network model?
   A. Profiting
   B. Planning
   C. Scheduling
   D. Controlling
   E. (b) and (c).

3. Which of the following is the slack for activity?
   A. \( ET_j - LT_j \)
   B. \( LT_j - Et_j \)
   C. \( LT_j - t_{ij} \)
   D. \( ET_j - t_{ij} \)
   E. \( t_{ij} - LT_j \).
19.6.2 SHORT ANSWER QUESTIONS

1. When an activity has zero slack, it means that it lies on the ________

2. The network analysis that deals with uncertain demand is known as _______

3. An action, project operation or task which is a time and resource required effort to complete a particular path of overall project is known as ________

4. When an event is the starting event of two or more activities, the event is known as ________

5. In a network diagram, the approach that eliminates the use of dummy activities is known as ________

6. Arrow diagram method has a better adaptation to ______ method of analysis.

7. The following estimates (t₀, tₚ and tₙ) are used in the computational procedure of ________

Use the following information to answer questions 8 and 9.
Using the following network diagram:

8. The possible routes are ______________

9. The critical path is ______________

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
REPLACEMENT ANALYSIS

20.0 LEARNING OBJECTIVES

After studying this chapter, readers should be able to understand:

- Meaning and purpose of replacement theory;
- Concept and technique of replacement;
- Replacement policy of those items that wear-out gradually;
- Replacement policy of the items that fail suddenly; and
- Difference between individual and group replacement policies.

20.2 INTRODUCTION

All equipments used in industries, military, and even at homes have a limited life span. By passage of time, these equipments fail suddenly or wear out gradually. As these machines or equipments are wearing out, the efficiency of their functions continues to decrease and in effect affects the production rates or economic/social benefits of the system. Also at the old age or the wearing-out stage, the equipments will definitely require higher operating costs and more maintenance costs due to repairing and replacement of some parts.

Examples of the equipments are transportation vehicles (such as cars, lorries or aircraft), machines used in industries, tires, highway tube lights, electric bulbs, and contact set, used by vehicles.

The essential reasons for the study of replacement analysis are:

(a) To ensure efficient functioning of the equipments; and

(b) To know when and how best the equipments can be replaced in order to minimise the total costs of maintaining them.
There are various types of replacing policies, but this chapter is concerned with the following:

(a) Replacement of equipments/items that deteriorate or wear-out gradually; and
(b) Replacement of equipments/items that fail suddenly.

20.2 Replacement of Equipments/Items that Deteriorate or Wear-Out Gradually

The efficiency of equipment/items that deteriorate with time will be getting low. As the items deteriorate, gradual failure sets in. The gradual failure is progressive in nature which affects the efficiency and resulted in:

(a) A decrease in its production capacity;
(b) The increasing maintenance and operating costs; and
(c) Decrease in the value of the re-sale price (or salvage) of the item.

Due to the above effects, it is economical to replace a deteriorating equipment/item with a new one. As the repair and maintenance costs are the determining factors in replacement policy, the following two policies are to be considered.

(a) **Replacement of items that Deteriorate and whose maintenance and repair costs increase with time, ignoring changes in the value of money during the period.**

This is a simple case of minimizing the average annual cost of an equipment when the maintenance cost is an increasing function time but the time value of money remains constant.

In order to determine the optional replacement age of a deteriorating equipment/item under the above conditions, the following mathematical formulae shall be used:

\[
\begin{align*}
C & = \text{Capital or purchase cost of the equipment/item} \\
S & = \text{Scrap (or salvage) value of the equipment/item at the end of } t \text{ years.} \\
TC(t) & = \text{Total cost incurred during } t \text{ years.} \\
ATC & = \text{Average annual total cost of the equipment/item}
\end{align*}
\]
\( n \) = Replacement age of the equipment/item; i.e. number of equipment years the equipment/item is to be in use.

\( f(t) \) = Operating and maintenance cost of the equipment/item at time \( t \).

**Case 1:** (When time “\( t \)” is a continuous variable)

Here, we need to find the value of \( n \) that minimizes the total cost incurred during the first \( n \) years.

\[
TC(t) = \text{Capital cost} - \text{scrap} + \text{maintenance cost}
\]

\[
= C - S + \int_0^1 f(t) \, dt
\]

But for average annual total cost, we have

\[
\text{ATC}(n) = \frac{1}{n} \left( C - S + \int_0^n f(t) \, dt \right)
\]

To obtain optimal value for \( n \) for which \( \text{ATC}(n) \) is minimum, we differentiate \( \text{ATC}(n) \) with respect to \( n \) and equate the result to zero so that.

\[
f(n) = \frac{1}{n} \left( C - S + \int_0^n f(t) \, dt \right), \quad n \neq 0
\]

\[
= \text{ATC}(n)
\]

Hence, the equipment/item is to be replaced when the average annual cost equals the current maintenance cost.

**Case 2** (When time “\( t \)” is a discrete variable).

In a similar manner, the total cost incurred during \( n \) years is

\[
TC(n) = C - S + \sum_{t=0}^n f(t) = \frac{1}{2} \left[ f(1) - S + \int_0^n f(t) \, dt \right]
\]

while the average annual cost on item is given as

\[
TC(n) = \frac{1}{n} \left[ C - S + \sum_{t=0}^n f(t) \right]
\]
\[
\frac{d[TC(n)]}{dn} = \frac{-1}{n^2} + \frac{S}{n^2} - \frac{e(t)}{n^2}
\]

For this case, we will use a tabular method for obtaining the value of \(n\).

**ILLUSTRATION 20-1**

An owner of a grinding machine estimates from his past records that cost per year of operating his machine is given below:

<table>
<thead>
<tr>
<th>Year</th>
<th>Operating Cost (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>550</td>
</tr>
<tr>
<td>3</td>
<td>850</td>
</tr>
<tr>
<td>4</td>
<td>1250</td>
</tr>
<tr>
<td>5</td>
<td>1850</td>
</tr>
<tr>
<td>6</td>
<td>2550</td>
</tr>
<tr>
<td>7</td>
<td>3250</td>
</tr>
<tr>
<td>8</td>
<td>4050</td>
</tr>
</tbody>
</table>

If the cost price is ₦12,300 and the scrap value is ₦250, when should the machine be replaced?

**SUGGESTED SOLUTION 20-1**

\(C = ₦12300\) and \(S = ₦250\)

<table>
<thead>
<tr>
<th>Year of Service (n)</th>
<th>Running Cost (₦)</th>
<th>Cumulative Running Cost (₦)</th>
<th>Depreciation Cost Price (C - S)</th>
<th>Total Cost (₦) TC</th>
<th>Average Cost (₦) ATC (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>COL 1</td>
<td>COL 2</td>
<td>COL 3</td>
<td>COL 4</td>
<td>COL 3 + COL 4 = COL5</td>
<td>COL 6 = COL5/n</td>
</tr>
<tr>
<td>1</td>
<td>250</td>
<td>250</td>
<td>12050</td>
<td>12300</td>
<td>12300</td>
</tr>
<tr>
<td>2</td>
<td>550</td>
<td>800</td>
<td>12050</td>
<td>12850</td>
<td>6425</td>
</tr>
<tr>
<td>3</td>
<td>850</td>
<td>1650</td>
<td>12050</td>
<td>13700</td>
<td>4566.67</td>
</tr>
<tr>
<td>4</td>
<td>1250</td>
<td>2900</td>
<td>12050</td>
<td>14950</td>
<td>3737.50</td>
</tr>
<tr>
<td>5</td>
<td>1850</td>
<td>4750</td>
<td>12050</td>
<td>16800</td>
<td>3360</td>
</tr>
<tr>
<td>6</td>
<td>2550</td>
<td>7300</td>
<td>12050</td>
<td>19350</td>
<td><strong>3225</strong> *</td>
</tr>
<tr>
<td>7</td>
<td>3250</td>
<td>10550</td>
<td>12050</td>
<td>22600</td>
<td>3228.57</td>
</tr>
<tr>
<td>8</td>
<td>4050</td>
<td>14600</td>
<td>12050</td>
<td>26650</td>
<td>3331.25</td>
</tr>
</tbody>
</table>

It is observed from the table that the average annual cost \(ATC(n)\) is minimum in the sixth year. Hence, the machine should be replaced at the end of sixth year of usage.
ILLUSTRATION 20-2

A company is considering replacement of a machine which cost N70000. The maintenance cost and resale values per year of the said machine are given below:

<table>
<thead>
<tr>
<th>Year</th>
<th>Maintenance Cost (₦)</th>
<th>Resale Cost (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9000</td>
<td>40000</td>
</tr>
<tr>
<td>2</td>
<td>12000</td>
<td>20000</td>
</tr>
<tr>
<td>3</td>
<td>16000</td>
<td>12000</td>
</tr>
<tr>
<td>4</td>
<td>21000</td>
<td>6000</td>
</tr>
<tr>
<td>5</td>
<td>28000</td>
<td>5000</td>
</tr>
<tr>
<td>6</td>
<td>37000</td>
<td>4000</td>
</tr>
<tr>
<td>7</td>
<td>47000</td>
<td>4000</td>
</tr>
<tr>
<td>8</td>
<td>59000</td>
<td>4000</td>
</tr>
</tbody>
</table>

When should the machine be replaced?

**SUGGESTED SOLUTION 20-2**

<table>
<thead>
<tr>
<th>Year of Service n</th>
<th>Resale value (₦) (S)</th>
<th>Purchase Price ressale value (₦) C-S</th>
<th>Annual Maintenance Cost f(t)</th>
<th>Cumulation of Maintenance cost (₦) [\sum_{t=0}^{n} f(t)]</th>
<th>Total Cost (₦) [C-S + \sum_{t=0}^{n} f(t)]</th>
<th>Average Cost (₦) [\frac{C-S + \sum_{t=0}^{n} f(t)}{n}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40000</td>
<td>30000</td>
<td>9000</td>
<td>9000</td>
<td>39000</td>
<td>39000</td>
</tr>
<tr>
<td>2</td>
<td>20000</td>
<td>50000</td>
<td>12000</td>
<td>21000</td>
<td>71000</td>
<td>35500</td>
</tr>
<tr>
<td>3</td>
<td>12000</td>
<td>58000</td>
<td>16000</td>
<td>37000</td>
<td>95000</td>
<td>31666.67</td>
</tr>
<tr>
<td>4</td>
<td>6000</td>
<td>64000</td>
<td>21000</td>
<td>58000</td>
<td>122000</td>
<td>30500</td>
</tr>
<tr>
<td>5</td>
<td>5000</td>
<td>65000</td>
<td>28000</td>
<td>86000</td>
<td>151000</td>
<td>30200 *</td>
</tr>
<tr>
<td>6</td>
<td>4000</td>
<td>66000</td>
<td>37000</td>
<td>123000</td>
<td>189000</td>
<td>33714.29</td>
</tr>
<tr>
<td>7</td>
<td>4000</td>
<td>66000</td>
<td>47000</td>
<td>170000</td>
<td>236000</td>
<td>31500</td>
</tr>
<tr>
<td>8</td>
<td>4000</td>
<td>66000</td>
<td>59000</td>
<td>229000</td>
<td>295000</td>
<td>36875</td>
</tr>
</tbody>
</table>

From the table, it is observed that the average cost ATC(n) is minimum in the fifth year. Hence, the machine should be replaced by the end of 5th year.

(b) Replacement of items that deteriorate and whose maintenance cost increases with time; and value of money also changes with time.

This replacement policy can be seen as value of money criterion. Here, the replacement decision will be based on the equivalent annual cost whenever we have time value of money effect.

Whenever the value of money decreases at constant rate, the issue of depreciation factor or ratio comes in as in the computation of present value (or worth). For example, if the interest rate on N100 is r percent
per year, the present value (or worth) of N100 to be spent after n years will be:

\[
D = \frac{100}{(1 + \frac{r}{100})^n} \quad \ldots \quad 20.2.7
\]

Where D is the discount rate or depreciation value. With the principle of the discount rate, determine the critical age at which an item should be replaced.

**ILLUSTRATION 20-4**

The yearly cost of two machines A and B, when money value is neglected, is given below:

<table>
<thead>
<tr>
<th>Year</th>
<th>Machine A (₦)</th>
<th>Machine B (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1400</td>
<td>24000</td>
</tr>
<tr>
<td>2</td>
<td>800</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>1100</td>
</tr>
</tbody>
</table>

If the money value is 12% per year, find the cost patterns of the two machines and which of the machine is more economical?

**SUGGESTED SOLUTION 20-4**

The discount rate per year = (d) = \( \frac{1}{1 + 0.12} \) = 0.89

The discounted cost patterns for machines A and B are shown below:

\[
\frac{1}{(1 + 0.12)^1}, \quad \frac{1}{(1 + 0.12)^2}
\]

<table>
<thead>
<tr>
<th>Year</th>
<th>Machine A (Discounted Cost in ₦)</th>
<th>Machine B (Discounted cost in ₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1400 x 0.89 = 712</td>
<td>24000 x 0.89 = 267</td>
</tr>
<tr>
<td>2</td>
<td>800 x 0.89 = 712</td>
<td>300 x 0.89 = 267</td>
</tr>
<tr>
<td>3</td>
<td>1000 x (0.89)^2 = 792.1</td>
<td>1100 x (0.89)^2 = 871.31</td>
</tr>
<tr>
<td></td>
<td>Total Cost (₦) 2,904.1</td>
<td>Total Cost (₦) 25,138.3</td>
</tr>
</tbody>
</table>

**Decision:** Machine A is more economical because its total cost is lower.
20.4 REPLACEMENT OF EQUIPMENTS/ITEMS THAT FAIL SUDDENLY

In a real life situation, we have some items that do not deteriorate gradually but fail suddenly. Good examples of these items are electric bulbs, contact set, plugs, resistor in radio, television, computer, etc. Majority of items in this category are not usually expensive, but a quick attention or preventive replacement should be given in order not to have a complete breakdown of the system.

The items that experience sudden failure normally give desired service at variant periods. Service periods follow some frequency distributions which may be random, progressive or retrogressive.

There are two types of policies in the sudden failure namely:

(a) The individual replacement policy; and
(b) The group replacement policy.

(a) The individual replacement policy

This is a policy in which an item is replaced immediately it fails. The life span of this item is uncertain and it is assumed that failure occurs only at the end of its life span (say period t). Therefore, an establishment of probability distribution for the failure (through the past experience) is required for the problem. This will enable us to find the period that minimizes the total cost involved in the replacement.

ILLUSTRATION 20-5

The computer sets that are used in an organisation have resistors with a life span of five months. The failure rates (in percentages) of these resistors are given in the following table.

<table>
<thead>
<tr>
<th>Months</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage Failures</td>
<td>10</td>
<td>30</td>
<td>35</td>
<td>20</td>
<td>05</td>
</tr>
</tbody>
</table>

Given that 798 resistors are fixed for use at a time, each resistor costs ₦14. If group replacement and ₦60 if replacement is done individually, determine the cost of individual monthly replacement.
**SUGGESTED SOLUTION 20-5**

The following steps are to be taken:

(i) Compute the cost of monthly individual replacement which is represented by $C$ i.e. $C = RK$, where $R$ is the average number of monthly replacement, $K$ is the cost per item.

(ii) If it is not directly possible to have $R$, it can be computed or obtained from the following relationship:

$$R = \frac{\text{Total number of items used}}{\text{Average life span of the item}}$$

i.e. $R = \frac{N}{t}$, where $N$ and $t$ represent total number of items used and average life span of item respectively

We are to calculate $t$, the average life span of the resistor, as follows:

<table>
<thead>
<tr>
<th>Month($x_i$)</th>
<th>Percentage failure</th>
<th>Probabilities ($P_i$)</th>
<th>$P_iX_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>0.30</td>
<td>0.60</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>0.35</td>
<td>1.05</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>0.20</td>
<td>0.80</td>
</tr>
<tr>
<td>5</td>
<td>05</td>
<td>0.05</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1.00</td>
<td>2.80</td>
</tr>
</tbody>
</table>

From the table

$$t = \frac{\sum P_iX_i}{\sum P_i} = \frac{2.8}{1} = 2.80 \text{ months}$$

Average number of monthly replacements equal to

$$R = \frac{N}{T} = \frac{798}{285} = 285$$

The individual cost replacement is equal to

$$C = RK$$

$$= 285 \times 60$$

$$= \text{₦}17,100$$
(b) The Group Replacement Policy

This is a policy where an en-masse replacement of items is made. It is cheaper and safer to apply the policy when there is a large number of identical items which are more likely to fail within a particular time period.

The usual practice in the group replacement policy is to fix a time interval that replacement can be made. That is, we make a replacement of all items at a fixed interval of time period t, whether the items failed or not, and at the same time to replace the individual failed items during fixed interval.

Next, we ensure that group replacement is made at the end of t\textsuperscript{th} period, if the cost of individual replacement for the t\textsuperscript{th} period is more than the average cost per unit time through the end of t period.

**ILLUSTRATION 20-6**

Use the data in Illustration 20.5 to determine

(a) The best interval period between group replacement; and

(b) The cost of group replacement.

**SUGGESTED SOLUTION 20-6**

Given that \( N = 798 \) and the computed \( P_i \) (\( i = 1, 2, \ldots, 5 \)) is shown as

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_1 )</td>
<td>( P_2 )</td>
<td>( P_3 )</td>
<td>( P_4 )</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.30</td>
<td>0.35</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Let \( N_i \) represent the number of items replaced at the end of \( i \text{th} \) month and \( N_0 = 798 \)

Therefore, we have the following replacements in the subsequent months:

\( N_0 = N_0 = 798 \) (initial)

At the end of 1\textsuperscript{st} Month, \( N_1 = N_0 P_1 = 798 \times 0.10 = 79.8 = 80 \)

At the end of 2\textsuperscript{nd} Month, \( N_2 = N_0 P_2 + N_1 P_1 = 798 \times 0.3 + 80 \times 0.1 = 247 \)

At the end of 3\textsuperscript{rd} Month, \( N_3 = N_0 P_3 + N_1 P_2 + N_2 P_1 = 798 \times 0.35 + 80 \times 0.30 + 247 \times 0.10 = 328 \)

At the end of 4\textsuperscript{th} Month, \( N_4 = N_0 P_4 + N_1 P_3 + N_2 P_2 + N_3 P_1 = 798 \times 0.20 + 80 \times 0.35 + 247 \times 0.30 + 328 \times 0.10 = 295 \)
At the end of 5th Month, \( N_5 = N_0 P_5 + N_1 P_4 + N_2 P_3 + N_3 P_2 + N_4 P_1 \)
\[ = 798(0.05)+80(0.20)+247(0.35)+328(0.30)+295(0.1) = 270 \]

At the end of 6th Month, \( N_6 = N_0 P_6 + N_1 P_5 + N_2 P_4 + N_3 P_3 + N_4 P_2 + N_5 P_1 \)
\[ = 798(0)+80(0.05)+247(0.20)+328(0.35)+295(0.30)+270(0.10) \]
\[ = 284 \]

From the computed values of \( N_i \), we could see that the number of resistors failing each month increases till third month, then decreases and again increases from sixth month. Hence, \( N_i \) will oscillate continuously until the system reaches a steady-state.

Now let's obtain the Total Cost of group replacement (TC) by:
\[ TC = \text{Number of group replacement} \times \text{Group Cost} + \text{Number of Individual replacement} \times \text{Individual Cost} \]

<table>
<thead>
<tr>
<th>End of Month</th>
<th>Total Cost of group replacement</th>
<th>Average cost per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>798 x 14 + 80 x 60 = 15972</td>
<td>15972</td>
</tr>
<tr>
<td>2</td>
<td>798 x 14 + 60(80 + 247) = 30792</td>
<td>15396*</td>
</tr>
<tr>
<td>3</td>
<td>798 x 14 + 60(80 + 247 + 328) = 50472</td>
<td>16824</td>
</tr>
<tr>
<td>4</td>
<td>798 x 14 + 60(80+247+328+295) = 68172</td>
<td>17043</td>
</tr>
<tr>
<td>5</td>
<td>798x14+60(80+247+328+295+270)=84372</td>
<td>16874</td>
</tr>
</tbody>
</table>

(a) From the table, it is the second month that we have the average minimum cost. Hence, it is optimal to have group replacement after every two months.

(b) The cost of the group replacement is ₦15,396.

### 20.4 SUMMARY AND CONCLUSIONS

The essential reasons for the study of replacement analysis are to
- Ensure efficient functioning of the equipments
- Know when and how best the equipments can be replaced in order to minimize the total costs of maintaining them.

The following replacing policies are considered:
- Replacement of equipments that deteriorate or wear-out gradually. The maintenance and repair costs increase with time.
- Replacement of equipments/items that fail suddenly. Examples of these items include electric bulbs, contact set, plugs, resistors in radio, television, e.t.c.
Two types of replacement policies under sudden failure are used.
- The individual replacement policy where an item is replaced immediately it fails.
- Group replacement policy where an en masse replacement of items is made.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

20.5 REVISE QUESTIONS

20.5.1 MULTIPLE CHOICE QUESTIONS

1. Which of the following is the reason for the study of replacement theory?
   A. To ensure efficient functioning of the equipments
   B. To know when and how best the equipments can be replaced
   C. To minimize the costs of maintenance
   D. (a) and (b) only
   E. (a), (b) and (c).

2. Which of the following is a policy in the replacement of equipments or items that fail suddenly?
   A. Gradual replacement policy
   B. Individual replacement policy
   C. Group replacement policy
   D. (b) and (c) only
   E. (a) and (b) only.

3. Which of the following is not a resulting effect of gradual failure or deterioration of items?
   A. The output of the equipment
   B. Its production capacity
   C. The maintenance and operating costs
   D. The value of the re-sale price of the item
   E. The efficiency of the equipment.

20.5.2 SHORT ANSWER QUESTIONS

Use the following information to answer questions 1, 2 and 3.

Given the following failure rates of certain item:

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probability of failure</td>
<td>0.05</td>
<td>0.08</td>
<td>0.12</td>
<td>0.18</td>
<td>0.25</td>
<td>0.20</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>Cumulative probability</td>
<td>0.05</td>
<td>0.13</td>
<td>0.25</td>
<td>0.43</td>
<td>0.68</td>
<td>0.88</td>
<td>0.96</td>
</tr>
</tbody>
</table>

If the total number of items is 1000 and the individual and group costs of replacement are ₦2.25 and 60kobo per item respectively, assuming a replacement of all items simultaneously at fixed intervals and the individual items replace as they fail. Then
1. The average number of failures per month is approximately ___________

2. The average cost of individual replacement is ______________

3. The best interval between group replacement is ____________

4. At the old age of an operating machine, state the reason why it will definitely require higher operating costs and more maintenance costs.

5. The two common replacing policies are replacement of equipment items that:
   (i) __________________________
   (ii) __________________________

6. State any two consequences of equipment/items that deteriorate with time having its efficiency getting low.

7. State two characterizing features of the policy that governs an equipment that is replaced immediately it fails.

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
TRANSPORTATION MODEL

21.0 LEARNING OBJECTIVES

After studying this chapter, readers should be able to:

◆ Understand the nature of transportation models;
◆ Know that transportation problem is a special class of linear programming problem;
◆ Understand the concept of balanced and unbalanced transportation model;
◆ Obtain the initial basic feasible solution using the following methods: northwest corner rule, least cost, Vogel's approximation;
◆ Understand the optimality test of transportation problem by the stepping stone algorithm method; and
◆ Solve assignment problem as a special class of transportation problem.

21.1 INTRODUCTION

21.1.1 Nature of Transportation Model

Transportation model is a special class of linear programming problem in which the objective is to transport or distribute a single commodity or goods from various sources/origins to different destinations at a minimum total cost.

In the statement of transportation problem, the total supply available at the origin \((a_i)\), the total quantity demanded by the destination \((b_j)\) and the cost \((C_{ij})\) of transporting or shipping a unit of the commodity from a known origin to a known destination are given.
With the above information, the mathematical model and the pattern of a Transportation Model are given below respectively:

Mathematical Model:
\[
\begin{align*}
\text{Minimize } Z &= \sum_{i=1}^{M} \sum_{j=1}^{N} C_{ij} X_{ij} \\
\text{Subject to } & \sum_{j=1}^{N} X_{ij} \leq a_i; \quad i = 1, 2, \ldots, m \quad (\text{supply constraints}) \\
& \sum_{i=1}^{M} X_{ij} \leq b_j; \quad j = 1, 2, \ldots, n \quad (\text{demand constraints}) \\
& X_{ij} \geq 0 \text{ for all } i \text{ and } j
\end{align*}
\]

Table 21.1: General Pattern of a Transportation Problem

<table>
<thead>
<tr>
<th>Destination</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...n</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin 1</td>
<td>C_{11}</td>
<td>C_{12}</td>
<td>C_{13}</td>
<td>...</td>
<td>C_{1n}</td>
</tr>
<tr>
<td>2</td>
<td>C_{21}</td>
<td>C_{22}</td>
<td>C_{23}</td>
<td>...</td>
<td>C_{2n}</td>
</tr>
<tr>
<td>3</td>
<td>C_{31}</td>
<td>C_{32}</td>
<td>C_{33}</td>
<td>...</td>
<td>C_{3n}</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>m</td>
<td>C_{m1}</td>
<td>C_{m2}</td>
<td>C_{m3}</td>
<td>...</td>
<td>C_{mn}</td>
</tr>
</tbody>
</table>

An example of a transportation problem is given below:

**Balanced Transportation Problem**

<table>
<thead>
<tr>
<th>Destination</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin 1</td>
<td>24</td>
<td>18</td>
<td>27</td>
<td>24</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>28</td>
<td>24</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>25</td>
<td>38</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>Demand</td>
<td>50</td>
<td>70</td>
<td>30</td>
<td>40</td>
<td>190</td>
</tr>
</tbody>
</table>

From the above mathematical model, we could see that a transportation problem is a special class of a linear programming problem.

Also, a transportation problem is said to be balanced if \( \sum a_i = \sum b_j \); otherwise (i.e. \( \sum a_i \neq \sum b_j \)) it is unbalanced.

It is a necessary condition that a transportation problem must be balanced before one solves it. That is, the total supply must be equal to the total demand. This condition is also called the “Rim Condition”.
When the problem is not balanced, there is the need to create a dummy origin (row) or destination (column) for the difference between total supply and demand with zero cost in order to create the balance.

A typical example of balancing an unbalanced problem is given below:

### Use of Dummy to effect Balancing

<table>
<thead>
<tr>
<th>Destination</th>
<th>Origin</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>18</td>
<td>25</td>
<td>20</td>
<td></td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>17</td>
<td>23</td>
<td>19</td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>23</td>
<td></td>
<td>40</td>
</tr>
<tr>
<td>Dummy</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Demand</td>
<td>40</td>
<td>80</td>
<td>20</td>
<td>20</td>
<td></td>
<td>160</td>
</tr>
</tbody>
</table>

In this table, the given total supply ($\sum a_i$) is 130 while that of total demand ($\sum b_j$) is 160. Therefore, a dummy row is created for the difference of 30 (as supply) with zero costs in order to create the balance.

The problem in Transportation Model is to choose that strategy of shipping programme that will satisfy the destination and supply requirements at the minimum total cost. Hence, there is the need to apply the Transportation methods of solving the model.

### 21.3 METHODS FOR SOLVING TRANSPORTATION PROBLEM

The solution to Transportation problem involves two phases. The first phase is to obtain the initial basic feasible solution; while obtaining the optimal solution is the second phase.

In the first phase, there are various methods of obtaining the initial basic feasible solution. However, this chapter is concerned with:

1. **North-West corner Rule**,
2. **Least cost Method**, and
3. **Vogel’s Approximation (or penalty) Method**.

Note: that any of the three methods must satisfy the following conditions in order to obtain the initial solution:

1. **The problem must be balanced** (i.e. $\sum a_i = \sum b_j$)
2. **The number of cell’s allocation must be equal to m + n-1**, where m and n are numbers of rows and columns respectively.
Any solution satisfying the two conditions is termed “Non-degenerate Basic Feasible solution” otherwise, it is called “Degenerate solution”.

(i) **North-West Corner Rule (NWCR)**

The method is the simplest but most inefficient as it has the highest total transportation cost in comparison to all other methods. The main reason that can be attributed to this is that the method does not take into account the cost of transportation for all the possible alternative routes.

The steps needed to solve a transportation problem by NWCR are:

**Step 1:** Begin by allocating to the North-West cell (i.e. variable $X_{11}$) of transportation matrix the allowable minimum of the supply and demand capacities of that cell. i.e. $\text{Min}(a_1, b_1)$

**Step 2:** Check if allocation made in the first step is equal to the supply (demand) available at the first row (column), then cross-out the exhausted row (column) so that no further assignment can be made to the said row (column). Move vertically (horizontally) to the next cell and apply Step1.

In case $a_1 = b_1$, move to cell (2,2). (i.e. variable $X_{22}$) and apply Step1.

**Step 3:** Step 2 should be continued until exactly one row or column is left uncrossed in the transportation matrix. Then make allowable allocation to that row or column and stop. Otherwise, return to Step1.

**ILLUSTRATION 21-1**

SAO company has 3 plants or locations (A, B, C) where its goods can be produced with production capacity of 50, 60, 50 per month respectively for a particular product. These units are to be distributed to 4 points (X, Y, W, Z) of consumption with the demand of 50, 70, 30 and 10 per month respectively.

The following table gives the transportation cost (in Naira) from various plants to the various points of consumption:
Obtain the Initial Basic Feasible solution by NWCR.

**SUGGESTED SOLUTION 21-1**

The NWCR algorithm applied to this problem resulted to the following table:

<table>
<thead>
<tr>
<th>Destination</th>
<th>X</th>
<th>Y</th>
<th>W</th>
<th>Z</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Plant B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>60</td>
</tr>
<tr>
<td>Plant C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Demand</td>
<td>50</td>
<td>70</td>
<td>30</td>
<td>10</td>
<td>160</td>
</tr>
</tbody>
</table>

**Explanation to the above allocation**

Beginning from North-West (i.e Cell X_{11}) corner

- Allocate 50 to cell X_{11} in order to satisfy the minimum of demand and supply capacities. Zero balance is left for both demand and supply. Therefore, row 1 and column 1 are crossed out.

- Move to cell X_{22} and allocate 60. The supply balance is zero while the demand balance is 10. Therefore, row 2 is crossed out.
Next, move to cell $X_{32}$ and allocate 10 giving the balance of zero for demand and 40 for the supply. Column 2 is therefore crossed out.

Then move to cell $X_{33}$ and allocate 30 to exhaust the demand and having 10 balance for supply. Therefore, column 3 is crossed out.

Finally, 10 is allocated to cell $X_{34}$

(ii) **Least Cost Method (LCM)**

In this method the cheapest route is always the focus for allocation. It is a better method compared to NWCR because costs are considered for allocation. The algorithm is stated thus:

**Step 1:** Assign as much as possible to the smallest unit cost (ties are broken arbitrarily). Also bear in mind the idea of allowable minimum of supply and demand capacities as done in NWCR.

**Step 2:** Cross-out the exhausted row or column and adjust the supply and demand accordingly. If both row and column are exhausted simultaneously, only one is crossed-out (in order to avoid degenerating case).

**Step 3:** Look for the smallest cost in the uncrossed row or column and assign the allowable quantity. Repeat this process until left with exactly one uncrossed row or column.

**ILLUSTRATION 21-2**

Use the data in illustration 21-1 to determine the Initial Basic Feasible solution by Least Cost Method.
**SUGGESTED SOLUTION 21-2**

Using the above algorithm, gives the following table:

<table>
<thead>
<tr>
<th>Destination</th>
<th>Plant</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>21</td>
<td>50</td>
</tr>
<tr>
<td>Y</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>Z</td>
<td>24</td>
<td>30</td>
</tr>
</tbody>
</table>

| Demand      | 50    | 70     | 30    | 10    | 160 |
|-------------| 10    | 20     | 0     | 0     |      |

Total Cost = 50(18) + 20(18) + 40(19) + 10(24) + 10(25) + 30(28)  
= ₦3350

Explanation to the above allocation

- The lowest cost cell is $X_{12}$ with 18. Then 50 is allocated to that cell in order to satisfy minimum of the demand and supply. Zero balance is left for supply while that of demand is 20. Therefore, row one is crossed out.

- For the remaining un-crossed cells, cell $X_{22}$ has lowest cost of 18. Then assign 20 to this cell in order to give zero balance for demand and 40 balance for supply. Therefore, column two is crossed out.

- The next cell with lowest cost, for the uncrossed cells, is $X_{21}$ with 19. Allocate 40 to this cell in order to give zero balance to supply and 10 to demand. Therefore, cross out row two.

- Move to cell $X_{31}$ because it has lowest cost. (24) among the uncrossed cells and then allocate 10. The balance of zero is obtained for demand while that of supply is 40. Column one is exhausted and it is crossed out.

- Lastly, allocate 30 and 10 respectively to the remaining two cells $X_{33}$ and $X_{34}$. 


(iii) **Vogel’s Approximation Method (VAM)**

VAM, which is also called penalty method, is an improvement on the LCM method that generates a better initial solution.

It makes use of opportunity cost (penalty) principles in order to make allocation to various cells by minimizing the penalty cost. The steps in this method are:

**Step 1:** Compute for each row(column) the penalty by subtracting the smallest unit (Cij) from the next smallest unit cost in the same row (column).

**Step 2:** Select the row or column with the highest penalty and then allocate as much as possible to the variable with least cost in the selected row or column. Any ties should be handled arbitrarily.

**Step 3:** Adjust the supply and demand and cross-out the exhausted row or column. If a row and a column are simultaneously exhausted, only one of the two is crossed-out and the other assigned zero supply (demand).

**Step 4:** Compute the next penalties by considering uncrossed rows (columns) and go to step 2.

**Step 5:** Exactly when one row (column) remains uncrossed, then allocate the leftover and stop.

**ILLUSTRATION 21-3**

Use the data in illustration 21-1 to obtain the initial solution by VAM.
**SUGGESTED SOLUTION 21-3**

Applying the above steps, gives the following tables:

<table>
<thead>
<tr>
<th>Destination</th>
<th>Plant</th>
<th>X</th>
<th>Y</th>
<th>W</th>
<th>Z</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>21</td>
<td>50</td>
<td>18</td>
<td>27</td>
<td>50 0</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>40</td>
<td>19</td>
<td>20</td>
<td>24</td>
<td>60 40 0</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>10</td>
<td>24</td>
<td>30</td>
<td>28</td>
<td>50 40 30 0</td>
</tr>
<tr>
<td>Demand</td>
<td>50</td>
<td>20</td>
<td>30</td>
<td>10</td>
<td>160</td>
<td></td>
</tr>
</tbody>
</table>

Penalty Table

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Rows</th>
<th>Columns</th>
<th>Allocation</th>
<th>X_{12} = 50</th>
<th>X_{22} = 20</th>
<th>X_{21} = 40</th>
<th>X_{31} = 10</th>
<th>X_{33} = 30</th>
<th>X_{34} = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>X_{12} = 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>* 1</td>
<td>1</td>
<td>X_{22} = 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>* 1</td>
<td>1</td>
<td>X_{21} = 40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>* *</td>
<td>1</td>
<td>X_{31} = 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The principle of obtaining the difference between the two least costs along row (column) is applied throughout the penalty table in order to make allocation as stated in step 2.

In the Penalty Table, the circled figures represent highest penalty in each iteration; * represents the crossed-out row or column, and (-) stands for penalty not possible.

Note that in the penalty table, 3 is obtained under row A (in iteration 1) by the difference of costs 21 and 18. This principle is continually applied to other rows (B and C) to obtain 1 and 1 respectively.

For the column also, 2 is obtained under column X (in iteration 1) by the difference of costs 21 and 19.

Total cost = 50(18) + 40(19) + 20(18) + 10(24) + 30(28) + 10(25)
= ₦3350
ILLUSTRATION 21-4

A company with three factories (X,Y,Z) and five warehouses (A,B,C,D,E) in different locations has the transportation costs (in Naira) from factories to warehouses. Factory capacities and warehouse requirements are stated below:

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>Factory Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>A  800</td>
<td>B  600</td>
</tr>
<tr>
<td>Y</td>
<td>C  800</td>
<td>D  600</td>
</tr>
<tr>
<td>Z</td>
<td>E  1100</td>
<td></td>
</tr>
</tbody>
</table>

Warehouse Requirements

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>Factory Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>A  350</td>
<td>B  425</td>
</tr>
<tr>
<td></td>
<td>C  500</td>
<td>D  650</td>
</tr>
<tr>
<td></td>
<td>E  575</td>
<td></td>
</tr>
</tbody>
</table>

Determine the initial feasible solution by

(b) North West Corner Method
(b) Least Cost Method
(b) Vogel’s Approximation Method

SUGGESTED SOLUTION 21-4

(a) **North West Corner Method**

Total Cost

\[= (350 \times 5) + (425 \times 8) + (25 \times 6) + (475 \times 8) + (125 \times 6) + (525 \times 5) + (575 \times 6) = ₦15,925\]

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>Factory Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>A  350</td>
<td>B  425</td>
</tr>
<tr>
<td></td>
<td>C  500</td>
<td>D  650</td>
</tr>
<tr>
<td></td>
<td>E  575</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Warehouse Requirements

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>Factory Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>A  350</td>
<td>B  425</td>
</tr>
<tr>
<td></td>
<td>C  500</td>
<td>D  650</td>
</tr>
<tr>
<td></td>
<td>E  575</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Explanation to the above allocation**

Beginning from North-West (i.e cell X_{11}) corner,

- Allocate 350 to cell X_{11} in order to satisfy the minimum of factory capacity and warehouse requirement. Zero balance is left for warehouse requirement while that of factory capacity along X_{11} is 450. Therefore, column one is crossed out.

- Move to cell X_{12} and allocate 425. The warehouse balance along cell X_{12} is zero while we have the balance of 25 for the factory capacity. Therefore, column two is crossed out.

- Next, move to cell X_{13} and allocate 25 giving the balance of zero for factory capacity and 475 for the warehouse along that cell (i.e. X_{13}). Row one is therefore crossed out.

- Move to cell X_{23} to allocate 475. It then gives a balance of zero for the warehouse while that of factory capacity is 125. Column three is crossed out.

- We also move to cell X_{24} and allocate 125 to that cell (X_{24}) giving the balance of zero for the factory capacity and 525 for the warehouse. Then cross-out row two.

- Finally, allocate 525 and 575 respectively to the remaining two cells X_{34} and X_{35}.

- LCM

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>Factory Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>x</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>y</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>z</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Warehouse Requirements</td>
<td>350</td>
<td>425</td>
</tr>
</tbody>
</table>

Total Cost = (225x4) + (575x3) + (350x4) + (250x8) + (425x4) + (250x7) + (425x5) = ₦11,600

**Explanation to the above allocation**

The cell that has lowest cost is X_{15}. Allocate 575 to that cell giving the balance of zero for warehouse requirement and 225 for factory capacity. Column five is crossed out.
For the remaining uncrossed cells, cells $X_{14}$, $X_{21}$ and $X_{32}$ have the lowest cost of 4. Arbitrarily, cell $X_{32}$ is allocated with 425 giving the balance of zero for warehouse requirement and 675 for the factory capacity. Column two is crossed-out.

Next, cell $X_{21}$ is allocated 350 giving the balance of zero for warehouse requirement and 250 for the factory capacity. Column 1 is also crossed-out. We allocate 225 to cell $X_{14}$ to give the balance of zero for factory capacity and 425 for the warehouse requirement. Row one is crossed out. The left-over cells are $X_{23}$, $X_{24}$, $X_{33}$ and $X_{34}$. We allocate 425 to cell $X_{34}$ to give the balance of zero for warehouse requirement and 250 for the factory capacity. Therefore, column four is crossed-out. Finally, the remaining two cells $X_{23}$ and $X_{33}$ are respectively allocated 250 each.

(C) VAM

<table>
<thead>
<tr>
<th>Factories</th>
<th>Warehouses</th>
<th>Factory Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>x</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>z</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>425</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Warehouse Requirements</th>
<th>350</th>
<th>425</th>
<th>500</th>
<th>650</th>
<th>575</th>
<th>525</th>
<th>2500</th>
</tr>
</thead>
</table>

Penalty Table

<table>
<thead>
<tr>
<th>Iteration</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>$X_{32} = 425$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>*</td>
<td>2</td>
<td>$X_{13} = 575$</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>*</td>
<td>$X_{21} = 350$</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>*</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>$X_{14} = 225$</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>$X_{24} = 250$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$X_{33} = 500$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$X_{34} = 175$</td>
</tr>
</tbody>
</table>

The principle of obtaining the difference between the two least costs along row (column) is applied throughout the penalty table in order to make allocation as stated in step 2.
In the Penalty Table, the circled figures represent highest penalty in each iteration.

(*) represents the crossed out row or column, and (-) stands for penalty not possible. Also, note in the penalty table that 1 is obtained under row X (in iteration 1) by the difference between 4 and 3. Similarly for row Y, 1 is obtained from the difference between 5 and 4. The difference computations continued in this manner for both rows and columns in each iteration.

Total Cost = 
\[(225 \times 4) + (575 \times 3) + (350 \times 4) + (250 \times 6) + (425 \times 4) + (500 \times 7) + (175 \times 5)\]
\[= \text{₦11,600}.\]

21.3 OPTIMALITY TEST

The second phase of solving Transportation problem is the optimality test. Here, it is also required that the obtained initial solution should be balanced and non-degenerated.

The two popular methods usually used for this test are the “stepping stone method” and the modified distribution (MOD I) method, but in this study pack more emphasis is placed on the former i.e. the stepping stone method.

21.3.1 Stepping Stone Algorithm Method

In carrying-out the optimality test, this method uses the principle of the Simplex method where the given solution minimizes the objective function only if the relative cost coefficients of the non-basic variables are greater than or equal to zero.

Succinctly put, the method consists of computing relative cost coefficients for each non-basic variables. The relative cost coefficient is obtained by increasing a non-basic variable by one unit and then compute the resulting change in the total transport cost. The most negative of the \((m-1)(n-1)\) relative cost coefficients will be taken as new basic variable.

To illustrate this method, let us consider the following example:
ILLUSTRATION 21-5

Given the following transportation problem:

<table>
<thead>
<tr>
<th>Source</th>
<th>Destination</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>Q</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

Demand:
| 20 | 25 | 35 | 80 |

(a) Allocate by NWCR
(b) Obtain the optimal solution from the allocation result in (a) by Stepping Stone method.

SUGGESTED SOLUTION 21-5

(a)

<table>
<thead>
<tr>
<th>Source</th>
<th>Destination</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>Q</td>
</tr>
<tr>
<td>A</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>15</td>
</tr>
</tbody>
</table>

Demand:
20, 0, 25, 15, 35, 0, 80

Total cost = (20 X 3) + (10 X 5) + (15 X 7) + (35 X 11) = N600

(b)

Let us increase the non-basic variable X_{21} from 0 to 1. Then we need to adjust some allocation in order to satisfy the demand and supply constraints.

Here, as X_{21} increases by 1, X_{11} decreases by 1, X_{12} increases by 1, and X_{22} decreases by 1. This is shown in the following diagram:

Fig.1

<table>
<thead>
<tr>
<th>Source</th>
<th>Destination</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>Q</td>
</tr>
<tr>
<td>A</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>B</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>

The relative cost coefficient (C_{ij}) is computed as:

\[ C_{21} = C_{21} \text{ (change in X}_{21}\text{) + C}_{11} \text{ (change in X}_{11}\text{) + C}_{12} \text{ (change in X}_{12}\text{) + C}_{22} \text{ (change in X}_{22}\text{)} \]

\[ = 4 \times +1 + (3 \times -1) + 5 \times +1 + 7 \times -1 \]

\[ = 4 - 3 + 5 - 7 = -1 \]

Similarly for the non-basic variable X_{13}, we have:

\[ = C_{13} \times -1 + C_{23} \times -1 + C_{22} \times +1 + C_{12} \times -1 \]

\[ = 7 \times +1 + 11 \times -1 + 7 \times +1 + 5 \times -1 \]

\[ = 7 - 11 + 7 - 5 = -2 \]
Since C13 is more negative than C21, X13 becomes new basic variable by adjusting the quantities in cells X13, X23, X22, and X12 so that one of old basic variables becomes non-basic. The adjustment is handled as follows:

\[ \theta = 10 \] will make X12 non basic, hence X13 = 10, X22 = 25, and X23 = 25, X12 = 0

Incorporating these new values in the overall table will give the following table:

<table>
<thead>
<tr>
<th>Sources</th>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>3</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>7</td>
<td>7</td>
<td>11</td>
</tr>
</tbody>
</table>

Demand

20  25  35  80

The new total cost = (20x3) + (10x7) + (25x7) + (25x11) = 580

Again, let us increase non-basic variable X21 by 1 in the new table, we have

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-1</td>
<td></td>
<td>+1</td>
</tr>
<tr>
<td>B</td>
<td>+1</td>
<td></td>
<td>-1</td>
</tr>
</tbody>
</table>

Therefore, the C21 = C21 (+1) + C11 (-1) + C13 (-1) + C23 (-1)  
= 4 (+1) + 3(-1) + 7(+1) + 11(-1)  
= 4 - 3 + 7 - 11  
= -3

Similarly for the non-basic variable X12, we have:
C12  =  C12 (+1) + C13 (-1) + C23 (+1) + C22 (-1)
=  5(1) – 7 + 11 – 7
= 2

From two computations, it is X_{21} that can be made basic variable as follows:

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20 - θ</td>
<td></td>
<td>10 + θ</td>
</tr>
<tr>
<td>B</td>
<td>+ θ</td>
<td>25 - θ</td>
<td>11</td>
</tr>
</tbody>
</table>

If θ = 20, X_{11} will be come a non-basic variable
Hence X_{21} = 20, X_{13} = 30, X_{23} = 5 and X_{11} = 0
These new values are now incorporated into the overall table as follows:

<table>
<thead>
<tr>
<th>Sources</th>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>30</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>7</td>
<td>11</td>
<td>50</td>
</tr>
</tbody>
</table>

The problem is now optimal with total cost of 520 with the following allocations:
X_{13} = 30, X_{21} = 20, X_{22} = 25 and X_{23} = 5.

21.4 PROFIT MAXIMIZATION PROBLEM (IN TRANSPORTATION MODEL)

Whenever a profit table is given, it can be solved in either of the following two ways:

(i) The profit table can be converted to cost table by multiplying by –1 and then apply all steps stated in the cost table. The total cost
will be multiplied back by –1 in order to obtain the answer as profit.

(ii) In the profit table, consider the highest profits and apply the steps given in the LCM and VAM. Note that highest profit will now replace lowest cost in the two methods.

21.5 ASSIGNMENT PROBLEM AS A SPECIAL CLASS OF THE TRANSPORTATION PROBLEM

The main difference between the assignment problem and transportation problem is that the demand and supply capacities in assignment problem are one while it is not in transportation problem.

The assignment problem can be formally defined as: Having n jobs, n facilities and the effectiveness of each facility for each job, the optimized measure of effectiveness is based on assigning each facility to one and only one job.

The mathematical expression of an assignment problem is given by

\[
\begin{align*}
\text{Minimize } Z &= \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} \\
\text{Subject to } \sum_{j=1}^{n} X_{ij} &= 1, \quad i = 1, 2, \ldots, m; \\
& \quad \sum_{i=1}^{m} X_{ij} = 1, \quad j = 1
\end{align*}
\]

The application areas of this problem are in:

(a) Assigning various jobs to various machines; and
(b) Assigning tractors (in different locations) to trailers (in different locations) in order to pick them up to centralized depot, etc.

21.5.1 Solution To Assignment Model

The common technique usually used to solve Assignment problem is called Hungarian Method or Reduced Matrix Method.

The method consists of the following steps for a cost table:

**Step 1:** Develop a cost table from the given problem and ensure that the table is balanced (i.e. the number of rows equals the number of columns). A dummy row or
column with zero cost is used to balance up if the problem is unbalanced.

**Step 2:** Determine the opportunity cost table as follows:

(a) Subtract the lowest entry in each column of the cost table from all entries in that column.
(b) Subtract the lowest entry in each row of the table obtained in 2(a) from all the entries in that row.

**Step 3:** Determine whether an optimal assignment has been made. The procedure is to draw straight line vertically or horizontally through the total opportunity cost in such a manner as to minimise the number of lines necessary to cover all zero cells.

An optimal assignment is made when the number of lines is equal to the number of rows or columns. One can then stop.

If an optimal assignment is not feasible, we modify the total cost table in the next step as follows

**Step 4:**

(a) Select the smallest number in the table that is not covered by a straight line and subtract this number from all numbers not covered by a straight line.

(b) Add the same lowest number (selected in 4(a)) to the number lying in the intersection of any two lines.

**Step 5:** Go to step 3.

A typical example of an assignment problem is given below:

**ILLUSTRATION 21-6**

A construction company has four construction Roads (A,B,C,D) on hand and wants to assign four site engineers (X,Y,W,Z) who will supervise the jobs. Based on the following table of cost (in thousand Naira) implications on each engineer, determine the optimal assignment.
**SUGGESTED SOLUTION 21-6**

Applying Step2(a) to the problem we have

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td>39</td>
<td>0</td>
<td>51</td>
<td>20</td>
</tr>
<tr>
<td>W</td>
<td>5</td>
<td>0</td>
<td>32</td>
<td>28</td>
</tr>
<tr>
<td>Z</td>
<td>19</td>
<td>29</td>
<td>22</td>
<td>27</td>
</tr>
</tbody>
</table>

Applying Step2(b) to table 1, we have

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td>39</td>
<td>0</td>
<td>51</td>
<td>20</td>
</tr>
<tr>
<td>W</td>
<td>5</td>
<td>0</td>
<td>32</td>
<td>28</td>
</tr>
<tr>
<td>Z</td>
<td>0</td>
<td>10</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Also, applying Step3 led to straight lines drawn on zero cells in table 2.

In table 2, we have 3 lines which are not equal to the number of rows or columns. Therefore we proceed to Step4 to give the following table:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>3*</td>
<td>12*</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td>39</td>
<td>0</td>
<td>48</td>
<td>17</td>
</tr>
<tr>
<td>W</td>
<td>5*</td>
<td>0</td>
<td>29</td>
<td>25</td>
</tr>
<tr>
<td>Z</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>5+</td>
</tr>
</tbody>
</table>

Rule 4b applies

* the smallest of the remaining uncrossed cells is **Now** subtracted from the uncrossed cell. (see Rule 4a)

Application of Step3 to table 3 resulted to the straight lines. Since the number of lines is not equal to number of rows, we move again to Step4.
**TRANSPORTATION MODEL**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>3</td>
<td>17*</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td>34</td>
<td>0</td>
<td>43</td>
<td>12</td>
</tr>
<tr>
<td>W</td>
<td>0+</td>
<td>0</td>
<td>24</td>
<td>20</td>
</tr>
<tr>
<td>Z</td>
<td>0</td>
<td>15*</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

**Note:**
Since the number of lines is equal to 4, the problem is optimal. We can then assign:

Engineer X to Road D (: only 1 zero entry happens in D unlike C).
Engineer Y to Road B
Engineer W to Road A (: the zero entry under B has been assigned to Y).
Engineer Z to Road C
Total cost = 34 + 36 + 46 + 76 = 192

### 21.6 MAXIMIZATION PROBLEM

Wherever we have a profit table, it is the maximization of assignment problem that we need to apply. The maximization problem can be handled in either of the following two ways:

a) The profit table can be changed to cost table by multiplying by –1 and then apply all the steps stated in section 23.5.1.

b) We can subtract profits along the row (column) from highest profit of corresponding row (column) in a similar manner up to step 2 and then go to step 3 for optimality test.

### 21.7 SUMMARY AND CONCLUSIONS

This is a special class of linear programming problem in which the Objective is to transport or distribute a single commodity from various sources to different destinations at a minimum cost. The initial basic feasible solution of a transportation problem can be obtained by the following methods:

(a) The North-West Corner Rule (NWCR);
(b) Least Cost Method (LCM); and
(c) Vogel’s Approximation Method (VAM).

The Stepping Stone method is used to obtain the optimal allocation; while the Hungarian method is used to solve the assignment, a special class of transportation problem.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.
21.8  REVISION QUESTIONS

21.8.1 MULTIPLE CHOICE QUESTIONS
Pick the appropriate and correct answers to the following.

1. The main objective of a transportation problem is to:
   A. Transport goods to the supply points.
   B. Allocate goods to people.
   C. Distribute a single good from various sources to different destinations at minimum total cost.
   D. Sell goods to consumers.
   E. To bring goods nearer to the people.

2. Which of the following is not a method of obtaining initial feasible solution of a transportation problem?
   A. NWCR
   B. Least cost method.
   C. Vogel’s approximation method.
   D. Stepping stone method.
   E. Simplex Method.

3. Least cost method is better than NWCR because it is
   A. Faster in computation and allocation.
   B. Concerned with handling of computation and allocation.
   C. Straight forward for allocation and computation.
   D. Needed for cost for allocation and computation.
   E. Having fewer iterations.

4. The rim condition in transportation problem
   A. Is the addition of dummy to destination of source.
   B. Means the total demand equals total supply.
   C. Means that the shipment to a dummy source represents surplus.
   D. Means all of the above.
   E. Is the absence of dummy.

5. With the supply capacities: \( a_1 = 120, a_2 = 70, a_3 = 60, a_4 = 80 \), and demand capacities: \( b_1 = 120, b_2 = 70, b_3 = 90, b_4 = 110 \); determine the amount of dummy capacity to add to the source or destination.
   A. 60 (dummy) for supply. (Source).
   B. 60 (dummy) for demand (destination).
   C. 70 (dummy) for supply (source).
   D. 70 (dummy) for demand (destination).
   E. 50 (dummy) for supply (source).

21.8.2 SHORT ANSWER QUESTIONS

1. The major difference between LCM and VAM is that VAM uses ________ costs.

2. If \( m \) and \( n \) are numbers of origins and destinations respectively, the numbers of cells allocation that does not conform to \( m+n-1 \) will lead to ____________ solution.
3. Stepping stone method consists of computation of _____________ coefficients.

4. The popular and common method usually used for assignment problem is called ________________ method.

5. By Least Cost Method, determine initial feasible solution for the following cost table (in Naira):

<table>
<thead>
<tr>
<th>Origin</th>
<th>Destination</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>I</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>II</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>III</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Demand</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

Refer to Suggested Solutions to Revision Questions in Appendix 1, page 483.
22.0 LEARNING OBJECTIVES

After studying this chapter participants should be able to:

- Understand IT tools for costing application;
- Apply such tools in cost accounting activity; and
- Demonstrate proficiency in computer communications using computers and Web conferencing, e-mail, and research of cost and management accounting issues.

22.1 INTRODUCTION

In management accounting, cost accounting establishes budget and actual cost of operations, processes, departments or product and the analysis of variances, profitability or social use of funds. Managers use cost accounting to support decision-making to reduce a company's costs and improve profitability. As a form of management accounting, cost accounting need not follow standards such as GAAP, because its primary use is for internal managers, rather than outside users, and what to compute is instead decided pragmatically.

Costs are measured in units of nominal currency by convention. Cost accounting can be viewed as translating the Supply Chain (the series of events in the production process that, in concert, result in a product) into financial values.

Cost accounting has long been used to help managers understand the costs of running a business. Modern cost accounting originated during the industrial revolution, when the complexities of running a large
scale business led to the development of systems for recording and tracking costs to help business owners and managers make decisions.

In the early industrial age, most of the costs incurred by a business were what modern accountants call "variable costs" because they varied directly with the amount of production. Money was spent on labor, raw materials, power to run a factory, etc. in direct proportion to production. Managers could simply total the variable costs for a product and use this as a rough guide for decision-making processes.

Some costs tend to remain the same even during busy periods, unlike variable costs which rise and fall with volume of work. Over time, the importance of these "fixed costs" has become more important to managers. Examples of fixed costs include the depreciation of plant and equipment, and the cost of departments such as maintenance, tooling, production control, purchasing, quality control, storage and handling, plant supervision and engineering. In all of these and in some complex situation determining these costs parameters and other related management decision points requires the application of computers. One handy computer application that can be used in computing various cost and management decision parameter is the electronic spreadsheet discussed in a companion book to this study pack (Information Technology).

For purposes of completeness, we shall recap the discussion on electronic spreadsheet and go head to give examples of its usage in cost accounting activity such as computing depreciation schedule.

22.2 ELECTRONIC SPREADSHEET – AN INTRODUCTION

A spreadsheet is a computer simulation program that simulates a paper worksheet. It displays multiple cells that together make up a grid consisting of rows and columns, each cell containing either alphanumeric or numeric values. A spreadsheet cell may alternatively contain a formula that defines how the contents of that cell is to be calculated from the contents of any other cell (or combination of cells) each time any cell is updated. Spreadsheets are frequently used for financial information modeling because of their ability to re-calculate the entire sheet automatically after a change to a single cell is made.

The related term spreadmart describes the situation that occurs when one or more business analysts develop a system of linked spreadsheets to perform a business analysis, then grow it to a size and degree of complexity that makes it nearly impossible to maintain. Re-
implementing the analysis using a datamart architecture often improves both the reliability of the analysis and its maintainability.

Visicalc is usually considered the first electronic spreadsheet (although this has been challenged), and it helped turn the Apple II computer into a success and greatly assisted in their widespread application. Lotus 1-2-3 was the leading spreadsheet when DOS was the dominant operating system. Excel is now considered to have the largest market share on the Windows and Macintosh platforms.

**Entering Data**

Cells can contain three things: labels, values and formulas. It's that simple. *Labels* (alpha characters) are the text headings, or titles, that describe what the numbers represent. *Values* are the numbers that you perform calculations on. *Formulas* perform the calculations by manipulating values to produce a result.

(a) To enter data into a cell, place the cell selector on the required cell.
(b) Type your entry (you’ll be in *edit mode*).
(c) Press [Enter] or click **Enter** on the Formula bar.

**Enter Data in a Range of Cells**

If you do a lot of data entry and don’t like navigating to the beginning of the next column or row each time you enter another record of information, there is an easier way.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Products</td>
<td>January</td>
<td>February</td>
<td>March</td>
<td>Total</td>
</tr>
<tr>
<td>2</td>
<td>Pentium 400</td>
<td></td>
<td></td>
<td></td>
<td>$0</td>
</tr>
<tr>
<td>3</td>
<td>Pentium 350</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Pentium 300</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>Monitors</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>Printers</td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>Total</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) First, select the range of cells for which you’ll be entering data.
(b) When you begin typing, your entry will be entered into the active cell.
(c) To move from cell to cell within the selected range, use the following keys to remain within the selected range.
   (i). `[Tab]` to move to the right;
   (ii). `Shift`[`Tab`] to move left;
(iii). [Enter] to go down a column; and
(iv). [Shift][Enter] to move up.

When you come to the end of a long row or column and need to move back to the beginning of the next, just press [Tab] or [Enter]. When you reach the end of the range, press [Tab] or [Enter] and you’ll return to the beginning of the selection.

**Labels**

Labels can be any text that contains at least one alpha character. It can be combined with numbers, but as long as at least one letter is included, Excel will consider this entry a label since no calculations can be performed on alpha characters.

At times you may want to use numbers for a text label. An example may be you are compiling data from the past several years and need to label your columns with the year (i.e. 2008). If you are using AutoSum to sum the column, this entry is automatically considered a value and will be summed as part of the column. To avoid this, change the cell format to a text label instead of a value.

The easy way to do this is to type an apostrophe before the numbers (i.e. ’1998). This will force the format to be text. Do not worry; the apostrophe will be hidden from view in the spreadsheet, although it is visible in the Formula Bar.

**Values**

Excel considers an entry to be a value if it consists of only numbers, no alpha characters, unless formatted otherwise as mentioned above.

**Formula**

A formula is an equation that performs operations on values. Formulas can perform mathematical operations, such as addition and multiplication, or they can compare worksheet values or join text. Formulas can refer to other cells on the same worksheet, cells on other sheets in the same workbook, or cells on sheets in other workbooks. You must use the proper syntax for Excel to calculate the correct results. Misplaced punctuation and incorrect spelling will result in an incorrect calculation or error message.
A formula in Microsoft Excel always begins with an equal sign (=). The equal sign tells Excel that the succeeding characters constitute a formula. Following the equal sign are the elements to be calculated (the operands), which are separated by mathematical operators. Each operand can be a value that does not change (a constant value), a cell or range reference, a label, a name, or a worksheet function. Excel calculates the formula from left to right, according to a specific order for each operator in the formula. You can change the order of operations by using parentheses.

Excel over the years has become a popular brand of electronic spreadsheet for business and home use. This may have been due to the aggressive marketing style of Microsoft, as each office suite of programs incorporate the excel program and other related office programs.

**Examples of application of excel programme in costing**

A good number of costing and management accounting decisions revolves around sensitivity analysis such as “What If ….Analysis”, “Scenario Management”, and “Goal Seek….Analysis”. The notes below show how excel programme can be used to demonstrate the application of these management and cost accounting situations.

**22.3 WHAT-IF ANALYSIS**

Excel allows you to perform a what-if analysis by using four different tools. These are:

(a) Data Tables;
(b) Scenarios;
(c) Goal Seek; and
(d) Solver.

Let us discuss all these tools in detail.

A data table is a range of cells that contains a list of values that are replaced in formulas to find out how the change affects the result of the formula. Data tables allow you to calculate, view, and compare the results of different variations of a value in a cell. Data tables are of two types:
(a) **One-variable Data Table** Enables you to change the value of one cell to a range of values in the data table and compare the results. For example, in the weekly report, Mr. Paul can use a data table to review the change in the total amount of discount allowed if the rate of discount is changed to 12.5%, 12.25% or 13%.

(b) **Two-variable Data Table** Enables you to change values of two cells in a data table. For example, Mr. Paul can find out the change in the total amount of discount allowed if both the discount amount and the sales price are changed to different values.

22.4 **SCENARIO**

A scenario is a view of a worksheet that contains a variation of a set of data in the worksheet. You can use a scenario for forecasting the outcome of the changing values in range of cells. You can create a number of scenarios in a worksheet and switch between them to view various results. The changes made to a worksheet by using a scenario are permanent. If you change the value of a cell that contains a formula in a scenario, the value is changed permanently and the formula is lost. For example, Mr. Paul can use a scenario to find out the effect on the total amount and discount amount if the values in the unit price and discount rate columns change.

The steps to create a scenario are:

(a) Click on the Scenarios option on the Tools menu.

The Scenario Manager dialog box is displayed (refer Figure 22.1)

![Scenario Manager](image)

Fig 22.1: Scenario Manager

(b) Click on the Add button.

The Add Scenario dialog box is displayed (refer to Figure 22.2)
As the changes made to a scenario are permanent, you can create a scenario that contains the original values and then create a number of scenarios with different values. Another method is to copy the data to another worksheet and create a number of scenarios on the data.

(c) Enter the name of the scenario in the scenario name field.

In Fig. 22.8, the name of the scenario is Original.

(d) Select the range of cells that contain the values that have to change in the Changing cells field by using the collapse button.

(e) Select the Prevent changes option to prevent other users from changing the scenario.

This option is effective only if the worksheet is protected.

(f) Select the Hide option if you do not want the name of the scenario to appear in the list of scenarios in the Scenario Manager Dialog box.

(g) Click on the OK button

The Scenario Values dialog box is displayed (refer to Fig. 22.3)
(h) If you are creating a scenario of the original values, click on the OK button.

For example, no values have been changed in the Scenario Values dialog box that is displayed in Fig. 22.9

(i) If you are creating a scenario with a different set of values enter or edit the values of a changing cell in the field that is next to the reference for the changing cell.

(j) Click on the OK button to create the scenario.

You can click on the Add button to add another scenario and display the Add Scenario dialog box.

Richard creates three scenarios, Original, Best Case and Worst Case. The Original scenario contains the original values. The Best Case scenario contains increased value of the unit price column and the discount rate of 11.75%. This scenario depicts the maximum amount of sales in the department. The Worst Case scenario contains decreased values of the unit price of all items and a discount rate of 12.5%. This scenario depicts the lowest sales that is acceptable from the department (refer Fig. 22.3 below).

Figure 22.3 A Scenario Table

22.5 Depreciation Calculation

Excel offers different functions to calculate depreciation of an asset over time.
Depreciating an asset places a value on the asset at a point in time, based on the original value and its life. The function that you choose depends on the type of depreciation method that you use.

<table>
<thead>
<tr>
<th>Depreciation Method</th>
<th>Function</th>
<th>Arguments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Straight Line Method</strong></td>
<td>SLN</td>
<td>Cost, Salvage, Life</td>
</tr>
<tr>
<td><strong>Declining/Reducing Balance</strong></td>
<td>DB</td>
<td>Cost, Salvage, Life, Period, [Month]</td>
</tr>
<tr>
<td><strong>Double-Declining Balance</strong></td>
<td>DDB</td>
<td>Cost, Salvage, Life, Period, Month, [Factor]</td>
</tr>
<tr>
<td><strong>Sum of Year’s Digits</strong></td>
<td>SYD</td>
<td>Cost, Salvage, Life, Period,</td>
</tr>
</tbody>
</table>

Arguments

- **Cost**: Original cost of the asset
- **Salvage**: Scrap value after being fully depreciated
- **Life**: Number of periods over which the asset will be depreciated
- **Period**: Period in life in which the calculation is being made
- **Month**: Number of months in the first year; if omitted, Excel uses 12
- **Factor**: Rate at which the balance declines; if omitted, it is assumed to be 2 (that is, double-declining)

Further application of excel spreadsheet can be seen from the charting tool in excel. Sometimes management will need a pictorial view of some information store as data tables; in this case the data need to be graphed as a chart.
22.6 CHARTING

A chart presents a table of numbers visually. Displaying data in a well conceived chart can make the data more understandable; and you often can make your point more quickly as a result.

Charts allow you to graphically display data stored in a worksheet. Charts are linked to the data from which they are inserted and are updated whenever the data is modified. A chart can be:

Embedded- is placed in a worksheet as an embedded object and is saved with that worksheet (refer to Figure 22.5)

![Figure 22.5 Embedded Chart](image)

(a) Chart sheet – is saved as a separate sheet in a workbook that contains only the chart (refer to Figure 22.6)

![Fig. 22.6 Chart Sheet](image)
Both these charts are updated when the worksheet data is modified. When you insert a chart, the values from the worksheet are automatically represented in the chart. While inserting a chart you can specify various options based on which the chart is inserted. For example, you can specify the titles for X-axis and Y-axis in a chart.

The steps to insert a chart in Excel are:

1. Select the cells that contain the data and the text that has to be displayed in a chart. To insert a chart on data of nonadjacent rows or columns, select a range of cells, press the Ctrl key, select another range of cells and release the Ctrl key.

Click on the Chart option on the Insert menu.

The Chart Wizard is invoked (refer Figure 22.7).

![](chart_wizard.png)

The first step in inserting a chart is to select the chart type from the Chart Type screen (refer to Figure 22.7) in the Chart Wizard. The Chart Wizard prompts you to select the type of chart that has to be inserted based on a set of data. Excel displays list of the various types of charts that can be inserted, in the Chart type option. These are:

(b) **Column**: Displays data changes over a period of time and compares the items in a group. The items are organized horizontally (x-axis) and values of the items are organized vertically (Y-axis) to demonstrate the variations between the values over a period of time.
(c) **Bar:** Represents comparison between items of a group. The items are organized vertically (Y-axis) and the values of the items are displayed horizontally (X-axis). A bar chart is different from a column chart as unlike a column chart the focus is on comparing values of the items and not on time. For example, you can use a Bar chart to show the comparison of sales values of items for a day and a Column Chart to show the weekly trend of sales for all the items.

(d) **Line:** Shows the trend of data at various intervals of time. It can be used to compare the trend of two or more types of data at an equal interval of time.

(e) **Pie:** Shows the size of an item in the selected data series. For example, 20% of the total sales (100%) in BC Supermarket comes from sales of Baby Products. This information can be displayed by using a Pie chart. A pie chart can be used to provide information about only one data series.

(f) **XY (Scatter):** Shows the relationships between the numeric values in various data series. It is generally used to illustrate scientific data. You can put the values for the axis in one row or column and the values of the Y-axis in the corresponding columns or rows.

(g) **Areas:** Illustrates the magnitude of change over a period. It is also used to signify the contribution of a part to the whole. For example, it can be used to plot a chart of sales of all stores of Essentials in the country and can show the contribution of each store.

(h) **Doughnut:** Shows the size of an item in a data series like a pie chart. The difference being that unlike a Pie chart more than one data series can be plotted by using a Doughnut chart. The Doughnut chart contains a number of rings where each ring denotes a data series.

(i) **Surface:** Finds out the maximum number of combinations between two values. For example, the Surface Chart can be used to find out the variations in profit if the cost price and the sales price are modified for an item in Essentials.
(j) **Bubble**: Displays relationships between numeric values in a data series. The Bubble chart is similar to the XY (Scatter) chart but in this case, the value of the third variable in the chart is indicated by the size of a bubble.

(k) **Stock**: Displays the difference between the high, low, close values of an item in the stock-market.

(l) **Cone, Cylinders and Pyramid**: Act as three-dimensional data markers in a chart.

You can also select from a list of user defined charts called Custom Type charts by clicking on the Custom. Types tab in the Chart Type screen (refer Figure 22.7).

2. Click on the Next button after selecting the type of chart that has to be inserted.

This Chart Source Data screen is displayed (refer Figure 22.8). Excel prompts you to select the data series for which the chart has to be inserted.

The Chart Source Data screen contains two tabs, Data Range and Series. If you have already selected the range of cells, the selected range is displayed in the Data range field in the Data Range tab. You can change the range by clicking on the collapse button in the Data range field.

The chart can be based on values of the rows or columns. By default, the chart is inserted based on the columns.

![Chart Source Data](image)  
*Fig. 22.8 Chart Source Data*
3. You can click on the Series tab to change particular values of the Series (refer Figure 22.9).

![Figure 22.9 Chart Source Data (Series Tab)](image)

4. Click on the Next button after making required selections.

The Chart Options screen is displayed that prompts you to select the various options to insert the chart (refer to Fig. 22.10)

![Fig 22.10 Chart Options](image)

The Chart options screen has six tabs. These are:

(a) **Titles**: Allows you to enter the title of the chart, the name of the group of the items in the X-axis and the name of the parameter on which the item is being measured in the Y-axis (refer to Fig. 22.10).

(b) **Axes**: Allows you to control the presentation of a chart (refer to Fig. 22.11). If you deselect the Category (X) axis option, the
names of the items in the group are not displayed in the chart. For example, if you deselect this option, the names of all the items on the X-axis. The three options in this category are: Automatic, Category and Time-scale. The Automatic and Category options provide similar functionalities. If any of these options is selected, all the items in the category are displayed. If the Time-scale option is selected, Excel displays the chart based on a randomly generated uniform time scale (refer to Fig. 22.11). Similarly, you can also control the display of parameters on the Y-axis by using the value (Y) axis option.

(c) **Gridlines:** Allows you to display gridlines on the chart. By default, Major gridlines option of Value (Y) axis area is selected. In Figure 22.12, Minor gridlines option on Value (Y) axis area has also been selected. This means that a gridline is shown for every number on the Y-axis.

(d) **Legends:** Provides you details about the data columns in a chart. You can choose to display the legend at various locations in the chart. By default, the legend is displayed at the right of the
chart. In Figure 22.13, the legend is displayed at the button of the chart.

(e) **Data Labels**: Allows you to add labels to the data depicted in the chart (refer to Fig 22.14). By default, none option is selected. If you want to display the actual value of the items sold then you can select the show value option. You can select the Show percent option, to display the percentage of the whole of all data points in Doughnut and Pie charts. You can display the category names for all data points in a chart by selecting the Show label option. Show Label and percent option is a combination of both Show Label and Show percent option. Show bubble sizes option is available for use in a Bubble chart, in which the size of the bubble is displayed based on the third parameter. In Fig. 22.14 the Show value option is selected.

(f) **Data Table**: Allows you to display the values for all data series in a table below the chart (refer to Figure 22.15). This tab is not
available for Pie, XY (Scatter), Doughnut, Bubble, and Surface charts.

![Chart Options (Data Tables)](image)

**Fig: 22.15 Chart Options (Data Tables)**

1. Click on the Next button after making required selections from the Chart Options screen.

The Chart Location screen is displayed (refer to Figure 2.16).

![Chart Location](image)

**Fig 22.16 Chart Location**

The Chart Location screen allows you to insert an embedded chart or a chart sheet. Select the As new sheet option, and type the name of the new worksheet to insert a chart sheet. To insert an embedded chart, select the As object in option and select the name of the sheet in which you want to embed the chart.

2. Click on the Finish button to insert the chart. A chart sheet called Weekly Status Chart is inserted (refer Fig. 22.17).
22.7 SUMMARY AND CONCLUSIONS

This chapter tried to discuss extensively the application of computers in cost and management accounting functions with particular emphasis on the use of excel with limited examples. This approach does not suggest that excel is the only programme. The use of excel in these examples is as a result of the fact that it is commonly used and very popular than other spreadsheet programmes such as VisiCalc, etc. Readers should also bear in mind that there are sophisticated simulation programmes that can be used to carry out complicated cost and management accounting routine such as in linear programming techniques using simplex algorithm. For further reading, you may consult Introduction to Operations Research by Tarha.

Refer to Comprehensive Questions and Suggested Solutions in Appendix II, page 501.

22.8 REVISION QUESTION

22.8.1 MULTIPLE CHOICE QUESTIONS
1. Owing to the ability to recalculate after change, in a single cell, the -------- software is frequently used for financial information processing.
   A. Word processing
   B. Database management software
   C. Spreadsheet
   D. Compiler
   E. Computer

2. To enter data into a spreadsheet,
   A. Press ALT + ENTER keys.
   B. Press the ENTER key
   C. Press the TAB key
   D. Check on any cell and start typing
   E. Press the ALT + TAB keys
3. Which of the following is an AutoSum function that will display the largest number in a selected range of cells?
   A. MaxSum
   B. Sum
   C. Min
   D. Average
   E. Max

4. Each of the following is true about Labels in Excel except
   A. They are right aligned
   B. They are not used in calculation
   C. They are left aligned
   D. They can include numerical calculations
   E. They can include numbers.

5. Which formula will be equivalent to $6^6 \times 6^6$?
   A. $=5^6$
   B. $=\text{sum}(6^5)$
   C. $=6^5$
   D. $6^5$
   E. $-\text{sum}(5^6)$

### SHORT ANSWER QUESTIONS

1. Each cell of a spreadsheet can accept Labels, ---------, and Values.

2. The four different tools allowed by Excel in performing a ‘What – If’ analysis are Data tables, Senarios, ---------, and Goalseeking.

3. In any Excel chart, Cones, Cylinders, and Pyramids act as three-dimensional ------.

4. The chart option screen of an Excel normally has six tabs. They are Titles, Axes, --------- ---, Data labels, Data tables, and Legends.

5. Absolute cell reference can be created by placing --------- before both the column letter and row number for a cell, using F4 function key on the keyboard.

Refer to Suggested Solutions to Revision Question in Appendix 1, page 483.


APPENDIX I

SUGGESTED SOLUTIONS
TO REVISION QUESTIONS

CHAPTER 1

MULTIPLE CHOICE

1. D
2. C
3. C
4. A
5. A

SHORT ANSWERS

1. Cost elements are the items of cost making up the total cost of a product or service. The items can be aggregated as Prime or Direct Costs, Total Cost.

2. The Accountant views cost as the value of economic resources used in the production of goods or services, income or profit.

3. The qualities of a good cost accounting system may be mnemonically determined by as TURROCC thus:
   ♦ Timeliness
   ♦ Understandable
   ♦ Relevant
   ♦ Reliable
   ♦ Objective
   ♦ Complete
   ♦ Comparable
   ♦ Direct Cost

4. Cost accounting sense refers to the need for organizations, profit and not-for-profit, to critically and objectively trace ALL cost elements related to their products or services before arriving at their selling prices, based on accurate identification and computation of actual total cost.
CHAPTER 2

MULTIPLE CHOICE

1. C
2. D
3. A
4. B
5. B

SHORT ANSWERS

1. Just-in-Time Purchasing
2. ABC is Activity Based Analysis Technique which is a selective method of inventory control that attempts to segregate and group materials according to total value
3. An efficient stock control, that is, efficient material procurement, storage, usage and accounting.
4. Stock Turnover is a means of evaluating or measuring management efficiency on its stock holding practices. It is denoted as:

\[
\frac{\text{Stock of Materials Consumed}}{\text{Average Stock Level}}
\]

5. Factors that determine Re-order level are:
   a. Consumption Rate;
   b. Minimum Level;
   c. Delivery Time; and
   d. Variation in delivery time.

CHAPTER 3

MULTIPLE CHOICE

1. A
2. A
3. B
4. B
5. C
SHORT ANSWERS

1. Job Evaluation

2. Remi’s Bonus Payment
   Standard Time for Producing 4500 units = \(\frac{4500}{3.2} = 1406.25\) hrs
   Time taken to Produce 4500 units = 1200 hrs
   Time saved = 206.25
   Hasley Weir’s bonus for the group = \(\frac{206.25 \times 5 \times 1}{3} = 343.75\)
   Remi’s share = \(\frac{343.73 \times 60}{1200} = 17.19\)

3. Advantages of Group Incentive Scheme are:
   (i) It creates strong loyalty within the group;
   (ii) It results in reduced absenteeism and increased output;
   (iii) It can be applied to all levels of workers.
   (Any two will do)

   Disadvantages of group incentive scheme are:
   (i) It may discourage individual initiative;
   (ii) It may be de-motivating as efficient and inefficient employees are paid the same amount of bonus.

4. Advantages of time based method of remuneration include:
   (i) Wages are simple to calculate
   (ii) Earnings are averagely stable
   (iii) It helps in the attainment of quality work
   (iv) It facilitates planning for labour cost

   Disadvantages
   (i) There is no incentive for efficient employees
   (ii) Supervision may be needed to avoid idle time
   (iii) Efficient and inefficient workers are paid at the same rate.

5. (A) Straight Piece Rate
   Wages are calculated at a fixed rate per unit of production multiplied by the units produced.

   (B) Differential Piece Rate
   This method encourages workers to increase output by offering an increased rate per unit of different levels of production.

CHAPTER 4
MULTIPLE CHOICE

1. B
   Overhead absorbed \((2200 \times N3) = 6,600\)
   Actual cost incurred \((4800 \times 60/144 \times N3) = 6,000\)
   Over-absorbed \(N\) 600
2. A
Sales revenue (2200 x N20) N44000
less Cost of Sales (2200 x N12) N26400
Overhead absorbed N6600 N33000
N11000
Overhead over-absorbed N600
Profit N11600

3. D
Using high-low method

<table>
<thead>
<tr>
<th>Activity</th>
<th>Cost (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>4000</td>
</tr>
<tr>
<td>Low</td>
<td>2500</td>
</tr>
<tr>
<td>1500</td>
<td>4000</td>
</tr>
</tbody>
</table>

Variable cost per unit = 4000/1500
= N2.667

Fixed cost = N20000 – (4000 x 2.667)
= 9333

Fixed overhead absorption rate = 9333/3000
= N3.11

Total overhead cost per unit = N2.67 + N3.11
= N5.78

PRODUCTION COST PER UNIT
Direct Material N12000/3000 N4.00
Direct Labour N 9000/3000 N3.00
Prime Cost N7.00
Overhead N5.78
Cost per unit N12.78

4. C
Overhead absorption rate = N5000/1250 = N4.00
Overhead absorbed = N4x 1100 = N4400
Actual overhead = N5250
Under absorbed = N850

5. C

SHORT ANSWERS

1. Activity Based Costing (ABC) is a system that suggests that costs should be pooled together on the basis of the activities that drive the costs, regardless of the department.

2. Use of a single (blanket rate) in cost absorption will be allowed where:
   a. Only one type of product is produced
   b. All the products use the services of all departments equally
c. Overheads form an insignificant part of total cost

3. The approaches to overhead absorption are:
   (a) The traditional approach – based on production volume
   (b) The activity based approach – taking into consideration the real factors that cause overhead to vary with production.

4. (a) Cost allocation is the assignment of a whole item of cost to a single cost unit, centre, account or time period.
   (b) Cost apportionment is the division of cost to two or more cost units, centres, accounts or periods.

5. (a) Cost centre is a department of an organization to which costs may be ascertained.
   (b) Overhead absorption is the allotment of overhead cost to cost units by means of predetermined rates.

CHAPTER 5
MULTIPLE CHOICE
1. B
2. D
3. C
4. D
5. A

SHORT ANSWERS
1. Incremental costs are those costs that are affected by changes in volume of output, product mix, and product redesign. Sunk costs are not affected by change in production level.
2. A cost accountant will, with cost estimation method, be able to determine semi-fixed costs and analyse them into their fixed and variable cost elements for effective decision making.
3. A Cost Accountant is concerned with the behavioural pattern of cost to determine:
   (i) Expected future costs for budgeting and planning purposes.
   (ii) Actual costs to compare with standards for appropriate corrective actions to be taken where variances occur.
4. Examples of independent variables are:
   I. Units of output, Sales units, Direct Labour Hours, Machine Hours, Kilowatts.
   II. Variable costs are directly related to volume of output or level of production. They rise in direct proportion with output.
5. Overhead costs.
CHAPTER 6

MULTIPLE CHOICE

1. C
2. B
3. C
4. C
5. A

SHORT ANSWERS

1. Job costing
2. Marginal Costing and Absorption costing
3. Specific order costing and unit costing
4. The features are:
   a. Set –up costs
   b. Unit cost decreases as the batch size increases
   c. Batches of dissimilar products are brought together
5. It is the lower of
   \[
   \begin{align*}
   \text{Value of work certified} & \times \text{Estimated Total Profit} \\
   \text{Total Contract Price} & \\
   \text{OR} \\
   \text{Cost to date} & \times \text{Estimated Total Profit} \\
   \text{Estimated Total Cost} &
   \end{align*}
   \]

CHAPTER 7

MULTIPLE CHOICE

1. A
2. A
3. C
4. B
5. C
SHORT ANSWERS
1. Costs were added uniformly; therefore equivalent units for the elements of cost will be the same.

<table>
<thead>
<tr>
<th>Physical</th>
<th>Equivalent Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening WIP</td>
<td>30,000</td>
</tr>
<tr>
<td>Units started and completed</td>
<td>184,000</td>
</tr>
<tr>
<td>Closing WIP</td>
<td>16,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>230,000</strong></td>
</tr>
</tbody>
</table>

2. Total cost (32210 + 55400 + 31300) = N118,910
3. Cost per Unit = N0.55
4. Cost of goods transferred: 
   \[ 205,000 \times 0.55 = 112,750 \]
   Add: Cost of opening WIP = 11,752
   \[ \text{Total} = 124,502 \]
5. Closing work in progress = 0.55 x 11200 = 6160

CHAPTER 8
MULTIPLE CHOICE
1. C
2. C
3. D
4. C
5. A

SHORT ANSWERS
1. Budget limiting factor is for instance, the lack or shortage of a resource or a restriction on sales demand at a particular price, which limits the activity of an entity.

2. Kaizen Budgeting. This is a budgeting system that incorporates continuous improvement in all aspects of an entity’s operation process at all levels in the budget.

3. The consideration of behavioural aspects of budget is to ensure:
   (i) Commitment to the achievement of budget target.
   (ii) Maintenance of confidence in the budget.
   (iii) Risk of breach of ethical standard is reduced.

4. Zero base budgeting
6. Cash budget is a budget that takes account of all projected receipts and disbursements for a future period, which shows estimated closing cash balance. Cash budget is a tool for cash management.

CHAPTER 9

MULTIPLE CHOICE

1. B
2. B
3. B
4. 
4. C
   Actual cost of materials = 3000 x 4 = 12,000
   Variance 1,500 (unfav.)
   Standard Price per unit = 10,500/3000 = N3.50
5. B

SHORT ANSWERS

1. Standard material 3x500 = 1,500
   Variance (300/2) = 150
   Actual material used = 1,650
   Add Closing stock = 400
   Less Opening Stock 100
   Material Purchases 1,950
2. Standard items worked 11,584 x 6.5 hrs = 75,246 hrs
   Variance = 36,000/5 = 7,200 hrs
   Actual Hours worked 82,446 hrs
3. Ideal Standard
4. Attainable standard is the standard that can be achieved under normal efficient operating conditions.
5. Sales Margin Variance
   Sales margin variance is the difference between the budgeted margin and the actual margin attained.

CHAPTER 10

MULTIPLE CHOICE

1. A
2. A
3. C
4. C
5. D

SHORT ANSWERS
1. 35,000 Units. If contribution margin is 30%, contribution will be N6.

\[
\text{B.E.P (units)} = \frac{210,000}{6} = 35,000 \text{ units}
\]

2. Units = \(\frac{210,000 + 60,000}{6} = \frac{270,000}{6} = 45,000 \text{ units}\)

3. Shade Ltd.
   N6.00. In decision making, what is more relevant is the current market value.

4. Consideration for deciding whether or not to accept a special order are:
   (i) Whether the price quoted by the customer covers the marginal cost of production.
   (ii) Whether the company has excess capacity.

5. The criterion in make-or-buy decision is whether the supplier’s quotation is less than the total variable cost of production of the company.

CHAPTER 11
MULTIPLE CHOICE
1. C
2. C
3. A
4. D
5. C

SHORT ANSWERS
1. **Backflush Accounting**
   This is a method of costing, which applies cost to the output of the process. Costs are allocated to output produced excluding work in progress on the assumption that these costs are a realistic measure of actual costs incurred.
2. **Life cycle costing**
   This is a system which ensures that there are physical asset cost records over assets’ lives so that decisions made on the assets will yield optimum assets usage at lowest possible cost.

3. **Materials Requirement Planning**
   This is a system that lists materials and components required to meet production schedule so that adequate stock level is maintained and items are available when needed.

4. **Business Process Re-engineering**
   Business Process Re-engineering is a process which has as its objectives the design of a more efficient production system by the identification of what a production system should be, given the level of quality of a product.

5. Differences between Standard Costing and Budgetary Control.
   (i) Budgets are stated in monetary or financial terms while standards need not be.
   (ii) Budgets are usually focused on cost centres while standards are usually for the various activities of an entity.

**CHAPTER 12**

**MULTIPLE CHOICE**

1. B
2. B
   Reconciling from financial accounts and cost accounts.
   Difference in opening stock 2,000
   Difference in closing stock (2,000)
   Interest Charged 12,500
   Dividend Income (7,000)
   **5,500**
3. D
4. B
5. D

**SHORT ANSWERS**

1. **Notional costs which may be found in costing records are:**
   (i) Notional interest on capital
   (ii) Notional rent.

2. In interlocking cost accounting system, separate ledgers are maintained for the financial accounting system and the cost accounting system while in integrated system there is no such separation but a common input of data is used for all accounting purposes.
3. Advantages of interlocking system.
   (i) It enables more detailed information to be provided.
   (ii) It will enable individual job accounts to be reconciled with the control accounts for all jobs.
   (iii) It records transfers between jobs which is ignored in financial account.

4. Ledgers contained in a Control Account are:
   (i) Stores Ledger Control Account
   (ii) Work in Progress Control Account
   (iii) Finished Goods Control Account
   (iv) Overhead Control Account

5. It presents information with more details for decision making purposes.

CHAPTER 13

MULTIPLE CHOICE

1. B
2. D
3. \( \frac{150}{x} = .25 \Rightarrow \frac{150}{125} = .20 \Rightarrow \text{Use month is May (A)} \)

SHORT ANSWERS

1. 80%
2. Base year is 2001
3. Index for 2002 with 2000 as Base year = \( \frac{100}{105} \times 100\% = 04.76 \)
4. 2000 2002 2004
   105 125 110
   84 100 88 (2002 = 100%)
   \% change from 2000 to 2004 is 4% increase
   \sum p_o q_o \times 00 = \frac{30 \times 5 + 10 \times 2}{20 \times 5 + 15 \times 2} = \frac{930}{600} = 55\%
5. \sum p_o q_o = \frac{30 \times 5 + 10 \times 2}{20 \times 0 + 15 \times 5} = 86\%
6. Fisher index = \( \sqrt{L \times P} = \sqrt{120 \times 8} = 18\% \)
CHAPTER 14

MULTIPLE CHOICE
1. C
2. B
3. A

SHORT ANSWERS
1. \((0.6)(0.3) = 0.18\)
2. \((0.4)(0.7) = 0.28\)
3. \(1 - .28 = 0.72\)
4. \(\frac{4}{10} = \frac{2}{5} = 0.4\)
5. \(\frac{4}{10} = \frac{2}{5} = 0.4\)
6. \(\frac{3}{10} = 0.3\)
7. \(\frac{6}{10} / \frac{9}{10} = \frac{2}{3}\)

CHAPTER 15

MULTIPLE CHOICE
1. C
2. D
3. D

SHORT ANSWERS
1. \(n(A \cup L) = 0 - = 0\)
2. \(10 + 5 - x = 0.2 \Rightarrow x = \)
3. \(10 - 3 = 7\)
4. \[ 15 - 3 = 12 \]

5. \[ A \cap \bar{c} \{A - B \text{ means elements in } A \text{ but not in } B \} \]

6. \[ A \cap B = 5, 6, 7 \]

7. \[ A \cup B = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \Rightarrow \text{Cardinality of } A \cup B = 11 \]

CHAPTER 16

MULTIPLE CHOICE

1. A

2. D

3. C \[ \Rightarrow \begin{vmatrix} 1 & 5 & 3 \\ -1 & 3 & 5 \end{vmatrix} = : \times - : \times = 5 \times 1 = 1 \]

SHORT ANSWERS

1. \[ A + B = \begin{pmatrix} 8 & 11 & 2 \\ 3 & -5 & 2 \end{pmatrix} \]

2. AB does not exist

\[ \because \text{The number of columns of left matrix} \neq \text{number of rows of right matrix} \]

3. \[ A^T B = \begin{pmatrix} 2 & 3 \\ 4 & -5 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 6 & 7 & 1 \\ 0 & 0 & 2 \end{pmatrix} \]
\[ = \begin{pmatrix} 12 & 14 & 8 \\ 4 & 28 & -6 \\ 6 & 7 & 1 \end{pmatrix} \]

4. \[ AB^T = \begin{pmatrix} 2 & 4 & 1 \\ 3 & -5 & 0 \end{pmatrix} \begin{pmatrix} 6 & 0 \\ 7 & 0 \\ 1 & 2 \end{pmatrix} \]
\[ = \begin{pmatrix} 41 & 2 \\ -7 & 0 \end{pmatrix} \]
\[ |AB^T| = 8 \]

5. \[ A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}, \quad b = \begin{pmatrix} 25 \\ 43 \end{pmatrix} \]
6. \[ A^{-1} = \frac{1}{10 - 12} \begin{pmatrix} 5 & -3 \\ -4 & 2 \end{pmatrix} \]
\[ = \frac{1}{-2} \begin{pmatrix} 5 & -3 \\ -4 & 2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -5 & 3 \\ 4 & -2 \end{pmatrix} \]

7. \[ X = A^{-1}b = \frac{1}{2} \begin{pmatrix} -5 & 3 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} 25 \\ 43 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \]

CHAPTER 17

MULTIPLE CHOICE

1. D
2. B
3. C

SHORT ANSWERS

1. Composite

2. Turning or stationary point

3. \[ \frac{dP}{dx} = \frac{1}{x} \cdot \frac{dP}{dP} \]
\[ P = 150 - 3x - x^2 \]
\[ \frac{dP}{dx} = -3 - 2x^2 = -(3 + 2x^2) \]
\[ \frac{P}{x} = \frac{150 + x + \frac{2}{x}}{x} = \frac{-x^2 + x - 50}{x} \]
\[ \therefore \frac{dP}{dx} = \frac{P}{x} \cdot \frac{1}{dP} = \frac{-x^2 + x - 50}{x} \cdot \frac{1}{-3x + \frac{x}{x}} \]
\[ e_d = \frac{x^2 + x - 50}{x(3 + x^2)} \]

At \( x = 5 \), we have
\[ e_d = \frac{25 + 5 - 50}{5(3 + 0)} \]
\[ = \frac{-10}{265} \]
\[ = -0.0375 \]
4. \[ TC = 18q - 5q^2 + q^3 \]
   \[ AC = \frac{TC}{q} = 18 - 5q + q^2 \]
   \[ \frac{d}{dq} (AC) = -5 + 2q = 0 \]
   \[ \Rightarrow 2q = 5 \]
   \[ q = 2.5 \]

5. \[ AC \text{ (at } q = 2.5) \]
   \[ = 18 - (2.5) + (2.5)^2 \]
   \[ = 18 - 12.5 + 6.25 \]
   \[ = 11.75 \]

6. \[ MC = \frac{dT}{dq} \]
   \[ = 18 - 10q + 3q^2 \]
   \[ \text{If } q = 2.5, MC = 18 - 10(2.5) + 3(2.5)^2 \]
   \[ = 18 - 25 + 18.75 = 11.75 \]

7. \[ y = 3(x^2 + 5x) \]
   \[ \frac{dy}{dx} = 3(2x + 5) \]
   \[ \text{At } x = 5, \quad \frac{dy}{dx} = 3(10 + 5) \]
   \[ = 45. \]

CHAPTER 18

MULTIPLE CHOICE
1. D
2. B
3. E

SHORT ANSWERS
1. Symmetric
2. Artificial
3. Initial basic feasible
4. Feasible Solution
5. Shadow price
6. Marginal profit
7. Feasible solution.
CHAPTER 19

MULTIPLE CHOICE

1. C
2. A
3. B

SHORT ANSWER

1. Critical path
2. PERT
3. Activity
4. burst
5. AON diagram
6. PERT

   ADFGI = 10 + 10 + 6 + 7 + 8 = 41
   ACEGI = 10 + 4 + 12 + 7 + 8 = 41
   ACHI = 10 + 4 + 20 + 8 = 42
   BEGI = 15 + 12 + 7 + 8 = 42
   BHI = 15 + 20 + 8 = 43
   The Critical path is B-H-I with 43.

CHAPTER 20

MULTIPLE CHOICE

1. E
2. D
3. B

SHORT ANSWERS

1. Expected Value = (1x0.05) + (2x0.08) + (3x0.12) + ...... (8x0.04)
   = 0.05 + 0.16 + 0.36 + 0.72 + 1.25 + 1.2 + 0.56 + 0.32
   = 4.62
   :. Average number of failures per month = $\frac{1000}{4.62} \approx 216$
2. Average cost of individual replacement = \( N(216 \times 2.25) \)
   \[ = N486 \]

3. Minimum cost of group replacement per month = \( N208.30 \) in the 3\textsuperscript{rd} Month

4. The reason is due to repairing and replacement of some parts.

5. (i) Deteriorate or wear-out gradually and
    (ii) fail suddenly

6. (i) A decrease in its production capacity
    (ii) The increasing maintenance and operating costs.
    (iii) Decrease in the value of the re-sale price (or salvage) of item.

7. (i) Item’s life span is uncertain
    (ii) It is as summed that failure occurs only at the end of its life span.

\textbf{CHAPTER 21}

1. C
2. D
3. D
4. B
5. A

\textbf{SHORT ANSWERS}

1. Penalty or opportunity
2. Degenerate
3. Relative cost
4. Hungarian method or reduced matrix method
5. The table is not balanced. Hence, we make it balanced by creating a dummy Destination for the demand
Total Cost = 30(5) + 20(6) + 20(0) + 20(3) + 20(7) + 60(8)
= 150 + 120 + 0 + 60 + 140 + 480
= ₦950.

CHAPTER 22

MULTIPLE CHOICE

1. C
2. D
3. E
4. A
5. C

SHORT ANSWERS

1. Formula
2. Solver
3. Data marker
4. Gridlines
5. $ (Dollar sign)
APPENDIX II

COMPREHENSIVE QUESTIONS AND SUGGESTED SOLUTIONS

QUESTION 1

On 1 January Alhaji Gogo started a small business buying and selling a special yarn. He invested his savings of ₦40,000 in the business and, during the next six months, the following transactions occurred:

<table>
<thead>
<tr>
<th>Date of Purchase</th>
<th>Quantity (Boxes)</th>
<th>Total Cost (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 Jan</td>
<td>200</td>
<td>7,200</td>
</tr>
<tr>
<td>8 Feb</td>
<td>400</td>
<td>15,200</td>
</tr>
<tr>
<td>11 March</td>
<td>600</td>
<td>24,000</td>
</tr>
<tr>
<td>12 April</td>
<td>400</td>
<td>14,000</td>
</tr>
<tr>
<td>15 June</td>
<td>500</td>
<td>14,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date of Sale</th>
<th>Quantity (Boxes)</th>
<th>Total Value (₦)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Feb</td>
<td>500</td>
<td>25,000</td>
</tr>
<tr>
<td>20 April</td>
<td>600</td>
<td>27,000</td>
</tr>
<tr>
<td>25 June</td>
<td>400</td>
<td>15,200</td>
</tr>
</tbody>
</table>

The yarn is stored in premises Alhaji Gogo has rented and the closing stock of yarn, counted on 30 June, was 500 boxes.

Other expenses incurred, and paid in cash, during the six-month period amounted to ₦2,300.

Required

a) Calculate the value of the material issued during the six-month period, and the value of the closing stock at the end of June, using the following methods of pricing:
   (i) First in, first out;
   (ii) Last in, first out; and
   (iii) Weighted average (calculations to two decimal places only).

(10 marks)

b) Calculate and discuss the effect each of the three methods of material pricing will have on the reported profit of the business, and examine the performance of the business during the first six-month period.

(12 marks)
SUGGESTED SOLUTION TO QUESTION 1

(a) Alhaji Gogo

STOCK VALUATION USING THE FIRST IN FIRST OUT METHOD

<table>
<thead>
<tr>
<th>Date</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 Jan</td>
<td>200</td>
<td>36</td>
<td>7,200</td>
<td>600</td>
<td>36</td>
<td>7,200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Feb</td>
<td>400</td>
<td>38</td>
<td>15,200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 Feb</td>
<td></td>
<td></td>
<td></td>
<td>200</td>
<td>36</td>
<td>7,200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Feb</td>
<td></td>
<td></td>
<td></td>
<td>300</td>
<td>38</td>
<td>11,400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 Feb</td>
<td></td>
<td></td>
<td></td>
<td>500</td>
<td>36</td>
<td>18,600</td>
<td>100</td>
<td>38</td>
<td>3,800</td>
</tr>
<tr>
<td>11 March</td>
<td>600</td>
<td>40</td>
<td>24,000</td>
<td>300</td>
<td>40</td>
<td>12,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 April</td>
<td>400</td>
<td>35</td>
<td>14,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 April</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
<td>38</td>
<td>3,800</td>
<td>1,100</td>
<td>38</td>
<td>41,800</td>
</tr>
<tr>
<td>20 April</td>
<td></td>
<td></td>
<td></td>
<td>500</td>
<td>40</td>
<td>20,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 June</td>
<td>500</td>
<td>28</td>
<td>14,000</td>
<td>600</td>
<td>40</td>
<td>23,800</td>
<td>1,000</td>
<td>40</td>
<td>32,000</td>
</tr>
<tr>
<td>15 June</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 June</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
<td>35</td>
<td>3,500</td>
<td>600</td>
<td>35</td>
<td>17,500</td>
</tr>
<tr>
<td>30 June</td>
<td>300</td>
<td>35</td>
<td>10,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 June</td>
<td>400</td>
<td>35</td>
<td>14,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 June</td>
<td>100</td>
<td>35</td>
<td>3,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Cost of Materials Issued:

<table>
<thead>
<tr>
<th>Date</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Feb</td>
<td>500</td>
<td>units</td>
<td>18,600</td>
</tr>
<tr>
<td>20 April</td>
<td>600</td>
<td>units</td>
<td>23,800</td>
</tr>
<tr>
<td>25 June</td>
<td>400</td>
<td>units</td>
<td>14,500</td>
</tr>
<tr>
<td>30 June</td>
<td>100</td>
<td>units (stock loss)</td>
<td>3,500</td>
</tr>
</tbody>
</table>

Closing stock = 500 units
Value = 500 units @ N28 = N14,000
## (ii) STOCK VALUATION USING THE LAST IN FIRST OUT METHOD

<table>
<thead>
<tr>
<th>Date</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 Jan</td>
<td>200</td>
<td>36</td>
<td>7,200</td>
<td>200</td>
<td>36</td>
<td>7,200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Feb</td>
<td>400</td>
<td>38</td>
<td>15,200</td>
<td>600</td>
<td>36</td>
<td>22,400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Feb</td>
<td></td>
<td></td>
<td></td>
<td>400</td>
<td>38</td>
<td>15,200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>36</td>
<td>3,600</td>
<td></td>
<td>36</td>
<td>3,600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 March</td>
<td>600</td>
<td>40</td>
<td>24,000</td>
<td>500</td>
<td>37.33</td>
<td>18,800</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 April</td>
<td>400</td>
<td>35</td>
<td>14,000</td>
<td></td>
<td>37.33</td>
<td>14,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 April</td>
<td>400</td>
<td>35</td>
<td>14,000</td>
<td>100</td>
<td>37.33</td>
<td>3,735</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>40</td>
<td>8,000</td>
<td></td>
<td>37.94</td>
<td>22,400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 June</td>
<td>500</td>
<td>28</td>
<td>14,000</td>
<td>600</td>
<td>32.97</td>
<td>19,600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 June</td>
<td></td>
<td></td>
<td></td>
<td>400</td>
<td>28</td>
<td>11,200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 June</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
<td>28</td>
<td>2,800</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Materials Issued:

- 10 Feb: 500 units @ N37.33 = N18,800
- 20 April: 600 units @ N37.94 = N22,764
- 25 June: 400 units @ N32.97 = N13,188

Closing Stock = 500 units

Value = 400 units @ N40 = N16,000

100 units @ N36 = N3,600

N19,600

## (iii) STOCK VALUATION USING WEIGHTED AVERAGE METHOD

<table>
<thead>
<tr>
<th>Date</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
<th>Qty</th>
<th>Price</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 Jan</td>
<td>200</td>
<td>36</td>
<td>7,200</td>
<td>200</td>
<td>36</td>
<td>7,200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Feb</td>
<td>400</td>
<td>38</td>
<td>15,200</td>
<td>600</td>
<td>37.33</td>
<td>22,400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Feb</td>
<td></td>
<td></td>
<td></td>
<td>500</td>
<td>37.33</td>
<td>18,665</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>700</td>
<td>39.62</td>
<td>27,735</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 March</td>
<td>600</td>
<td>40</td>
<td>24,000</td>
<td>100</td>
<td>37.94</td>
<td>3,735</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 April</td>
<td>400</td>
<td>35</td>
<td>14,000</td>
<td></td>
<td>39.62</td>
<td>27,735</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 April</td>
<td>600</td>
<td>37.94</td>
<td>22,764</td>
<td>500</td>
<td>37.94</td>
<td>18,971</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 June</td>
<td>500</td>
<td>28</td>
<td>14,000</td>
<td>1,000</td>
<td>32.97</td>
<td>32,971</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 June</td>
<td></td>
<td></td>
<td></td>
<td>400</td>
<td>32.97</td>
<td>13,188</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 June</td>
<td></td>
<td></td>
<td></td>
<td>100</td>
<td>32.97</td>
<td>3,297</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Materials Issued:

- 10 Feb: 500 units @ N37.33 = N18,800
- 20 April: 600 units @ N37.94 = N22,764

N54,800
25 June  400 units @ N32.97 = 13,188
30 June  100 units @ N32.97 (stock loss) = 3,297

Closing stock = 500 units
Value = 500 units @ N32.97 = N16,485

PROFIT AND LOSS

<table>
<thead>
<tr>
<th></th>
<th>FIFO</th>
<th>LIFO</th>
<th>WAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>67,200</td>
<td>67,200</td>
<td>67,200</td>
</tr>
<tr>
<td>Cost of sale</td>
<td>(60,400)</td>
<td>(54,800)</td>
<td>(57,914)</td>
</tr>
<tr>
<td>Gross profit</td>
<td>6,800</td>
<td>12,400</td>
<td>9,286</td>
</tr>
<tr>
<td>Less other expenses</td>
<td>2,300</td>
<td>2,300</td>
<td>2,300</td>
</tr>
<tr>
<td>Net profit</td>
<td>4,500</td>
<td>10,100</td>
<td>6,986</td>
</tr>
</tbody>
</table>

The LIFO method shows the highest profit. This is because the business is undergoing a period of falling prices making the value of stocks issued to production of cheap rates. The LIFO method also has the highest closing stock figure. The FIFO method has the least profit while the WAM is somewhere between them. The WAM tends to smooth wide variation in prices. In a period of rising prices, the positions of the FIFO and LIFO methods will be interchanged, that is, FIFO will tend to show a higher profit and closing stock than LIFO.

QUESTION 2

A company is preparing its production overhead budgets and determining the apportionment of those overheads to products. Cost centre expenses and related information have been budgeted as follows:

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Machine shop A</th>
<th>Machine shop B</th>
<th>Assembly</th>
<th>Canteen</th>
<th>Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indirect wages</td>
<td>N78,560</td>
<td>N8,586</td>
<td>N9,190</td>
<td>N15,674</td>
<td>N29,650</td>
<td>N15,460</td>
</tr>
<tr>
<td>Consumable material</td>
<td>N16,900</td>
<td>N6,400</td>
<td>N8,700</td>
<td>N1,200</td>
<td>N600</td>
<td>-</td>
</tr>
<tr>
<td>(including, Maint.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rent and Rate</td>
<td>N16,700</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Buildings Insurance</td>
<td>N2,400</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Power</td>
<td>N8,600</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Heat and Light</td>
<td>N3,400</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Depreciation of machinery</td>
<td>N40,200</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Value of machinery</td>
<td>N402,000</td>
<td>N201,000</td>
<td>N179,000</td>
<td>N22,000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Other information:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power usage – Technical Estimate %</td>
<td>100</td>
<td>55</td>
<td>40</td>
<td>3</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>Direct labour (hours)</td>
<td>N35,000</td>
<td>N8,000</td>
<td>N6,200</td>
<td>N20,800</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Machine usage (hours)</td>
<td>N25,200</td>
<td>N7,200</td>
<td>N18,000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Area (sq.ft)</td>
<td>N45,000</td>
<td>N10,000</td>
<td>N12,000</td>
<td>N15,000</td>
<td>N6,000</td>
<td>N2,000</td>
</tr>
</tbody>
</table>
Required

(a) Determine budgeted overhead absorption rates for each of the production departments, using bases of apportionment and absorption which you consider most appropriate from the information provided.  (13 marks)

(b) On the assumption that actual activity was as follows:

<table>
<thead>
<tr>
<th></th>
<th>Machine shop A</th>
<th>Machine shop B</th>
<th>Assembly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct labour hrs.</td>
<td>7,200</td>
<td>6,500</td>
<td>21,900</td>
</tr>
<tr>
<td>Machine usage hrs</td>
<td>7,300</td>
<td>18,700</td>
<td>-</td>
</tr>
</tbody>
</table>

And total production overhead expenditure was ₦176,533, prepare the production overhead control account for the year (you are to assume that the company has a separate cost accounting system).

(c) Explain the meaning of the word ‘control’ in the title of the account prepared in answer to (b)

SUGGESTED SOLUTION TO QUESTION 2

(a) A COMPANY

<table>
<thead>
<tr>
<th>Item</th>
<th>Basic</th>
<th>Total</th>
<th>Machine Shop A</th>
<th>Machine Shop B</th>
<th>Assembly</th>
<th>Canteen</th>
<th>Maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indirect Wages</td>
<td>Allocation</td>
<td>78,560</td>
<td>8,586</td>
<td>9,190</td>
<td>15,674</td>
<td>29,650</td>
<td>15,460</td>
</tr>
<tr>
<td>Consumable and materials</td>
<td>Allocation</td>
<td>16,900</td>
<td>6,400</td>
<td>8,700</td>
<td>1,200</td>
<td>600</td>
<td>-</td>
</tr>
<tr>
<td>Rent and Rate Building</td>
<td>Area (sq fit)</td>
<td>16,700</td>
<td>3,711</td>
<td>4,453</td>
<td>5,567</td>
<td>2,227</td>
<td>742</td>
</tr>
<tr>
<td></td>
<td>Area (sq fit)</td>
<td>2,400</td>
<td>533</td>
<td>640</td>
<td>800</td>
<td>320</td>
<td>107</td>
</tr>
<tr>
<td>Insurance</td>
<td>Power estimate</td>
<td>8,600</td>
<td>4,730</td>
<td>3,440</td>
<td>258</td>
<td>-</td>
<td>172</td>
</tr>
<tr>
<td>Heat &amp; Light Depreciation</td>
<td>Area (sq fit)</td>
<td>3,400</td>
<td>756</td>
<td>907</td>
<td>1,133</td>
<td>453</td>
<td>151</td>
</tr>
<tr>
<td>of machinery</td>
<td>Value of Machine</td>
<td>40,200</td>
<td>20,100</td>
<td>17,900</td>
<td>2,200</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>166,760</td>
<td>44,816</td>
<td>45,230</td>
<td>26,832</td>
<td>33,250</td>
<td>16,632</td>
</tr>
<tr>
<td>Canteen’s maintenance costs</td>
<td>Direct labour cost</td>
<td>7,600</td>
<td>5,890</td>
<td>19,760</td>
<td>(33,250)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Maintenance cost</td>
<td>Machine hour</td>
<td>4,752</td>
<td>11,880</td>
<td>-</td>
<td>-</td>
<td>(16,632)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>57,168</td>
<td>63,000</td>
<td>46,592</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Workings:

Overhead Absorption Rates

\[
\text{Machine Shop A} = \frac{\text{Budgeted Overhead}}{\text{Machine Hours}}
\]

\[
= \frac{₦57,168}{7,200} = ₦7.94/\text{hrs}
\]

Machine Shop B

\[
= \frac{\text{Budgeted Overhead}}{\text{Machine Hours}}
\]
PRODUCTION OVERHEAD CONTROL A/C

<table>
<thead>
<tr>
<th></th>
<th>₦</th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ledger control A/c</td>
<td>176,533</td>
<td></td>
</tr>
<tr>
<td>(Overhead incurred)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W-I-P</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Machine Shop A</td>
<td>57,962</td>
<td></td>
</tr>
<tr>
<td>(7,300 x 7.94)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Machine Shop B</td>
<td>65,450</td>
<td></td>
</tr>
<tr>
<td>(18,700 x 3.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Assembly</td>
<td>49,056</td>
<td></td>
</tr>
<tr>
<td>(21,900 x 2.24)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Under absorption – P&amp;L</td>
<td>4,065</td>
<td></td>
</tr>
<tr>
<td></td>
<td>176,533</td>
<td>176,533</td>
</tr>
</tbody>
</table>

(c) Control accounts are total accounts. They collect together all the actual overhead incurred (by function - this time production) and how they are transferred to that is, recovered from each production cost centre to form total production cost.

QUESTION 3

One of the building contracts currently engaged in by a construction company, which it commenced 15 months ago, remains unfinished. The following information relating to work on the contract has been prepared for the year just ended:

<table>
<thead>
<tr>
<th></th>
<th>₦’000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract price</td>
<td>2,100</td>
</tr>
<tr>
<td>Value of work certified at end of year</td>
<td>1,840</td>
</tr>
<tr>
<td>Cost of work not yet certified</td>
<td>35</td>
</tr>
<tr>
<td>Costs incurred:</td>
<td></td>
</tr>
<tr>
<td>Opening balances</td>
<td></td>
</tr>
<tr>
<td>Cost of work completed</td>
<td>250</td>
</tr>
<tr>
<td>Materials on site (physical stock)</td>
<td>10</td>
</tr>
<tr>
<td>During the year</td>
<td></td>
</tr>
<tr>
<td>Materials delivered to site</td>
<td>512</td>
</tr>
<tr>
<td>Wages</td>
<td>487</td>
</tr>
<tr>
<td>Hire of plant</td>
<td>96</td>
</tr>
<tr>
<td>Other expenses</td>
<td>74</td>
</tr>
<tr>
<td>Closing balance</td>
<td></td>
</tr>
<tr>
<td>Materials on site (physical stock)</td>
<td>18</td>
</tr>
</tbody>
</table>

As soon as materials are delivered to the site, they are charged to the contract account. A record is also kept of materials as they are actually used on the contract. Periodically a stock check is made and any discrepancy between book stock and physical stock is transferred to a general contract materials discrepancy account. This is absorbed back into each contract.
currently at a rate of 0.4% of materials booked. The stock check at the end of the year revealed a stock shortage of N4,000.

In addition to the direct charges listed above general overheads of the company are charged to contracts at 5% of the value of work certified. General overheads of N13,000 had been absorbed into the cost of work completed at the beginning of the year.

It has been estimated that further costs to complete the contract will be N215,000. This estimate includes the cost of materials on site at the end of the year just finished, and also a provision rectification.

**Required:**
(a) Explain briefly the distinguishing features of contract costing (4 marks)
(b) Determine the profitability of the above contract, and recommend how much profit (to the nearest N000) should be taken for the year just ended. (Provide a detailed schedule of costs.) (14 marks)
(c) State how your recommendation in (b) would be affected if the contract price was N3,500,000 (rather than N2,100,000) and if no estimate has been made of costs to completion. (4 marks)

**SUGGESTED SOLUTION TO QUESTION 3**

(a) The distinguishing features of contract costing include:
(i) It is usually carried out to customer’s specification.
(ii) Work is usually carried out at client site rather than within the business premises.
(iii) Work is usually for a long period which may straddle two or more accounting periods.
(iv) Architect’s or engineer’s valuation is usually required.
(v) Since the contract usually takes long time, straddling more than one accounting periods profit has to be taken in more than one financial year in compliance with the matching concept.

(b) **A CONSTRUCTION COMPANY**

<table>
<thead>
<tr>
<th>CONTRACT ACCOUNT</th>
<th>N000</th>
<th>N000</th>
</tr>
</thead>
<tbody>
<tr>
<td>W.I.P b/f</td>
<td>250</td>
<td></td>
</tr>
<tr>
<td>Material b/f</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Materials delivered to site</td>
<td>512</td>
<td></td>
</tr>
<tr>
<td>Wages</td>
<td>487</td>
<td></td>
</tr>
<tr>
<td>Wire of plant</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td>Other expenses</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>Stock shortage absorbed</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>(0.4x500)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Overheads</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td><strong>Stock shortage</strong></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td><strong>Material c/f</strong></td>
<td>18</td>
<td></td>
</tr>
<tr>
<td><strong>Cost to date</strong></td>
<td>1,488</td>
<td>1,488</td>
</tr>
<tr>
<td><strong>Work in progress</strong></td>
<td>1,835</td>
<td></td>
</tr>
<tr>
<td><strong>Profit</strong></td>
<td>347</td>
<td>1,835</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1,510</td>
<td>1,510</td>
</tr>
<tr>
<td><strong>Profit to date</strong></td>
<td>1,488</td>
<td></td>
</tr>
<tr>
<td><strong>Profit</strong></td>
<td>347</td>
<td>1,835</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1,835</td>
<td>1,835</td>
</tr>
</tbody>
</table>
Calculation of Estimated Total Profit ETP/(Notional Profit)

<table>
<thead>
<tr>
<th></th>
<th>₦</th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract Price</td>
<td>2,100,000</td>
<td></td>
</tr>
<tr>
<td>Estimated Total Cost</td>
<td></td>
<td>1,488,000</td>
</tr>
<tr>
<td>Cost to date</td>
<td></td>
<td>215,000</td>
</tr>
<tr>
<td>Cost to complete</td>
<td></td>
<td>215,000</td>
</tr>
<tr>
<td>(1,703,000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimated Total profit</td>
<td></td>
<td>397,000</td>
</tr>
</tbody>
</table>

Since the contract is fairly close to completion, the cost to completion being able to be estimated with certainty, profit can be taken on the contract. This is calculated as follows:

Calculation of Profit Received to date:
Value of work certified x ETP
Contract Price

\[
\frac{1,840 \times \text{₦}397,000}{2,100} = \text{₦}347,848
\]

Or

\[
\frac{\text{Cost to date} \times \text{Notional Profit}}{\text{Estimated Total Cost}}
\]

\[
\frac{1,488 \times \text{₦}397,000}{1,703} = \text{₦}346,879
\]

The lower profit figure may be chosen

Profit Taken in Current Period:

<table>
<thead>
<tr>
<th></th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit realised to date</td>
<td>346,879</td>
</tr>
<tr>
<td>Profit b/f</td>
<td></td>
</tr>
<tr>
<td></td>
<td>346,879</td>
</tr>
</tbody>
</table>

(c) If the contract price were ₦3,500,000 and it was not possible to estimate the cost to completion, then it would not be possible to calculate the estimated total profit. It would, therefore, not seem reasonable to take any profit on the contract.

QUESTION 4

Sammy Manufacturing Company makes a product called “vait”. Some of the manufacturing expenses are easily identified as fixed or directly variable with production. The cost accountant of the company is confronted with problem of preparing a flexible budget for the coming of the year and wishes to determine the fixed and variable elements of the mixed factory overhead.
The following details are provided for the first 10 months of the year.

<table>
<thead>
<tr>
<th>Month</th>
<th>No. of units Produced (x)</th>
<th>Mixed factory overhead (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,500</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>2,000</td>
<td>1,000</td>
</tr>
<tr>
<td>3</td>
<td>3,000</td>
<td>1,350</td>
</tr>
<tr>
<td>4</td>
<td>2,500</td>
<td>1,250</td>
</tr>
<tr>
<td>5</td>
<td>3,000</td>
<td>1,300</td>
</tr>
<tr>
<td>6</td>
<td>2,500</td>
<td>1,200</td>
</tr>
<tr>
<td>7</td>
<td>3,500</td>
<td>1,400</td>
</tr>
<tr>
<td>8</td>
<td>3,000</td>
<td>1,250</td>
</tr>
<tr>
<td>9</td>
<td>2,500</td>
<td>1,150</td>
</tr>
<tr>
<td>10</td>
<td>1,500</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>25,000</td>
<td>11,500</td>
</tr>
</tbody>
</table>

Determine the fixed and variable elements of the mixed factory overhead using:

(a) The high and low method: and (4 marks)
(b) The method of least squares (12 marks)
(c) Differentiate between correlation coefficient \( r \) and coefficient of determination. (4 marks)

**Total 20 marks**

**SUGGESTED SOLUTION TO QUESTION 4**

(a) **SAMMY MANUFACTURING COMPANY**

High and Low Method

<table>
<thead>
<tr>
<th>Units</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>3,500</td>
<td>1,400</td>
</tr>
<tr>
<td>Low</td>
<td>1,500</td>
<td>800</td>
</tr>
<tr>
<td>Difference</td>
<td>2,000</td>
<td>600</td>
</tr>
</tbody>
</table>

Variable overhead rate: \( \text{₦600/2,000 units} = \text{₦0.30/unit} \)

To obtain the fixed element

<table>
<thead>
<tr>
<th>Units</th>
<th>High</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>350</td>
<td>350</td>
</tr>
</tbody>
</table>

Mixed overhead observed

| Units | 1,400 | 800 |

Less variable overhead

| Units | 1,050 | 450 |

(No. 30/unit)

\( \therefore \) Fixed Overhead

| Units | 350 |

Therefore, the formula is \( \text{₦350 fixed} + \text{₦0.30 per unit} \)

In other words \( Y(\text{total overhead}) = 0.3x + 350 \)

(b) Since the number are too voluminous, we will use the alternative formula for \( b \), that is,

\[
b = \frac{\sum(x-x̄)(y-ȳ)}{\sum(x-x̄)^2}
\]
\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Number of units produced} & \text{Mixed factory overhead} & \times x & y & (x \times \bar{y}) & (y \times \bar{y}) & (x \times \bar{y})^2 \\
\hline
1,500 & 800 & -1,000 & -350 & 350,000 & 1,000,000 \\
2,000 & 1,000 & -500 & -150 & 75,000 & 250,000 \\
3,000 & 1,350 & 500 & 200 & 100,000 & 250,000 \\
2,500 & 1,250 & 0 & 100 & 0 & 0 \\
3,000 & 1,300 & 500 & 150 & 75,000 & 250,000 \\
2,500 & 1,200 & 0 & 50 & 0 & 0 \\
3,500 & 1,400 & 100 & 250 & 250,000 & 1,000,000 \\
3,000 & 1,250 & 500 & 100 & 50,000 & 250,000 \\
2,500 & 1,150 & 0 & 0 & 0 & 0 \\
\hline
25,000 & 11,500 & -1000 & -350 & 350,000 & 1,000,000 \\
\hline
\end{array}
\]

\(\bar{x} = 2,500 \quad \bar{y} = 1,15\)

Therefore, \(b = \frac{1,250,000}{4,000,000} = 0.3125\)

\(a = \frac{11,500 - (0.3125 \times 25,000)}{10} = 1,150 - 781.25 = 368.75\)

Thus, the cost–volume formula is \(\text{N}\,368.75\) fixed, plus \(\text{N}\,0.3125\) per unit.

(c) The correlation coefficient \(r\) measures the degree of correlation between \(y\) and \(x\). The range of values it takes on is between \(-1\) and \(+1\). While the coefficient of determination, designated \(r^2\) (read as \(r\)-squared). Simply put \(r^2\) tells us how good the estimated regression equation is. In other words, it is a measure of “goodness of fit” in the regression. Therefore, the higher the \(r^2\), the more confidence we can have in our estimated cost for units.

**QUESTION 5**

Rantak Nigeria Limited, a textile company makes one of its product “ASO” in 3 main processes:

(a) Whapping;
(b) Knitting; and
(c) Finishing.

In a particular period, the cost of production were as follows:

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{(W)} & \text{(K)} & \text{(F)} & \text{Total} \\
\hline
\text{Whapping} & \text{Knitting} & \text{Finishing} & \text{Total} \text{ N} \\
\hline
\text{Direct materials} & 10,000 & 16,000 & 17,310 & 43,310 \\
\text{Direct labour} & 15,000 & 20,000 & 25,000 & 60,000 \\
\text{Direct expenses} & 2,500 & 1,130 & - & 3,630 \\
\text{Production overhead} & - & - & - & 30,000 \\
\hline
\end{array}
\]

10,000 units at \(\text{N}\,5\) each were introduced to whapping process, normal loss for W, K, F processes were estimated at 10%, 5% and 10% respectively.
Output from whapping, knitting and finishing process were 9,200 units, 8,700 units and 8,000 units respectively, with the possibility of sales of scrap from W, K, and F at ₦3, ₦5, and ₦6 respectively.

Stock of materials and work-in-progress are the same both at opening and closing production overhead is absorbed by each process on a basis of 50% of Direct Labour cost.

You are required to:
(a) Prepare accounts for the three processes.
(b) Draw up an abnormal loss/gain account and explain how the gains and loss were arrived at.

(18 marks)

**SUGGESTED SOLUTION TO QUESTION 5**

(a) **RANTAK NIGERIA LIMITED**

**Process Accounts**

**(Whapping Process)**

<table>
<thead>
<tr>
<th>Units</th>
<th>Unit Cost</th>
<th>Value</th>
<th>Units</th>
<th>Unit Cost</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qty</td>
<td>₦</td>
<td>₦</td>
<td>Qty</td>
<td>₦</td>
<td>₦</td>
</tr>
<tr>
<td>Input Direct Material</td>
<td>10,000</td>
<td>5.00</td>
<td>50,000</td>
<td>Normal Loss</td>
<td>1,000</td>
</tr>
<tr>
<td>Direct Material</td>
<td>10,000</td>
<td>Knitting process</td>
<td>9,200</td>
<td>9.111</td>
<td>83,822</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>15,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct Labour</td>
<td>2,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenses Production Overheads</td>
<td>7,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abnormal Gain</td>
<td>200</td>
<td>9.111</td>
<td>1,822</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gain</td>
<td>10,200</td>
<td>86.822</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Knitting Process**

<table>
<thead>
<tr>
<th>Units</th>
<th>Unit Cost</th>
<th>Value</th>
<th>Units</th>
<th>Unit Cost</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qty</td>
<td>₦</td>
<td>₦</td>
<td>Qty</td>
<td>₦</td>
<td>₦</td>
</tr>
<tr>
<td>Whapping process Direct Material</td>
<td>9,200</td>
<td>9.111</td>
<td>83,822</td>
<td>Normal loss</td>
<td>460</td>
</tr>
<tr>
<td>Direct Material</td>
<td>16,000</td>
<td>Finishing process</td>
<td>8,700</td>
<td>14.72</td>
<td>128,063</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>20,000</td>
<td>Abnormal loss</td>
<td>40</td>
<td>14.72</td>
<td>589</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>1,130</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenses Production Overheads</td>
<td>10,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9,200</td>
<td>130,952</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9,200</td>
<td>130,952</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Finishing Process

<table>
<thead>
<tr>
<th>Units</th>
<th>Unit Cost</th>
<th>Value</th>
<th>Units</th>
<th>Unit Cost</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qty</td>
<td>N</td>
<td>N</td>
<td>Qty</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Knitting Process</td>
<td>8,700</td>
<td>14.72</td>
<td>128,063</td>
<td>Normal loss</td>
<td>870</td>
</tr>
<tr>
<td>Direct Material</td>
<td>17,310</td>
<td>Finishing stock</td>
<td>8,000</td>
<td>22.69</td>
<td>181,510</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>25,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Production Overheads</td>
<td>12,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abnormal Gain</td>
<td>170</td>
<td>22.69</td>
<td>3,857</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 8,870 | 186,730 | 8,870 | 186,730 |

(b) **Abnormal Loss Account**

<table>
<thead>
<tr>
<th>£</th>
<th>£</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knitting Process 40 x 14.72</td>
<td>589</td>
</tr>
<tr>
<td>Cash 40 x 5.00</td>
<td>200</td>
</tr>
<tr>
<td>Profit &amp; Loss Account</td>
<td>389</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>589</td>
</tr>
</tbody>
</table>

(c) **Abnormal Gain Account**

<table>
<thead>
<tr>
<th>£</th>
<th>£</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whapping Process 200 x 3.0</td>
<td>600</td>
</tr>
<tr>
<td>Whapping Process 200 x 9.11</td>
<td>1,822</td>
</tr>
<tr>
<td>Finishing Process 170 x 6.00</td>
<td>1,020</td>
</tr>
<tr>
<td>Finishing Process 170 x 22.70</td>
<td>3,857</td>
</tr>
<tr>
<td>Profit and Loss Account</td>
<td>4,059</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>5,679</td>
</tr>
</tbody>
</table>
Statement Showing Gain or Loss from each Process

**Whapping Process**

<table>
<thead>
<tr>
<th>Units</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>10,000 Normal Cost</td>
</tr>
<tr>
<td>10% Normal Loss</td>
<td>1,000 Scrap Value @ N3</td>
</tr>
<tr>
<td>Normal Output</td>
<td>9,000</td>
</tr>
<tr>
<td>Actual Output</td>
<td>9,200</td>
</tr>
<tr>
<td>Abnormal Gain</td>
<td>200</td>
</tr>
</tbody>
</table>

**Knitting Process**

<table>
<thead>
<tr>
<th>Units</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>9,200 Normal Cost</td>
</tr>
<tr>
<td>5% Normal Loss</td>
<td>460 Scrap Value @ N5</td>
</tr>
<tr>
<td>Normal Output</td>
<td>8,740</td>
</tr>
<tr>
<td>Actual Output</td>
<td>8,700</td>
</tr>
<tr>
<td>Abnormal Loss</td>
<td>40</td>
</tr>
</tbody>
</table>

**Finishing Process**

<table>
<thead>
<tr>
<th>Units</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>8,700 Normal Cost</td>
</tr>
<tr>
<td>10% Normal Loss</td>
<td>870 Scrap Value @ N6</td>
</tr>
<tr>
<td>Normal Output</td>
<td>7,830</td>
</tr>
<tr>
<td>Actual Output</td>
<td>8,000</td>
</tr>
<tr>
<td>Abnormal Gain</td>
<td>170</td>
</tr>
</tbody>
</table>

**QUESTION 6**

A chemical company carries on production operations in two processes. Materials first pass through Process 1, where a compound is produced. A loss in weight takes place at the start of processing. The following data, which can be assumed to be representative, relates to the month just ended:

**Quantities**

<table>
<thead>
<tr>
<th>Kilos</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material input</td>
</tr>
<tr>
<td>Opening work in process (half processed)</td>
</tr>
<tr>
<td>Work completed</td>
</tr>
<tr>
<td>Closing work in process (two-thirds processed)</td>
</tr>
</tbody>
</table>

**Costs**

<table>
<thead>
<tr>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material input</td>
</tr>
<tr>
<td>Processing costs</td>
</tr>
<tr>
<td>Opening work in process – materials</td>
</tr>
<tr>
<td>– processing costs</td>
</tr>
</tbody>
</table>

Any quantity of the compound can be sold for N1.60 per kilo. Alternatively, it can be transferred to process 2 for further processing and packing to be sold as ‘Solo’ for N2.00 per kilo. Further materials are added in Process 2 such that for every kilo of compound used, two kilos of ‘Solo’ result.
Of the 160,000 kilos per month of work completed in Process 1, 40,000 kilos are sold as compound and 120,000 kilos are passed through Process 2 for sale as 'Solo'. Process 2 has facilities to handle up to 160,000 kilos of compound per month if required. The monthly costs incurred in Process 1 (other than the cost of the compound) are:

<table>
<thead>
<tr>
<th>120,000 kilos of compound input</th>
<th>160,000 kilos of compound input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>120,000</td>
</tr>
<tr>
<td>Processing costs</td>
<td>120,000</td>
</tr>
</tbody>
</table>

**Required**

(a) Determine using the average method, the cost per kilo of compound in Process 1, and the value of both work completed and closing work in process for the month just ended.  
(b) Demonstrate that it is worth while further processing 120,000 kilos of compound.  
(c) Calculate the minimum acceptable selling price per kilo, if a potential buyer could be found for the additional output of 'Solo' that could be produced with the remaining compound.

**SUGGESTED SOLUTION TO QUESTION 6**

‘Solo’

(a) **Equivalent units calculation**

<table>
<thead>
<tr>
<th>Units</th>
<th>Process I Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening stock</td>
<td>40,000</td>
</tr>
<tr>
<td>Fully worked 160,000</td>
<td>40,000</td>
</tr>
<tr>
<td>(160,000 – 40,000)</td>
<td>120,000</td>
</tr>
<tr>
<td>Closing stock</td>
<td>30,000</td>
</tr>
<tr>
<td>Total</td>
<td>190,000</td>
</tr>
<tr>
<td>Costs</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening stock</td>
<td>20,000</td>
</tr>
<tr>
<td>Added</td>
<td>75,000</td>
</tr>
<tr>
<td>Total</td>
<td>95,000</td>
</tr>
</tbody>
</table>

Cost per unit = $0.50 + $0.60 = $1.10

Value of work completed = 160,000 x 1.10 = $176,000

Closing stock = Process I Materials = 30,000 x 0.5 = $15,000
Conversion = 20,000 x 0.6 = $12,000

Closing stock = $27,000
(b) From Process 1, 120,000 @ ₦1.60
   (Opportunity cost) 192,000
   Further Materials added (120,000 kilos) 120,000
   Further Processing costs 120,000
   Total Costs of producing 240,000 kilos of compound 432,000
   Sales value (240,000 @ ₦2.00) 480,000
   Gain 48,000

Clearly, it is profitable to put 120,000 kilos through Process 2 as this will result in an additional gain of ₦48,000

(c) Costs of processing 40,000 kg further:

<table>
<thead>
<tr>
<th></th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>40,000</td>
</tr>
<tr>
<td>Processing costs</td>
<td>20,000</td>
</tr>
<tr>
<td>From Process 1 at opportunity cost</td>
<td>60,000</td>
</tr>
<tr>
<td>40,000 x ₦1.60</td>
<td>64,000</td>
</tr>
<tr>
<td>Total costs</td>
<td>124,000</td>
</tr>
<tr>
<td>Quantity of ‘Solo’ produced</td>
<td>80,000 kg</td>
</tr>
<tr>
<td>Therefore, Minimum price</td>
<td>₦1.55</td>
</tr>
</tbody>
</table>

**QUESTION 7**

In the absence of the Accountant you have been asked to prepare a month’s cost accounts for Reflex Limited which operates a batch costing system, fully integrated with the financial accounts. The cost clerk has provided you with the following information, which he thinks is relevant:

<table>
<thead>
<tr>
<th></th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balances at beginning of month:</td>
<td></td>
</tr>
<tr>
<td>Stores Ledger control account</td>
<td>24,175</td>
</tr>
<tr>
<td>Work in progress control account</td>
<td>19,210</td>
</tr>
<tr>
<td>Finished goods control account</td>
<td>34,164</td>
</tr>
<tr>
<td>Prepayments of production overheads</td>
<td></td>
</tr>
<tr>
<td>Brought forward from previous month</td>
<td>2,100</td>
</tr>
<tr>
<td><em>Transactions during the month:</em></td>
<td></td>
</tr>
<tr>
<td>Materials purchased</td>
<td>76,150</td>
</tr>
<tr>
<td>Materials issued: to production</td>
<td>26,350</td>
</tr>
<tr>
<td>For factory maintenance</td>
<td>3,280</td>
</tr>
<tr>
<td>Material transferred between batches</td>
<td>1,450</td>
</tr>
<tr>
<td><strong>Direct workers</strong></td>
<td></td>
</tr>
<tr>
<td>Total wages paid: (net)</td>
<td>17,646</td>
</tr>
<tr>
<td>Employee’s deductions</td>
<td>4,364</td>
</tr>
<tr>
<td><strong>Indirect workers</strong></td>
<td></td>
</tr>
<tr>
<td>Total wages paid: (net)</td>
<td>3,342</td>
</tr>
<tr>
<td>Employee’s deductions</td>
<td>890</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct wages charged to batches from work tickets</td>
<td>15,236</td>
</tr>
<tr>
<td>Recorded non-productive time of direct workers</td>
<td>5,230</td>
</tr>
<tr>
<td>Direct wages incurred on production of</td>
<td></td>
</tr>
</tbody>
</table>
Capital equipment for use in the factory 2,670
Selling and distribution overheads incurred 5,240
Other production overheads incurred 12,200
Sales 75,400
Cost of finished goods sold 59,830
Cost of goods completed and transferred into Finished goods store during the month 62,130
Physical stock value of work in progress at End of month 24,360

The production overhead absorption rate is 150% of direct wages and it is the policy of the company to include a share of production overheads in the cost of capital equipment in the factory.

Required

(a) Prepare the following accounts for the month:
   (i) stores ledger control account
   (ii) work in progress control account
   (iii) finished goods control account
   (iv) production overhead control account
   (v) profit/loss account  (12 marks)

(b) Identify any aspects of the accounts which you consider should be investigated.  (4 marks)

(c) Explain why it is necessary to value a company’s stocks at the end of each period and also why, in a manufacturing company, expense items such as factory rent, wages of direct operatives, power costs etc are included in the value of work in progress and finished goods stocks.  (6 marks)

(Total 22 marks)

SUGGESTED SOLUTION TO QUESTION 7

REFLEX LTD

(a)(i) STORES LEDGER CONTROL ACCOUNT

<table>
<thead>
<tr>
<th></th>
<th>£</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening balances b/f</td>
<td>24,175</td>
</tr>
<tr>
<td>Creditors – materials purchased</td>
<td>76,150</td>
</tr>
<tr>
<td>Materials issued:</td>
<td></td>
</tr>
<tr>
<td>Work in progress control</td>
<td>26,350</td>
</tr>
<tr>
<td>Production overhead control</td>
<td>3,280</td>
</tr>
<tr>
<td>Closing stock c/f</td>
<td>70,695</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100,325</strong></td>
</tr>
<tr>
<td><strong>Opening balances b/f</strong></td>
<td><strong>100,325</strong></td>
</tr>
</tbody>
</table>
(ii) **WORK IN PROGRESS CONTROL ACCOUNT**

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening balance b/f</td>
<td>N 19,210</td>
</tr>
<tr>
<td>Stores ledger - materials issued</td>
<td></td>
</tr>
<tr>
<td>Wages control - direct wages</td>
<td>N 26,350</td>
</tr>
<tr>
<td>Production overhead control – overhead</td>
<td></td>
</tr>
<tr>
<td>absorbed (N 15,236x150%)</td>
<td>N 22,854</td>
</tr>
<tr>
<td>P&amp;L a/c – stock gain</td>
<td>N 2,840</td>
</tr>
<tr>
<td>(see note (4))</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>86,490</strong></td>
</tr>
<tr>
<td></td>
<td><strong>86,490</strong></td>
</tr>
</tbody>
</table>

(iii) **FINISHED GOODS CONTROL ACCOUNT**

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening balance b/f</td>
<td>N 34,164</td>
</tr>
<tr>
<td>Work in progress – cost of goods sold</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>96,294</strong></td>
</tr>
<tr>
<td></td>
<td><strong>96,294</strong></td>
</tr>
</tbody>
</table>

(iv) **PRODUCTION OVERHEAD CONTROL ACCOUNT**

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prepayment b/f</td>
<td>N 2,100</td>
</tr>
<tr>
<td>Stores ledger – materials issued for repairs</td>
<td></td>
</tr>
<tr>
<td>Wages control – idle time of direct workers</td>
<td></td>
</tr>
<tr>
<td>Wages control – indirect workers’ wages (3,342+890)</td>
<td>N 4,232</td>
</tr>
<tr>
<td>Cash/creditors – other overheads incurred</td>
<td>N 12,200</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>27,042</strong></td>
</tr>
<tr>
<td></td>
<td><strong>27,042</strong></td>
</tr>
</tbody>
</table>

(v) **PROFIT AND LOSS ACCOUNT**

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finished goods control – cost of goods sold</td>
<td>N 59,830</td>
</tr>
<tr>
<td>Gross profit c/f</td>
<td>N 15,570</td>
</tr>
<tr>
<td></td>
<td><strong>75,400</strong></td>
</tr>
<tr>
<td></td>
<td><strong>75,400</strong></td>
</tr>
<tr>
<td>Selling &amp; distribution overheads</td>
<td></td>
</tr>
<tr>
<td>Production overhead control-under-absorbed overhead</td>
<td>N 5,240</td>
</tr>
<tr>
<td></td>
<td>N 183</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>18,410</strong></td>
</tr>
<tr>
<td></td>
<td><strong>18,410</strong></td>
</tr>
</tbody>
</table>
Tutorial Notes

1. The value of materials transferred between batches will be recorded on the batch records and will not affect the cost accounts.

2. It is sometimes advisable to produce a ledger account not specifically requested for in the question. Here, it is useful to produce the direct wages control account.

<table>
<thead>
<tr>
<th>DIRECT WAGES CONTROL</th>
<th>£</th>
<th>£</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Direct wages:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net</td>
<td>17,646</td>
<td>15,236</td>
</tr>
<tr>
<td>Employees’ contribution</td>
<td>4,364</td>
<td>2,670</td>
</tr>
<tr>
<td>Balance (see below)</td>
<td>1,126</td>
<td>5,230</td>
</tr>
<tr>
<td></td>
<td><strong>23,136</strong></td>
<td><strong>23,136</strong></td>
</tr>
</tbody>
</table>

The balance may be regarded either as a wages accrual, or as a favourable labour rate variance. In the latter case, your profit should be increased by £1,126.

However, this option is less likely; labour rate variances only arise in a standard costing system, which does not appear to be in operation here.

3. Direct wages incurred on the production of the capital equipment and the production overheads absorbed will be debited to the capital equipment under Construction Account and ultimately to the appropriate fixed asset account.

4. This is a balance figure. In view of part (c) of the question, it is assumed that it represents a stock gain arising on the valuation of closing stocks at the end of the period.

You might prefer to treat the balancing £2,840 as ‘other direct charges’; if so, your net profit figure will be £2,840 less at £10,147.

(b) (i) Closing stock has increased at all three stages of production, but frighteningly so in the case of raw materials. The company must ensure that demand in the near future is sufficient to justify this high stock level or that measures are taken to reduce it.

(ii) Direct workers incurred idle time costing £5,230 in the month, nearly one-quarter of the total direct wages cost. Management must investigate the reasons for the non-productive time and prevent its recurrence.

(iii) The company must ensure that the value of the capital equipment on completion is not over-stated by the inclusion of production overheads.

(iv) The wages information does not reconcile. The reasons for this should be obtained. Possibly, too much time is being charged to batches or, indeed, the payroll department has miscalculated wages payable.

(c) It is necessary to value a company’s stocks at the end of each period for the following reasons:

(i) to derive a stock figure for the balance sheet;
(ii) to calculate the cost of stocks used or sold in the period and hence to calculate profit
(iii) to have a value which can be compared with expected sales demand, budgeted production etc to see if stock levels should be built up or run down;
(iv) to see if too much of a company’s capital is being tied up in its stocks.
(v) a stock-take, and a stock calculation based on the stock-take, will enable management to check the accuracy of its stock records in the cost accounts, or amend the stock records accordingly, transferring a stock gain or write-off to the P&L account to balance the accounts.

QUESTION 8

Global System Limited manufactures three products X, Y, and Z. Relevant cost data relating to the products are as follows:

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>UNIT COSTS (Kobo)</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>55</td>
<td>5</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>50</td>
<td>3</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>35</td>
<td>-</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>60</td>
<td>-</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>80</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

No losses occurred in the use of materials A, B, C, and D the standard yield of material E is 90%. This is an ideal standard. The expected yield is 80%.

During the four-week period budgeted sales are:

<table>
<thead>
<tr>
<th>Product</th>
<th>Sales Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>12,000</td>
</tr>
<tr>
<td>Y</td>
<td>15,000</td>
</tr>
<tr>
<td>Z</td>
<td>10,000</td>
</tr>
</tbody>
</table>

It is anticipated that 5% of the production of product Y will not pass inspection and will be disposed off immediately.

The stocks on hand at the beginning of the period are expected to be:

<table>
<thead>
<tr>
<th>FINISHED GOODS</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1,800</td>
</tr>
<tr>
<td>Y</td>
<td>2,000</td>
</tr>
<tr>
<td>Z</td>
<td>1,600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20,000</td>
</tr>
<tr>
<td>B</td>
<td>30,000</td>
</tr>
<tr>
<td>C</td>
<td>15,000</td>
</tr>
<tr>
<td>D</td>
<td>5,000</td>
</tr>
<tr>
<td>E</td>
<td>9,000</td>
</tr>
</tbody>
</table>

It is planned to increase finished goods stocks in order to satisfy order more quickly. Production is period 1 will be sufficient to increase stocks by 10% by the end of the period.
Materials stocks however, are considered to be high and a reduction of 10% is planned by the end of period 1.

You are required to prepare budgets for:
(a) Production (in quantity);
(b) Materials usage (in quantity); and
(c) Materials purchased (in quantity and value).

(16 marks)

SUGGESTED SOLUTION TO QUESTION 8

GLOBAL SYSTEMS LIMITED

(i) PRODUCTION BUDGET (QUANTITY)

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>12,000</td>
<td>15,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Add:</td>
<td>1,980</td>
<td>2,200</td>
<td>1,760</td>
</tr>
<tr>
<td>Less:</td>
<td>13,980</td>
<td>17,200</td>
<td>11,760</td>
</tr>
</tbody>
</table>

Add: Normal Loss

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

Gross Production

|       | 12,180| 15,200| 10,160|

(ii) MATERIALS USAGE BUDGET (QUANTITY)

<table>
<thead>
<tr>
<th>Materials</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product X</td>
<td>60,900</td>
<td>36,540</td>
<td>-</td>
<td>-</td>
<td>12,180</td>
</tr>
<tr>
<td>Y</td>
<td>64,000</td>
<td>32,000</td>
<td>48,000</td>
<td>16,000</td>
<td>16,000</td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td>60,960</td>
<td>50,800</td>
<td>40,640</td>
<td>-</td>
</tr>
</tbody>
</table>

Loss in Production

| Loss in Production |       |       |       |       | 7,045 |

Qty Required

|       | 124,900| 129,500| 98,800| 56,640| 35,225|

(iii) MATERIAL PURCHASE BUDGET

<table>
<thead>
<tr>
<th>Materials</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Usage</td>
<td>124,900</td>
<td>129,500</td>
<td>98,800</td>
<td>56,640</td>
<td>35,225</td>
</tr>
<tr>
<td>Add: Closing Stock</td>
<td>18,000</td>
<td>27,000</td>
<td>13,500</td>
<td>4,500</td>
<td>8,100</td>
</tr>
<tr>
<td>Less: Opening Stock</td>
<td>142,900</td>
<td>156,500</td>
<td>112,300</td>
<td>61,140</td>
<td>43,325</td>
</tr>
</tbody>
</table>

| Purchase Quantity | 122,900| 126,500| 97,300| 56,140| 34,325|
| Price (₦)        | 0.55   | 0.50   | 0.35  | 0.60  | 0.80  |
| VALUE (₦)        | 67,595 | 63,250 | 34,055| 33,684| 27,460|

QUESTION 9

From the information given below prepare a cash budget for SINGA Nigeria Limited for the first six months of 2005. Assume a nil opening balance.
COSTING AND QUANTITATIVE TECHNIQUES

Sales | Purchases
---|---
Nov. 2004 | 150,000 | 70,000
Dec. 2004 | 160,000 | 80,000
Jan. 2005 | 180,000 | 80,000
Feb. 2005 | 160,000 | 90,000
Mar. 2005 | 160,000 | 70,000
Apr. 2005 | 120,000 | 70,000
May 2005 | 140,000 | 60,000
June 2005 | 180,000 | 80,000

(a) Sales are realized 40% in cash, two thirds of the balance after 30 days and the remainder after another 30 days.
(b) Purchases are paid for 50% immediately and the balance two months after.
(c) Office running cost of ₦22,000 monthly up to April 1998 and ₦24,000 thereafter are settled during the month.
(d) Computers are to be acquired in December 1997 for ₦45,000 for a three-month instalmental payment starting with delivery.
(e) A new generator costing ₦20,000 is to be purchased in March 1998 under a trade-off agreement on the existing one with a value of ₦12,000.
(f) Companies Income Tax of ₦28,000 payable in equal quarterly installments has been agreed with the Revenue Service first installment being due in January 1998.
(g) It is proposed to pay dividends to shareholders in May 1998 amounts due is ₦40,000.
(h) Annual rent on the company premises is due in June. The Estate Agents have written to inform the company that 20% increase on the annual rent of ₦60,000 is inevitable.
(i) The company expects to sell unserviceable equipment in February 1998 for ₦18,000. Show all workings.

SUGGESTED SOLUTION TO QUESTION 9

SINGA NIGERIA LIMITED
CASH BUDGET FOR JAN – JUNE 2005

<table>
<thead>
<tr>
<th>Receipts:</th>
<th>JAN</th>
<th>FEB</th>
<th>MAR</th>
<th>APR</th>
<th>MAY</th>
<th>JUNE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>166,000</td>
<td>168,000</td>
<td>164,000</td>
<td>144,000</td>
<td>136,000</td>
<td>152,000</td>
</tr>
<tr>
<td>Equipment sales</td>
<td>18,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>166,000</td>
<td>186,000</td>
<td>164,000</td>
<td>144,000</td>
<td>136,000</td>
<td>152,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Payments:</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases</td>
<td>75,000</td>
<td>85,000</td>
<td>75,000</td>
<td>80,000</td>
<td>65,000</td>
<td>75,000</td>
</tr>
<tr>
<td>Office costs</td>
<td>22,000</td>
<td>22,000</td>
<td>22,000</td>
<td>22,000</td>
<td>24,000</td>
<td>24,000</td>
</tr>
<tr>
<td>Computer purchase</td>
<td>15,000</td>
<td>15,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generator purchase</td>
<td></td>
<td></td>
<td>8,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income Tax</td>
<td>7,000</td>
<td></td>
<td></td>
<td>7,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

552
Dividends - - - - 40,000 - 
Rent - - - - - 72,000 
**TOTAL** 119,000 122,000 105,000 109,000 129,000 171,000 

Net Receipts: 47,000 64,000 59,000 35,000 7,000 (19,000)  
Opening bal. - 47,000 111,000 170,000 205,000 212,000  
Closing bal. 47,000 111,000 170,000 205,000 212,000 193,000  

**CALCULATION OF SALES PROCEEDS**

<table>
<thead>
<tr>
<th></th>
<th>Nov. '000</th>
<th>Dec. '000</th>
<th>Jan. '000</th>
<th>Feb. '000</th>
<th>March. '000</th>
<th>April. '000</th>
<th>May. '000</th>
<th>June. '000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov</td>
<td>150</td>
<td>60</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>40%</td>
<td></td>
<td>-</td>
<td>60</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20%</td>
<td></td>
<td></td>
<td>-</td>
<td>30</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dec</td>
<td>160</td>
<td></td>
<td>64</td>
<td>64</td>
<td>32</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Jan</td>
<td>180</td>
<td></td>
<td>-</td>
<td>72</td>
<td>72</td>
<td>36</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Feb</td>
<td>160</td>
<td></td>
<td>-</td>
<td>-</td>
<td>64</td>
<td>64</td>
<td>32</td>
<td>-</td>
</tr>
<tr>
<td>Mar</td>
<td>160</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>64</td>
<td>64</td>
<td>32</td>
</tr>
<tr>
<td>April</td>
<td>120</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>48</td>
<td>48</td>
<td>24</td>
</tr>
<tr>
<td>May</td>
<td>140</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>56</td>
<td>56</td>
</tr>
<tr>
<td>June</td>
<td>180</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>124</td>
<td>166</td>
<td>168</td>
<td>164</td>
<td>144</td>
<td>136</td>
<td>152</td>
</tr>
</tbody>
</table>

**CALCULATION OF PURCHASES PAYMENTS**

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>March</th>
<th>April</th>
<th>May</th>
<th>June</th>
<th>July</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nov</td>
<td>70</td>
<td>35</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dec</td>
<td>80</td>
<td>-</td>
<td>40</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Jan</td>
<td>80</td>
<td>40</td>
<td>-</td>
<td>40</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Feb</td>
<td>90</td>
<td>-</td>
<td>45</td>
<td>-</td>
<td>45</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>March</td>
<td>70</td>
<td>-</td>
<td>-</td>
<td>35</td>
<td>-</td>
<td>35</td>
<td>-</td>
</tr>
<tr>
<td>April</td>
<td>70</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>35</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>May</td>
<td>60</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>30</td>
<td>-</td>
<td>30</td>
</tr>
<tr>
<td>June</td>
<td>80</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>85</td>
<td>75</td>
<td>80</td>
<td>65</td>
<td>75</td>
<td>30</td>
</tr>
</tbody>
</table>

**QUESTION 10**

The Sales Director of Musa Nasarawa Box Fabricators has become aware of the disadvantages of static budget. The director asks you, as the Management Accountant to prepare a flexible budget for October, 2005 for its main brand of boxes.

The following data are available for the actual operation in September 2005.

- Boxes produced and sold: 4,500
- Direct Materials costs: 180,000
- Direct Manufacturing Labour costs: 135,000
- Depreciation and other fixed Manufacturing costs: 101,400
- Average selling price per box: 140
- Fixed marketing costs: 162,700
Assume no stock of boxes at the beginning or end of the period. A 10% increase in the selling price is expected in October. The only variable marketing cost is a commission of ₦0.50K per unit paid to the manufacturer’s representatives, who bear all their own costs of traveling, entertaining customers, etc. A patent royalty of ₦2 per box manufactured is paid to an independent design firm. Salary increases that will become effective in October are ₦12,000 per year for the production supervisor and ₦15,000 per year for Sales Manager.

A 10% increase in direct material prices is expected to become effective in October. No changes are expected in direct manufacturing labour wages rates or in the productivity of the direct manufacturing labour personnel.

The company uses a marginal costing system and does not have standard costs for any of its inputs.

You are required to:

Prepare a flexible budget for October 2005 showing budgeted amounts at each of these output levels of boxes, 4000 units, 5000 units and 6000 units.

(16 marks)

**SUGGESTED SOLUTION TO QUESTION 10**

**MUSA NASARAWA BOX FABRICATORS LIMITED**

**FLEXIBLE BUDGET FOR THE MONTH OF OCTOBER 2001**

<table>
<thead>
<tr>
<th>Activity Level</th>
<th>4,000 units</th>
<th>5,000 units</th>
<th>6,000 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (A)</td>
<td>₦616,000</td>
<td>₦770,000</td>
<td>₦924,000</td>
</tr>
<tr>
<td>Direct material cost</td>
<td>₦176,000</td>
<td>₦220,000</td>
<td>₦264,000</td>
</tr>
<tr>
<td>Direct labour cost</td>
<td>₦120,000</td>
<td>₦150,000</td>
<td>₦180,000</td>
</tr>
<tr>
<td>Marketing variable cost</td>
<td>₦2,000</td>
<td>₦2,500</td>
<td>₦3,000</td>
</tr>
<tr>
<td>Royalties</td>
<td>₦8,000</td>
<td>₦10,000</td>
<td>₦12,000</td>
</tr>
<tr>
<td>Variable costs (B)</td>
<td>₦306,000</td>
<td>₦382,500</td>
<td>₦459,000</td>
</tr>
<tr>
<td>Contribution C (A - B)</td>
<td>₦310,000</td>
<td>₦387,500</td>
<td>₦465,000</td>
</tr>
<tr>
<td>Less: Fixed costs (D)</td>
<td>₦266,350</td>
<td>₦266,350</td>
<td>₦266,350</td>
</tr>
<tr>
<td>NET PROFIT E (C - D)</td>
<td>₦43,650</td>
<td>₦121,150</td>
<td>₦198,650</td>
</tr>
</tbody>
</table>

**WORKINGS**

(i) **STATEMENT OF COST**

<table>
<thead>
<tr>
<th></th>
<th>September</th>
<th>October</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct material cost</td>
<td>40.00</td>
<td>44.00</td>
</tr>
<tr>
<td>Direct labour cost</td>
<td>30.00</td>
<td>30.00</td>
</tr>
<tr>
<td>Marketing variable cost</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Royalties</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Total variable cost</td>
<td>72.50</td>
<td>76.50</td>
</tr>
<tr>
<td>Fixed Cost - Depreciation</td>
<td>101,400</td>
<td>101,400</td>
</tr>
<tr>
<td>Marketing</td>
<td>162,700</td>
<td>162,700</td>
</tr>
<tr>
<td>Increase in salary - Production</td>
<td>-</td>
<td>1.000</td>
</tr>
<tr>
<td>Increase in salary - Marketing</td>
<td>-</td>
<td>1.250</td>
</tr>
<tr>
<td></td>
<td>264,100</td>
<td>266,350</td>
</tr>
<tr>
<td>Selling Price</td>
<td>140</td>
<td>154</td>
</tr>
</tbody>
</table>
**QUESTION 11**

As cost accountant at Memuna Block Industry Limited, you produced the following information relating to the department manufacturing concrete blocks for the three month period ended 31 December 2005:

<table>
<thead>
<tr>
<th></th>
<th>₦</th>
<th>₦</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales (217,600 units)</strong></td>
<td>272,000</td>
<td></td>
</tr>
<tr>
<td><strong>Direct costs:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Materials</td>
<td>89,000</td>
<td></td>
</tr>
<tr>
<td>Labour</td>
<td>62,000</td>
<td>151,000</td>
</tr>
<tr>
<td><strong>Total direct costs:</strong></td>
<td>151,000</td>
<td></td>
</tr>
<tr>
<td><strong>Variable overheads:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Production</td>
<td>14,000</td>
<td></td>
</tr>
<tr>
<td>Selling</td>
<td>12,000</td>
<td>26,000</td>
</tr>
<tr>
<td><strong>Total variable overheads</strong></td>
<td>26,000</td>
<td></td>
</tr>
<tr>
<td><strong>Fixed overheads:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Production</td>
<td>16,000</td>
<td></td>
</tr>
<tr>
<td>Selling</td>
<td>10,000</td>
<td>26,000</td>
</tr>
<tr>
<td><strong>Total fixed overheads:</strong></td>
<td>26,000</td>
<td></td>
</tr>
<tr>
<td><strong>Total costs</strong></td>
<td>203,000</td>
<td></td>
</tr>
<tr>
<td><strong>Net profit</strong></td>
<td></td>
<td>69,000</td>
</tr>
</tbody>
</table>

You are informed that in the quarter ended 31 March 2006 direct labour rates are likely to rise by ten percent.

You are required to calculate:

(a)  (i) The breakeven point as revealed by the accounts for the three months ended 31 December 2005; and
     (ii) the additional sales required for the three months ended 31 March 2006 to maintain the same amount of net profit as in the three months ended 31 December 2005, on the assumption that selling price will not be raised.

(b) Demonstrate, with the aid of graph, the manner in which breakeven analysis could be used by Progress Block Ltd to show the effect of changes in costs and revenues; and Biliyaminu Aminu

(c) Comment on the limitations of breakeven analysis.

SUGGESTED SOLUTION TO QUESTION 11

**Memuna Block Industry Ltd.**

(a)  (i) Break even point (BEP)

In units, **BEP** = \( \frac{\text{Fixed costs}}{\text{Contribution per unit}} \)

Contribution per unit = \( \frac{\text{Total contribution}}{\text{Total units}} \)

\[
= \frac{\text{₦}(272,000 - 151,000 - 26,000)}{217,600 \text{ units}}
\]

= è1.115

\( \text{BEP} = \frac{\text{₦}69,000}{\text{₦1.115}} \)

= 61,852.63 units
Therefore, BEP = \frac{26,000}{43.658 \text{ k per unit}} = 59,553.7 \text{ units i.e. } 59,554 \text{ units}

(ii) Additional sales
New contribution per unit = \frac{N(95,000 - 6,200)}{217,600} = 0.408

For a profit of N69,000 or a contribution of N95,000; sales must rise to: \frac{95,000}{0.408} = 232,843 \text{ units}

This represents additional sales of 15,243 units.

(b) Break even charts
The type of graph which might be used by Dample Limited to show the effects of changes in costs and revenues (or perhaps more appropriately changes in unit costs and selling prices as in (a) (ii) above) is a break even chart (though a PV graph would have the same effect). Here a series of cost or revenue lines could be shown corresponding to various unit costs or selling prices. The chart below illustrates the type of problem suggested in (a) above but using a 50% increase in labour rates.

The graph shows that the break even point moves when the variable cost rises. Any alteration in selling price, variable cost or fixed overheads can be catered for by plotting the new cost or revenue line on the graph. At existing prices the revenue, costs and profit at any level of activity can be read from the graph.
(c) **The limitations of breakeven analysis**

The use of simple break even analysis assumes that all costs can be split into fixed and variable components and furthermore that one ‘activity base’ (units produced, units sold, hours worked) will be suitable for describing all variable costs and revenues.

Linear relationships are assumed. This means that fixed costs are assumed to be fixed at all levels of activity, and that variable cost per unit and sales price per unit are constants and independent of the level of activity.

In practice, fixed costs will only be constant over the ‘relevant range’. Over a larger range most fixed costs are stepped. If necessary, this can be adjusted for on the break even chart.

It is more difficult to show how variable costs will behave at various activity levels, but the cost accountant’s linear model is bound to be incorrect over a wide range. For example, as activity increases variable cost may fall because it is possible to buy raw materials cheaper in bulk, or because of increases in worker efficiency. On the other hand, scarcity of resources at high level of activity may lead to higher variable costs.

The linear relationship used for revenue totally ignores the sales price/demand relationship for a product. The market may not be perfect, and in order to increase sales of the product it may be necessary to reduce its price.

Simple break even analysis also assumes that there is a constant sales mix, or that sales of only one product are being considered. Furthermore, it assumes that there are no changes in stock levels and that units produced equals units sold. This last point will not matter if stock is valued at variable cost, but if an absorption costing system is in use it becomes more difficult to predict profits at different levels of activity if stock levels are changing.

Simple break even analysis is therefore most useful when predictions are made within the range of the company’s normal activity and when there no significant building up or running down of stocks.
QUESTION 12

Vokito Plc is comparing budget and actual data for the last three months.

<table>
<thead>
<tr>
<th></th>
<th>Budget</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Sales</td>
<td>950,000</td>
<td>922,500</td>
</tr>
<tr>
<td>Cost of sales</td>
<td>133,000</td>
<td>133,000</td>
</tr>
<tr>
<td>Raw materials</td>
<td>152,000</td>
<td>150,500</td>
</tr>
<tr>
<td>Direct labour</td>
<td>100,700</td>
<td>96,300</td>
</tr>
<tr>
<td>Variable production</td>
<td>125,400</td>
<td>115,300</td>
</tr>
<tr>
<td>overheads</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed production</td>
<td>511,100</td>
<td>495,100</td>
</tr>
<tr>
<td>overheads</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td>438,900</td>
<td>427,400</td>
</tr>
</tbody>
</table>

The budget was prepared on the basis of 95,000 units produced and sold, and actual production and sales for the three-month period were 90,000 units. Volto uses standard costing and absorbs fixed production overheads on a machine hour basis. A total of 28,500 standard machine hours were budgeted. A total of 27,200 machine hours were actually used in the three-month period.

Required

(a) Prepare a revised budget at the new level of activity using a flexible budgeting approach. (4 marks)

(b) Calculate the following
(i) raw material total cost variance;
(ii) direct labour total cost variance
(iii) fixed overhead efficiency variance;
(iv) fixed overhead expenditure variance;

(c) Suggest possible explanations for the following variances;
(i) raw materials total cost variance;
(ii) fixed overhead efficiency variance;
(iii) fixed overhead expenditure variance;

(d) Explain three key purposes of a budgeting system.

SUGGESTED SOLUTION TO QUESTION 12

**VOKITO PLC**

(a) The flexed budget will be based on the actual activity level of 90,000 units.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales: N950,000 x 90/95</td>
<td>900,000</td>
</tr>
<tr>
<td>Cost of sales</td>
<td></td>
</tr>
<tr>
<td>Raw materials: 133,000 x 90/95</td>
<td>126,000</td>
</tr>
<tr>
<td>Direct labour: 152,000 x 90/95</td>
<td>144,000</td>
</tr>
<tr>
<td>Variable production overheads: 100,700 x 90/95</td>
<td>95,400</td>
</tr>
<tr>
<td>Fixed production overheads: 125,400</td>
<td>490,800</td>
</tr>
<tr>
<td></td>
<td>409,200</td>
</tr>
</tbody>
</table>
(b) Raw materials cost total variance = 126,000 – 130,500 = N4,500 A
Direct labour cost variance = 144,000 – 153,000 = N9,000 A

Fixed overhead absorption rate = 125,400/28,500 = N4.40/ machine hour
Standard machine hours for actual production = 28,500 x 90/95 = 27,000

Standard fixed overhead (actual production) = 27,000 x 4.4 = N118,800
Fixed overhead absorbed on actual hours = 27,200 x 4.4 = N119,680
Fixed overhead efficiency variance = 118,800 – 119,680 = N880 A

Fixed overhead absorbed on actual hours = 27,200 x 4.4 = N119,680
Fixed overhead absorbed on budgeted hours = 28,500 x 4.4 = N125,400
Fixed overhead capacity variance = 119,680 – 125,400 = N5,720 A

Budgeted overhead expenditure = N125,400
Actual overhead expenditure = N115,300
Fixed overhead expenditure variance = 125,400 – 115,300 = N10,100 F

(c) Raw materials cost variance
The budgeted raw material cost for production of 95,000 units was N1.40 per unit (133,000/95,000) but the actual raw material cost for production of 90,000 units was N1.45 per unit (130,500/90,000). The raw material cost per unit may have increased either because more raw material per unit was used than budgeted, or because the price per unit of raw material was higher than budgeted. Calculation of the raw material price and usage sub-variances would indicate where further explanations should be sought.

Fixed overhead efficiency variance
The fixed overhead efficiency variance measures the extent to which more or less standard hours were used for the actual production than budgeted. In this case, a total of 27,200 machine hours were actually used, when only 27,000 standard machine hours should have been used. The difference may be due to poorer production planning than expected or to machine break downs.

Fixed overhead expenditure
The fixed overhead expenditure variance measures the extent to which budgeted fixed overhead differs from actual fixed overhead. Here, actual fixed overhead is N10,100 less than budgeted. This could be an error in forecasting fixed production overheads such as rent and power costs, or to a decrease in fixed production overheads, such as changing to a cheaper cleaning contractor.

(d) Key purposes of a budgeting system that could be discussed include planning, co-ordination, communication, control, motivation and performance evaluation. Students were required only to discuss three key purposes.

Planning
One of the key purposes of a budgeting system is to require planning to occur. Strategic planning cover several years but a budget represent a financial plan covering a shorter period, i.e. a budget is an operational plan. Planning helps an organisation to anticipate key changes in the business environment that could potentially impact on business activities and to prepare appropriate
responses. Planning also ensures that the budgeted activities of the organisation will support the attainment of the organisation’s objectives.

Co-ordination
Many organisations undertake a number of activities which need to be co-ordinated if the organisation is to meet its objectives. The budgeting system facilitates this co-ordination since organisational activities and the links between them are thoroughly investigated during budget preparation, and the overall coherence between the budgeted activities is reviewed before the master budget is agreed by senior managers. Without the framework of the budgeting system, individual managers may be tempted to make decisions that are not optimal in terms of achieving organisational objectives.

Communication
The budgeting system facilitates communication within the organisation both vertically (for example between senior and junior managers) and horizontally (for example between different organisational functions). Vertical communication enables senior managers to ensure that organisational objectives are understood by employees at all levels. Communication also occurs at all stages of the budgetary control process, for example during preparation and during investigation of end-of-period variances.

Control
One of the most important purposes of a budgeting system is to control through the comparison of budgeted costs and actual costs. Variances between budgeted and actual costs can be investigated in order to determine the reason why actual performance has deviated from what was planned. Corrective action can be introduced, if necessary, in order to ensure that organisational objectives are achieved. A budgeting system also facilitates management by exception, whereby only significant differences between planned and actual activities are investigated.

Motivation
The budgeting system can influence the behaviour of managers and employees, and may motivate them to improve their managerial performance if the target represented by the budget is set at an appropriate level. An inappropriate target has the potential to be demotivating, however, and a key factor here is the degree of participation in the budget-setting process. It has been shown that an appropriate degree of participation can have a positive motivational effect.

Performance evaluation
Managerial performance is often evaluated by the extent to which budgetary targets for which individual managers are responsible has been achieved. Managerial rewards such as bonuses or performance-related pay can also be linked to achievement of budgetary targets. Managers can also use the budget to evaluate their own performance and clarify how close they are to meeting agreed targets.
QUESTION 13

The standard cost card of a process for the month of January showed the following:

Material input

<table>
<thead>
<tr>
<th>Material</th>
<th>Units</th>
<th>Cost per Unit</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
<td>$60</td>
<td>$1,800</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>$40</td>
<td>$1,000</td>
</tr>
<tr>
<td>C</td>
<td>45</td>
<td>$50</td>
<td>$2,250</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td>$5,050</td>
</tr>
</tbody>
</table>

Normal loss 10 units

During the month, the following transactions actually took place:

<table>
<thead>
<tr>
<th>Material</th>
<th>Units</th>
<th>Cost per Unit</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>135</td>
<td>$65</td>
<td>$8,775</td>
</tr>
<tr>
<td>B</td>
<td>130</td>
<td>$35</td>
<td>$4,550</td>
</tr>
<tr>
<td>C</td>
<td>235</td>
<td>$45</td>
<td>$10,575</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td></td>
<td>$23,900</td>
</tr>
</tbody>
</table>

The output for the period is 480 units.

From the above information, you are required to compute the following material variances:

(a) Price Variance
(b) Mixture Variance
(c) Yield Variance

SUGGESTED SOLUTION TO QUESTION 13

(a) Price Variance

<table>
<thead>
<tr>
<th>Product</th>
<th>Units</th>
<th>Cost per Unit</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>135</td>
<td>$60 - $65</td>
<td>$675(A)</td>
</tr>
<tr>
<td>B</td>
<td>130</td>
<td>$40 - $35</td>
<td>$650(F)</td>
</tr>
<tr>
<td>C</td>
<td>235</td>
<td>$50 - $45</td>
<td>$1,175(F)</td>
</tr>
</tbody>
</table>

(b) Mixture Variance

Standard price of the standard mixture

\[
\text{\$5,050} \times \frac{500}{100} = \text{\$25,250}
\]

Standard price of actual mixture

<table>
<thead>
<tr>
<th>Material</th>
<th>Units</th>
<th>Cost per Unit</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>135</td>
<td>$60</td>
<td>$8,100</td>
</tr>
<tr>
<td>B</td>
<td>130</td>
<td>$40</td>
<td>$5,200</td>
</tr>
<tr>
<td>C</td>
<td>125</td>
<td>$50</td>
<td>$6,250</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td></td>
<td>$19,550</td>
</tr>
</tbody>
</table>

(c) Yield Variance

Expected yield is 90% of input

Expected output from 500 units (90% of 500) = (450)

Actual output 480

Yield variance (units) 30 units

Yield variance in $ = \$5,050 \times 30

\[\frac{90}{90} = \text{\$1,683(F)}\]
QUESTION 14

FALOLA Ventures Limited manufactures plastic buckets and projects sale of 120,000 units at N\$25.00 each in the coming year.

To produce each bucket, the company needs two units of material A and one unit each of B and C. The cost per unit of materials is

- A - N\$4.00;
- B - N\$3.50; and
- C - N\$4.50

The following additional information is relevant:
(a) In an eight-hour day, the machinists are expected to collectively produce 240 buckets, and their joint wages for a normal week of five working days is N\$2,400.00.
(b) the factory supervisor is on an annual salary of N\$180,000.00
(c) direct expenses is projected at 50 kobo per bucket.
(d) overall fixed costs are N\$210,000.00 per year.

The company received a special order for plastic chairs to be delivered within the same 12 months period under the following terms. 2000 units per month at N\$22.00 per unit. To fulfill this additional order:
(i) One unit each of materials A and B and two units of C are needed for the manufacture of a chair.
(ii) The machinists would have to work for four Saturdays to fulfill the order for a month. They are to be paid on double rate.
(iii) Because of the increased quantity of materials now needed, the company would enjoy 5% discount on all purchases.
(iv) Direct expenses for the special order is N\$5.00 per unit.
(v) The company needs to construct a new mould for the chairs at a cost of N\$100,000 but it could be sold for N\$60,000.00 after completing the order.

You are required to prepare a columnar costs and profit statement for the original projection, the new total projections and the difference. Show all workings. 

(20 marks)

SUGGESTED SOLUTION TO QUESTION 14

FALOLA VENTURES LIMITED
PROJECT RESULTS

<table>
<thead>
<tr>
<th></th>
<th>Normal Operations</th>
<th>With Additional Order</th>
<th>Additional Revenue/Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>3,000,000</td>
<td>3,528,000</td>
<td>528,000</td>
</tr>
<tr>
<td>Direct Material</td>
<td>1,920,000</td>
<td>2,200,200</td>
<td>280,200</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>420,000</td>
<td>466,080</td>
<td>46,080</td>
</tr>
<tr>
<td>Direct Exp.</td>
<td>60,000</td>
<td>180,000</td>
<td>120,000</td>
</tr>
<tr>
<td></td>
<td>2,400,000</td>
<td>2,846,280</td>
<td>446,280</td>
</tr>
<tr>
<td>Contribution</td>
<td>600,000</td>
<td>681,720</td>
<td>81,720</td>
</tr>
<tr>
<td>Fixed Costs</td>
<td>210,000</td>
<td>250,000</td>
<td>40,000</td>
</tr>
<tr>
<td>Profit</td>
<td>390,000</td>
<td>431,720</td>
<td>41,720</td>
</tr>
</tbody>
</table>
WORKINGS

<table>
<thead>
<tr>
<th></th>
<th>Normal Operations N</th>
<th>With Additional Order N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>120,000 x 25</td>
<td>3,000,000</td>
</tr>
<tr>
<td>Special Order</td>
<td>2,000 x 22 x 12</td>
<td>528,000</td>
</tr>
<tr>
<td>Direct Materials</td>
<td>3,000,000</td>
<td>3,528,000</td>
</tr>
<tr>
<td>Normal</td>
<td>16 x 120,000</td>
<td>1,920,000</td>
</tr>
<tr>
<td></td>
<td>(4.00 + 3.50) +</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.00 x 4.50)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 16.50 x 24,000</td>
<td>396,000</td>
</tr>
<tr>
<td></td>
<td>Less: 5% discount</td>
<td>115,800</td>
</tr>
<tr>
<td>Direct Labour</td>
<td>(2,400/5/240) x 120,000</td>
<td>240,000</td>
</tr>
<tr>
<td>Normal</td>
<td>Factory Supervisor</td>
<td>180,000</td>
</tr>
<tr>
<td></td>
<td>420,000</td>
<td>420,000</td>
</tr>
<tr>
<td>Special Order</td>
<td>Daily Rate 480</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Saturday x 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>960 x 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 3,840 x 12</td>
<td>46,080</td>
</tr>
<tr>
<td>Direct Expenses</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>(120,000 x 50k)</td>
<td>60,000</td>
</tr>
<tr>
<td>Special Order</td>
<td>- 24,000 x 5</td>
<td>120,000</td>
</tr>
<tr>
<td></td>
<td>60,000</td>
<td>180,000</td>
</tr>
<tr>
<td>Fixed Costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>210,000</td>
<td>210,000</td>
</tr>
<tr>
<td>Special Order</td>
<td>(New Mould</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100,000 - 60,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40,000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>210,000</td>
<td>250,000</td>
</tr>
</tbody>
</table>

QUESTION 15

Alaka Abayomi Plc manufactures and sells a single product. The following data have been extracted from the current year’s budget:

- Contribution per unit N8
- Total weekly fixed costs N10,000
- Weekly profit N22,000
- Contribution to sales ratio 40%

The company’s production capacity is not being fully utilized in the current year and three possible strategies are under consideration. Each strategy involves reducing the unit selling price on all units sold with a consequential effect on the budgeted volume of sales. Details of each strategy are as follows:
COSTING AND QUANTITATIVE TECHNIQUES

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Reduction in unit selling price</th>
<th>Expected increase in weekly sales volume over budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>C</td>
<td>7</td>
<td>25</td>
</tr>
</tbody>
</table>

The company does not hold stock of finished goods.

Required:
(a) Calculate for the current year:
(i) the selling price per unit for the product; and
(ii) the weekly sales (in units).  

(b) Determine, with supporting calculations, which one of the three strategies should be adopted by the company in order to maximize weekly profits.

(c) Briefly explain the practical problems that a management accountant might encounter in separating costs.

SUGGESTED SOLUTION TO QUESTION 15

ALAKA ABAYOMI PLC

(a) (i) Selling price per unit = N8/0.40 = N20
(ii) Weekly contribution = 10,000 + 22,000 = N32,000
    Weekly sales = 32,000/8 = 4,000 units

(b) Strategy  | A  | B  | C
Units        | 4,400 | 4,720 | 5,000
N/unit       | 19.60 | 19.00 | 18.60
Less Variable cost | (12.00) | (12.00) | (12.00)
Contribution | 7.60  | 7.00  | 6.60
N            |     |     |     
Total contribution | 33,440 | 33,040 | 33,000

Contribution and therefore profit is maximised when A is adopted.

(c) Some costs do not fall clearly into being either variable or fixed. They are the costs that are a mix of variable and fixed – sometimes called semi-variable or mixed costs.

The following techniques could be used to separate the fixed and variable components of semi-variable or mixed costs:

| ♦ The high and low method |
| ♦ Linear regression |

Many costs are a mix of variable and fixed elements, for example power costs (gas or electricity). The tariffs for power costs often consist of a fixed charge irrespective of the amount of power consumed and a variable charge per unit of consumption.
QUESTION 16

The following table shows the number (’000) of cheques per annum cleared in Ibadan Bankers Clearing House between 1970 and 1994.

No of cheques cleared (’000)

<table>
<thead>
<tr>
<th>No of cheques cleared (’000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>356 409 453 469 549 560</td>
</tr>
<tr>
<td>529 501 437 476 522 495</td>
</tr>
<tr>
<td>454 470 550 597 640 628</td>
</tr>
<tr>
<td>669 641 764 764 671 687</td>
</tr>
</tbody>
</table>

Source: CBN Statistical Bulletin Volume 6, No 2 December 1995

(a) Use the intervals of 300 – 399, 400 – 499 etc to construct the frequency distribution of the data.
(b) Indicate on a horizontal line the class boundaries and the class mark.
(c) Plot a histogram of the data.
(d) Construct the frequency polygon of the data.
   (i) Construct a table of cumulative frequency
   (ii) Plot the ogive
   (iii) Find the probability that the number of cheques that were cleared in 1975 is at least 500,000

SUGGESTED SOLUTION QUESTION 17

<table>
<thead>
<tr>
<th>Interval</th>
<th>Tally</th>
<th>Frequency</th>
<th>Class mark</th>
<th>Class boundaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 – 399</td>
<td>1</td>
<td>1</td>
<td>349.5</td>
<td>299.5 – 399.5</td>
</tr>
<tr>
<td>400 – 499</td>
<td>8</td>
<td>8</td>
<td>449.5</td>
<td>399.5 – 499.5</td>
</tr>
<tr>
<td>500 – 599</td>
<td>7</td>
<td>7</td>
<td>549.5</td>
<td>499.5 – 599.5</td>
</tr>
<tr>
<td>600 – 699</td>
<td>6</td>
<td>6</td>
<td>649.5</td>
<td>599.5 – 699.5</td>
</tr>
<tr>
<td>700 – 799</td>
<td>2</td>
<td>2</td>
<td>749.5</td>
<td>699.5 – 799.5</td>
</tr>
</tbody>
</table>

(b) Class mark

<table>
<thead>
<tr>
<th>Class mark</th>
<th>Class boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>z</td>
</tr>
</tbody>
</table>
A Histogram (c). (i)

Mode = 484.5

(d). (i)

<table>
<thead>
<tr>
<th>Interval</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
<th>Class boundaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 – 399</td>
<td>1</td>
<td>1</td>
<td>299.5 – 399.5</td>
</tr>
<tr>
<td>400 – 499</td>
<td>8</td>
<td>9</td>
<td>399.5 – 499.5</td>
</tr>
<tr>
<td>500 – 599</td>
<td>7</td>
<td>16</td>
<td>499.5 – 599.5</td>
</tr>
<tr>
<td>600 – 699</td>
<td>6</td>
<td>22</td>
<td>599.5 – 699.5</td>
</tr>
<tr>
<td>700 – 799</td>
<td>2</td>
<td>24</td>
<td>699.5 – 799.5</td>
</tr>
</tbody>
</table>

24
(ii)

- The first quartile = 6\textsuperscript{th} item in the distribution i.e (25\% of 24)
  \[ Q_1 = 6 \Rightarrow Q_1 = 469.5 \text{ (from the ogive)} \]
- The second quartile = 12\textsuperscript{th} item in the distribution (50\% of 24)
  \[ Q_2 = 12 \Rightarrow Q_2 = 574.5 \text{ (from the ogive)} \]
- The third quartile = 18\textsuperscript{th} item in the distribution (75\% of 24)
  \[ Q_3 = 18 \Rightarrow Q_3 = 629.5 \text{ (from the ogive)} \]

\[ \text{d) iii. Number of years with less than 500,000 cheques cleared = 9.25} \]
\[ \text{from ogive is shown as A).} \]
\[ \text{Number of years with more than 500,000 cheques} = 24 - 9.25 = 14.75 \]
\[ \therefore P(\text{More than 500,000 cheques are closed in a year} = \dfrac{14.75}{24} = 0.61} \]

\[ \text{a) Let the assumed mean, } \bar{x} = 549.5, \text{ scaling factor, } c = 100 \]

<table>
<thead>
<tr>
<th>Intervals</th>
<th>( x )</th>
<th>( f )</th>
<th>( d = x - A )</th>
<th>( h = d/c )</th>
<th>( fh )</th>
<th>( ft )</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 – 399</td>
<td>349.5</td>
<td>1</td>
<td>-200</td>
<td>-2</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>400 – 499</td>
<td>449.5</td>
<td>8</td>
<td>-100</td>
<td>-1</td>
<td>-8</td>
<td>8</td>
</tr>
<tr>
<td>500 – 599</td>
<td>549.5</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>600 – 699</td>
<td>649.5</td>
<td>6</td>
<td>100</td>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>700 – 799</td>
<td>749.5</td>
<td>2</td>
<td>200</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \bar{x} = A + c \times \sum fh )</td>
<td>( = 49.5 + )</td>
<td>( \sum f = 26 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\sigma = \sqrt{\frac{\sum fh^2 - \frac{\sum fh^2}{\sum f}}{\sum f}} \times c^2
\]
\[
= \frac{66 - 60}{24} \times 90^2 = 0.08333 \times 9000
\]
\[
= 0.8333
\]
\[
\text{Standard deviation} = \sqrt{\sigma}
\]
\[
= \sqrt{0.8333}
\]
\[
\therefore \text{Standard deviation} = 0.0408
\]
\[
\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100
\]
\[
= \frac{0.0408}{549.5} \times 100
\]
\[
= 8.94\%
\]

b) Using Pearson’s method of finding coefficient of skewness, we have;
\[
\text{Coefficient of skewness} = \frac{\text{mean} - \text{mode}}{\text{standard deviation}}
\]
\[
= \frac{549.5 - 84.5}{104.08}
\]
\[
= \frac{65}{104.08}
\]
\[
\Rightarrow \text{Coefficient of skewness} = 1.62
\]

**Comment:** The coefficient of skewness above shows that the data is positively skewed. That is, it is not symmetric. It is a small positive skewness.

**QUESTION 17**

(a) The following Table shows the annual turnover of three manufacturers A, B and C of competitive products.

<table>
<thead>
<tr>
<th>Year</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>25</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>2002</td>
<td>40</td>
<td>45</td>
<td>42</td>
</tr>
<tr>
<td>2003</td>
<td>65</td>
<td>55</td>
<td>60</td>
</tr>
<tr>
<td>2004</td>
<td>80</td>
<td>85</td>
<td>75</td>
</tr>
</tbody>
</table>
**Draw:**

(i) A pie chart of the annual turnover of Company B

(ii) A simple bar chart of the annual turnover of Company A

(iii) A compound bar chart of the annual turnover of the three Companies with Company bars side by side.

(iv) A component bar chart of the annual turnover of the companies with the company bar above the other.

(b) The following table shows the distribution of net output of a sample of small and medium sized enterprises (SME’s) by size and number of establishments in 2003.

<table>
<thead>
<tr>
<th>Size</th>
<th>No of establishment</th>
<th>Net output (000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 10</td>
<td>45</td>
<td>2,000</td>
</tr>
<tr>
<td>10 but Less than 20</td>
<td>40</td>
<td>2,500</td>
</tr>
<tr>
<td>20 but Less than 30</td>
<td>35</td>
<td>3,350</td>
</tr>
<tr>
<td>30 but Less than 40</td>
<td>20</td>
<td>3,125</td>
</tr>
<tr>
<td>40 but Less than 50</td>
<td>18</td>
<td>2,400</td>
</tr>
<tr>
<td>50 but Less than 60</td>
<td>14</td>
<td>4,225</td>
</tr>
<tr>
<td>60 but Less than 70</td>
<td>21</td>
<td>18,385</td>
</tr>
</tbody>
</table>

Analyse the data by means of a Lorenz curve.

**SUGGESTED SOLUTION TO QUESTION 17**

(a).

<table>
<thead>
<tr>
<th>Year</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>25</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>2002</td>
<td>40</td>
<td>45</td>
<td>42</td>
</tr>
<tr>
<td>2003</td>
<td>65</td>
<td>55</td>
<td>60</td>
</tr>
<tr>
<td>2004</td>
<td>80</td>
<td>85</td>
<td>75</td>
</tr>
<tr>
<td><strong>210</strong></td>
<td><strong>200</strong></td>
<td><strong>197</strong></td>
<td></td>
</tr>
</tbody>
</table>

For company B:

2001: \( \frac{15}{200} \times 60^\circ = 7^\circ \)

2002: \( \frac{45}{200} \times 60^\circ = 1^\circ \)

2003: \( \frac{55}{200} \times 60^\circ = 9^\circ \)

2004: \( \frac{85}{200} \times 60^\circ = 53^\circ \)
(i) Pie chart

(ii) A simple bar chart

(iii) A compound bar chart of companies A, B, C

(v) A component bar chart of companies A, B, C
(b)

<table>
<thead>
<tr>
<th>Size (personnel)</th>
<th>No. of establishments</th>
<th>Net output (₦'000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 10</td>
<td>45</td>
<td>2000</td>
</tr>
<tr>
<td>10 but less than 20</td>
<td>40</td>
<td>2500</td>
</tr>
<tr>
<td>20 but less than 30</td>
<td>35</td>
<td>3350</td>
</tr>
<tr>
<td>30 but less than 40</td>
<td>20</td>
<td>3125</td>
</tr>
<tr>
<td>40 but less than 50</td>
<td>18</td>
<td>2400</td>
</tr>
<tr>
<td>50 but less than 60</td>
<td>14</td>
<td>4225</td>
</tr>
<tr>
<td>60 but less than 70</td>
<td>21</td>
<td>18385</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No. of establishment</th>
<th>Cumulative total</th>
<th>Cumulative percent</th>
<th>Net output</th>
<th>Cumulative total</th>
<th>Cumulative percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>45</td>
<td>23.3</td>
<td>2000</td>
<td>2000</td>
<td>5.6</td>
</tr>
<tr>
<td>40</td>
<td>85</td>
<td>44.0</td>
<td>2500</td>
<td>4500</td>
<td>12.5</td>
</tr>
<tr>
<td>35</td>
<td>120</td>
<td>62.2</td>
<td>3350</td>
<td>7860</td>
<td>21.8</td>
</tr>
<tr>
<td>20</td>
<td>140</td>
<td>72.5</td>
<td>3125</td>
<td>10975</td>
<td>30.5</td>
</tr>
<tr>
<td>18</td>
<td>158</td>
<td>81.9</td>
<td>2400</td>
<td>13375</td>
<td>37.2</td>
</tr>
<tr>
<td>14</td>
<td>172</td>
<td>89.1</td>
<td>4225</td>
<td>17600</td>
<td>48.9</td>
</tr>
<tr>
<td>21</td>
<td>193</td>
<td>100</td>
<td>18385</td>
<td>35985</td>
<td>100</td>
</tr>
</tbody>
</table>

Lorentz curve: Net output of SME’s
QUESTION 18

(a) Explain each of the following concepts in Probability Theory.
   (i) Random experiment
   (ii) Sample Space of a random experiment.
   (iii) Mutually Exclusive events
   (iv) Independent events

(b) Some distribution outlets of Nigerian Brewery Products were classified by the level of sales for the month of January 2005 as High, Medium or Low as well as the size of labour force in each outlet.

The following table shows the results:

<table>
<thead>
<tr>
<th>Level of Sales</th>
<th>Size of labour force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>----------------</td>
<td>------</td>
</tr>
<tr>
<td>5-14</td>
<td>5</td>
</tr>
<tr>
<td>15-24</td>
<td>8</td>
</tr>
<tr>
<td>25-34</td>
<td>13</td>
</tr>
</tbody>
</table>

A distributor is chosen at random from the group. Compute the probability that in the month of January 2005:
   (i) Its level of sales is low;
   (ii) It employed between 25 – 34 people;
   (iii) Its level of sales is high and it employed 15-24 people;
   (iv) Its level of sales is low or it employed 5-14 people; and
(v) Its level of sales is medium given that it employed 15-24 people.
(c) A prominent distribution outlet within the group of distributors in (b) above has observed that on the average it makes high sales 25% of the months, compute that out of the three months July, August and September 2005, high sales will be recorded in (i) July only (ii) August and September only (iii) exactly 2 months (iv) all the 3 months (v) none of the 3 months (vi) at least one month.

**SUGGESTED SOLUTION TO QUESTION 18**

(a) (i) **Random Experiment:**
A random experiment is an experiment whose outcome cannot be predicted in advance. In other words, it is an experiment whose result cannot be predetermined.

(ii) **Sample Space of a Random Experiment:**
A Sample space $S$, is defined as the set of all possible outcomes of a random experiment.

(iii) **Mutually Exclusive Events:**
Two events $A$ and $B$ are mutually exclusive if the occurrence of one precludes (prevents) the occurrence of the other. That is, both events cannot occur at the same time.

(iv) **Independent events**
Two events $A$ and $B$ are independent if the occurrence of one does not affect the occurrence of the other. Or $P(A \cap B) = P(A)P(B)$.

(b) **level of sales**

<table>
<thead>
<tr>
<th></th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-14</td>
<td>5</td>
<td>18</td>
<td>22</td>
<td>45</td>
</tr>
<tr>
<td>15-24</td>
<td>8</td>
<td>12</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>25-34</td>
<td>13</td>
<td>15</td>
<td>12</td>
<td>40</td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
<td>45</td>
<td>39</td>
<td>110</td>
</tr>
</tbody>
</table>

(i) $P(\text{low level}) = \frac{\text{39}}{\text{110}} = 0.35$

(ii) $P(\text{25 -34 people were employed}) = \frac{\text{40}}{\text{110}} = 0.36$

(iii) $P(\text{high level } \& \ (15 -24)) = \frac{\text{8}}{\text{110}} = 0.07$

(iv) $P(\text{low level or (5 - 14) employed}) = P(\text{low level}) + P(\text{(5 - 14) employed}) - P(\text{low and 5 - 14}) = \frac{\text{39}}{\text{110}} + \frac{\text{45}}{\text{110}} - \frac{\text{22}}{\text{110}} = \frac{\text{62}}{\text{110}} = 0.56$

(v) $P(\text{Medium level } / (15 -24) \text{ employed}) = \frac{P[\text{medium and (15 -24)}]}{P[\text{(15 -24) employed}]}$
(c) Let $H =$ High scale, $P(H) = 0.25 => P(H) = 0.75$

(i). July Aug Sept $P(H) P(H) P(H)$ $(0.25) \times (0.75) \times (0.75) = 0.1406$

(ii). July Aug Sept $P(H) P(H) P(H)$ $(0.75) \times (0.25) \times (0.25) = 0.0469$

(iii). July Aug Sept $P(H) P(H) P(H)$ $P(H) P(H) P(H)$ $P(H) P(H) P(H)$ $(0.25)^3 \times (0.75) \times 3 = 0.1406$

(iv). $P(H) P(H) P(H) = (0.25)^3 = 0.0156$

(v). $P(H) P(H) P(H) = (0.75)^3 = 0.4219$

(vi). $P(\text{at least one}) = 1 - P(\text{None}) = 1 - 0.4219 = 0.5781$

**QUESTION 19**

(a) Monthly economic environment of the commercial areas of Apapa in Lagos can be classified as Excellent (E), Very good (VG), Good (G), Normal (N) and poor (P).

The following table shows the number (in percentage) of month for which each category of economic environment has been experienced in recent times and the net profit recorded by BANJO trading enterprises over the same period of time.

<table>
<thead>
<tr>
<th>Environment</th>
<th>E</th>
<th>VG</th>
<th>G</th>
<th>N</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of time</td>
<td>15</td>
<td>20</td>
<td>35</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Net profit (X) of Banjo (Nm)</td>
<td>10</td>
<td>12</td>
<td>6</td>
<td>-2</td>
<td>-15</td>
</tr>
</tbody>
</table>

(i) Find $E(X)$  (ii) $E(X^2)$  (iii) Var(X)  (iv) Coefficient of variation of (X).

(b) Out of 40 randomly selected banks in the country, 12 of them were classified by CBN as having high liquidity ratio.

(i) Can we conclude that by the CBN classification, about 17% of the banks in the country have high liquidity ratio?

(ii) Construct a 95% confidence interval for the proportion of all banks in the country with high liquidity ratio.

(c) The capacity utilization (%) of 7 industrial plants in Ikeja area of Lagos are as follows:

<table>
<thead>
<tr>
<th>Industrial plants</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity utilization (%)</td>
<td>35</td>
<td>45</td>
<td>55</td>
<td>20</td>
<td>35</td>
<td>60</td>
<td>30</td>
</tr>
</tbody>
</table>
The President of the Manufacturers Association claims that the true capacity utilization of all industrial plants in Ikeja is about 45%.

(i) Set up a null and alternative hypotheses to test the truth or otherwise of the President’s claim and test it at 5% level of significance.

(ii) Construct a 95% confidence interval for the level of capacity utilization of all industries in Ikeja.

**SUGGESTED SOLUTION TO QUESTION 19**

<table>
<thead>
<tr>
<th>Environment</th>
<th>E</th>
<th>VG</th>
<th>G</th>
<th>N</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of time</td>
<td>15</td>
<td>20</td>
<td>35</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>Net profit (x) Of Banjo (Nm)</td>
<td>10</td>
<td>12</td>
<td>6</td>
<td>-2</td>
<td>-15</td>
</tr>
<tr>
<td>P(X=x) = f(x)</td>
<td>0.15</td>
<td>0.2</td>
<td>0.35</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>x*f(x)</td>
<td>1.5</td>
<td>2.4</td>
<td>2.1</td>
<td>-0.24</td>
<td>-2.7</td>
</tr>
<tr>
<td>x²f(x)</td>
<td>15</td>
<td>28.8</td>
<td>12.6</td>
<td>0.48</td>
<td>40.5</td>
</tr>
</tbody>
</table>

(i) $E(X) = \sum x f(x) = 3.06$

(ii) $E(X^2) = \sum x^2 f(x) = 97.38$

(iii) $\text{Var}(X) = E(X^2) - [E(X)]^2$

$= 97.38 - (3.06)^2 = 97.38 - 9.364$

$= 88.016 = 88.02$

(iv) Standard deviation of X = $\sigma_x = \sqrt{88.02} = 9.38$

$\therefore \text{CV}(X) = \frac{\sigma_x}{E(X)} \times 100 = \frac{9.38}{3.06} \times 100 = 0.07\%$

(b) A test involving Proportion

$H_0 : P = P_o = .17$

$H_1 : P \neq P_o = .17$

$P_o = .17$

$\hat{P} = \frac{12}{40} = .3$

$q = .7$

$\alpha = .05$
Since $Z_{\text{cal}} > Z_{\text{tab}}$ (2.19 > 1.96)

We reject $H_0$. Hence, and conclude that the CBN classification is wrong.

(ii) A 95% confidence for $P$ is given by

$$\hat{p} \pm z_{\alpha/2} \cdot \text{SE}(\hat{p})$$

$\alpha = 0.05; \quad n = 40$

$$\hat{p} = 0.3, \quad \hat{q} = 0.7$$

$$\text{SE}(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.3 \times 0.7}{40}}$$

$$= 0.072$$

$$z_{\alpha/2} = 1.96$$

95% C.I for $P$ is $\sqrt{\hat{p}} \pm \frac{z_{\alpha/2}}{2} \cdot \text{SE}(\hat{P})$

$$= 0.3 \pm 0.96 \times 0.072$$

$$= 0.3 \pm 0.14 \text{ i.e. } (0.3 - 0.14, 0.3 + 0.14)$$

i.e. C.I is (0.16, 0.44)
4(c). (i) \( \mu \) = true mean capacity utilization
\( H_0 : \mu = 45, \ H_1 : \mu \neq 45 \ \alpha = 0.05 \)
(ii)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x - \bar{x} )</th>
<th>( \bar{t} - \bar{t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>-5</td>
<td>25</td>
</tr>
<tr>
<td>45</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>55</td>
<td>15</td>
<td>225</td>
</tr>
<tr>
<td>20</td>
<td>-20</td>
<td>400</td>
</tr>
<tr>
<td>35</td>
<td>-5</td>
<td>25</td>
</tr>
<tr>
<td>60</td>
<td>-20</td>
<td>400</td>
</tr>
<tr>
<td>30</td>
<td>-10</td>
<td>100</td>
</tr>
<tr>
<td><strong>280</strong></td>
<td></td>
<td><strong>1200</strong></td>
</tr>
</tbody>
</table>

\( \bar{x} = 0 \)

\( s^2 = \frac{\Sigma (x - \bar{x})^2}{n-1} = 1200 \)

\( \bar{x} = 0 \)

\( s^2 = \frac{1200}{6} = 200 \)

\( t_{cal} = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{10 - 5}{\sqrt{200}/\sqrt{6}} = -0.935 \)

\( |t_{cal}| = 0.935 \)
\( \alpha = 0.05, \ \alpha/2 = 0.025, \ df = n-1 = 6, \ t_{tab} = 2.571 \)

Hence \( t_{cal} < t_{tab} \) => Accept \( H_0 \)

\[ \therefore \text{The mean capacity utilization in Ikeja is 45%} \]

**QUESTION 20**

The Director General of the WAZOBIA Securities Exchange Commission (NSEC) claims that from available records, 40% of registered stock broker practitioners are less than 35 years old. Compute the probability that out of 100 practitioners attending a public lecture,

(a) At most 45 (ii) At least 36 (iii) At least 38 and at most 43 are less than 35 years old.

(b) The following table shows the annual production cost (Nm) and the annual gross revenue (Nm) of Kofi Industries Ltd over a period of 8 consecutive months. Production cost and revenue of Kofi Industries Ltd.

<table>
<thead>
<tr>
<th>Production Cost (Nm) ((x))</th>
<th>18</th>
<th>32</th>
<th>40</th>
<th>51</th>
<th>60</th>
<th>72</th>
<th>83</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Revenue (Nm) ((y))</td>
<td>152</td>
<td>192</td>
<td>380</td>
<td>390</td>
<td>280</td>
<td>389</td>
<td>264</td>
<td>230</td>
</tr>
</tbody>
</table>

(i) Plot a scatter diagram of the data and comment on the type of association between production cost and gross revenue of the industry.
(ii) Compute the Pearson’s correlation co-efficient between production cost and gross revenue and reconcile this with your comment in (i) above.

(iii) Obtain the regression equation of gross revenue on production cost.

(iv) What is the expected revenue when the production cost is ₦950m?.

**SUGGESTED SOLUTION TO QUESTION 20**

5

(a) \( p = 40\% = 0.4 \)

\( n = 100 \)

Using normal approximation to Binomial:

\[ \mu = np = 0.4 \times 100 = 40 \]

\[ \sigma = \sqrt{npq} = \sqrt{100(0.4)(0.6)} = 3.9 \]

(ii) \( P(\text{At most 45}) \)

\( P(\tilde{X} \leq 15) \)

\[ Z = \frac{45 - 40}{3.9} = \frac{5}{3.9} = 1.28 \]

\[ P(X \leq 45) = P(z < 1.28) = 0.5 + 0.3997 = 0.8997 \]

(iii) \( P(\text{At least 36}) \)

\[ P(\tilde{X} \geq 6) \]

\[ Z = \frac{36 - 40}{3.9} = \frac{-4}{3.9} = -1.02 \]

\[ P(Z \geq -1.02) = 0.5 + 0.8438 = 0.8438 \]
(iv) \( P(\text{At least 38 and at most 43}) = P(38<X<43) \)

For \( X=38 \):

\[
Z = \frac{38 - 0}{4.9} = 7.76 
\]

For \( X=43 \):

\[
Z = \frac{43 - 0}{4.9} = 8.78 
\]

\[
P(8 \leq X \leq 43) = P(-41 \leq Z \leq .61) 
\]

= .1591 + .2291

= .3882

(b)

<table>
<thead>
<tr>
<th>Product cost (x)</th>
<th>18</th>
<th>32</th>
<th>40</th>
<th>51</th>
<th>60</th>
<th>72</th>
<th>83</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross revenue (y)</td>
<td>152</td>
<td>192</td>
<td>380</td>
<td>390</td>
<td>280</td>
<td>389</td>
<td>264</td>
<td>230</td>
</tr>
</tbody>
</table>

(i). Scatter diagram

◆ Comment:

The scatter diagram shows evidence of positive linear association between production cost and gross revenue of the industry.
The correlation coefficient is positive. This shows a positive linear relationship between production cost and gross revenue. This is in line with the above pictorial observation from the scatter diagram.
\[ Y = \alpha \beta + \epsilon \rightarrow \text{regression model} \]

\[ \hat{Y} = a + bx \rightarrow \text{fitted model} \]

where

\[ b = \text{estimate of } \beta \]

\[ a = \text{estimate of } \alpha \]

\[ b = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2} \]

\[ = \frac{8(131390) - 446(2277)}{8(29322) - 446^2} \]

\[ = \frac{051120 - 015542}{234576 - 98916} \]

\[ = \frac{35578}{35660} \]

\[ = 1.998 \]

(iii) \[ \bar{x} = \frac{446}{8} = 5.75; \bar{y} = \frac{2277}{8} = 84.625 \]

\[ \therefore a = \hat{y} - bx \]

\[ = 84.625 - 1.998 \times 5.75 \]

\[ = 84.625 - 5.6385 \]

\[ = 78.98 \]

The regression equation is:

\[ \hat{Y} = 78.99 + 1.998X \]

(iv). When production cost (X) is N950, the expected gross revenue is obtained by

\[ \hat{Y} = 88.99 + 1.998(950) \]

\[ = 88.99 + 488.1 \]

\[ = 576.09 \]

\[ = 1237090 \]

\[ \therefore \text{the expected revenue} = N1237.09 \]

**QUESTION 21**

(a) The following table shows the amount of loan (N\text{,}000) per working day given out to 10 members of a co-operative society over a period of 4 consecutive weeks in the first quarter of 2005.
(i) Plot the data as a time series and comment on the components of time series exhibited by the data.

(ii) Compute five-day moving averages and super-impose these on the time series plot.

(iii) Compute the seasonal variations and the daily seasonal adjustments.

(iv) By suitably extending your moving average plot to Monday of the 5th week, find the seasonally adjusted forecast for the expected loan on Monday of the 5th week.

(b) The following table shows the unit price and quantity required by a household in Enugu in each of the specified years.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Unit Price (₦’00)</th>
<th>Quantity</th>
<th>Unit Price (₦’00)</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread (Loaf)</td>
<td>0.9</td>
<td>90</td>
<td>1.2</td>
<td>105</td>
</tr>
<tr>
<td>Milk (tin)</td>
<td>0.3</td>
<td>25</td>
<td>0.7</td>
<td>32</td>
</tr>
<tr>
<td>Meat (Kg)</td>
<td>120</td>
<td>150</td>
<td>250</td>
<td>170</td>
</tr>
<tr>
<td>Egg (crate)</td>
<td>1,500</td>
<td>3</td>
<td>2,200</td>
<td>5</td>
</tr>
</tbody>
</table>

Compute

(i) Laspeyre’s

(ii) Paasche’s


SUGGESTED SOLUTION TO QUESTION 21

<table>
<thead>
<tr>
<th>Day of the week</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Week</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thur</th>
<th>Fri</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>15</td>
<td>21</td>
<td>32</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td>30</td>
<td>45</td>
<td>49</td>
<td>52</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
<td>48</td>
<td>55</td>
<td>62</td>
<td>69</td>
</tr>
<tr>
<td>4</td>
<td>61</td>
<td>66</td>
<td>69</td>
<td>73</td>
<td>81</td>
</tr>
</tbody>
</table>
Comment: The plot reveals an upward trend.

<table>
<thead>
<tr>
<th>Day(x)</th>
<th>Observations (Y)</th>
<th>5-day moving totals</th>
<th>5-day moving averages (T)</th>
<th>SV=Y-T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mon</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tue</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wed</td>
<td>21</td>
<td>119</td>
<td>23.8</td>
<td>-2.8</td>
</tr>
<tr>
<td>Thur</td>
<td>32</td>
<td>140</td>
<td>28.0</td>
<td>4</td>
</tr>
<tr>
<td>Fri</td>
<td>40</td>
<td>155</td>
<td>31.0</td>
<td>9</td>
</tr>
<tr>
<td>Mon</td>
<td>32</td>
<td>179</td>
<td>35.8</td>
<td>-3.8</td>
</tr>
<tr>
<td>Tue</td>
<td>30</td>
<td>196</td>
<td>39.2</td>
<td>-9.2</td>
</tr>
<tr>
<td>Wed</td>
<td>45</td>
<td>208</td>
<td>41.6</td>
<td>3.4</td>
</tr>
<tr>
<td>Thur</td>
<td>49</td>
<td>220</td>
<td>44.0</td>
<td>5</td>
</tr>
<tr>
<td>Fri</td>
<td>52</td>
<td>238</td>
<td>47.6</td>
<td>4.4</td>
</tr>
<tr>
<td>Mon</td>
<td>44</td>
<td>248</td>
<td>49.6</td>
<td>-5.6</td>
</tr>
<tr>
<td>Tue</td>
<td>48</td>
<td>261</td>
<td>52.2</td>
<td>-4.2</td>
</tr>
<tr>
<td>Wed</td>
<td>55</td>
<td>278</td>
<td>55.6</td>
<td>-0.6</td>
</tr>
<tr>
<td>Thur</td>
<td>62</td>
<td>295</td>
<td>59.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Fri</td>
<td>69</td>
<td>313</td>
<td>62.6</td>
<td>6.4</td>
</tr>
<tr>
<td>Mon</td>
<td>61</td>
<td>327</td>
<td>65.4</td>
<td>-4.4</td>
</tr>
<tr>
<td>Tue</td>
<td>66</td>
<td>338</td>
<td>67.6</td>
<td>-1.6</td>
</tr>
<tr>
<td>Wed</td>
<td>69</td>
<td>350</td>
<td>70</td>
<td>-1</td>
</tr>
<tr>
<td>Thur</td>
<td>73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fri</td>
<td>81</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(iii).

<table>
<thead>
<tr>
<th>Week</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thur</th>
<th>Fri</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-2.8</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>-3.8</td>
<td>-9.2</td>
<td>3.4</td>
<td>5</td>
<td>4.4</td>
</tr>
<tr>
<td>3</td>
<td>-5.6</td>
<td>-4.2</td>
<td>-0.6</td>
<td>3</td>
<td>6.4</td>
</tr>
<tr>
<td>4</td>
<td>-4.4</td>
<td>-1.6</td>
<td>-1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Totals</td>
<td>-13.8</td>
<td>-15</td>
<td>-1</td>
<td>12</td>
<td>19.8</td>
</tr>
<tr>
<td>S.A</td>
<td>-14.2</td>
<td>-15.4</td>
<td>-1.4</td>
<td>11.6</td>
<td>19.4</td>
</tr>
</tbody>
</table>

Each seasonal adjustment should be further adjusted by $2/5 = 0.4$ leading to the following final daily adjustment:

- **Mon**: $3.8 - 0.4 = 4.2$
- **Tue**: $5 - 0.4 = 5.4$
- **Wed**: $- - 0.4 = -4$
- **Thur**: $12 - 0.4 = 1.6$
- **Fir**: $19.8 - 0.4 = 9.4$

(iv).

**Forecast**

On Monday of fifth week, the forecast loan (from the MA) projection is 81 (by projection of the MA plot).
(b) 1999 | 2004
---|---
**Commodity** | **Unit price ($N)** | **quantity** | **Unit price ($N)** | **quantity** |
Bread (loaf) | 0.9 | 90 | 1.2 | 105 |
Milk (tin) | 0.7 | 32 | 7.5 | 32 |
Meat (Kg) | 150 | 5 | 2200 | |
Egg (crate) | 0.3 | 25 | 0.7 | 32 |
**Totals** | | | | |
**1999 = 100** | **22588.5** | **28004.1** | **44225.5** | **53648.4** |

(i). Laspeyre's index = \[
\frac{\sum p_0q_o}{\sum p_oq_o} \times 100
\]
\[
= \frac{14225.5}{22588.5} \times 100
\]
\[
= 95.79
\]

(ii) Paache's index = \[
\frac{\sum p_oq_1}{\sum p_0q_1} \times 100
\]
\[
= \frac{43648.4}{28004.1} \times 100
\]
\[
= 91.57
\]

(iii) Fisher's index = \[
\sqrt{195.79 \times 91.57}
\]
\[
= \sqrt{37507.4903} = 93.67
\]

**QUESTION 22**

(a) A research consultant was given the assignment of evaluating students' performances in the May 2004 Module B of ICAN Foundation examinations. The consultant took a random sample of 35 students and found that 21 students passed QA, 18 Law and 11 Management. Further, 12 passed QA and Law, 7 QA and Management while 3 passed Law and Management while 2 passed all the three papers.

Represent the consultant's findings in an Euler-Venn diagram. Use your Euler-Venn diagram to find the number of candidates that passed

(i) QA but not Law
(ii) Exactly two papers.
(iii) None of the 3 papers.

(b) A manufacturing company produces three products E,F,G from its raw materials L,M,N and P. Each unit of E is produced from 4 units of L, 3 units of M, 2 units of N and 5 units of P, further a unit of F is produced from 3 units of L, 2 units of M, 1 unit of N and 3 units of P while a unit of G is produced from 5 units of L, 3 units of M, 2 units of N and 6 units of P.
COSTING AND QUANTITATIVE TECHNIQUES

Raw materials cost ₦2 per unit of L, ₦3 per unit of M, ₦5 per unit of N and ₦6 per unit of P. Other production costs per unit of E, F, and G respectively are ₦16, ₦11, and ₦14 while the selling prices for each unit of the products E, F, G are respectively ₦75, ₦50, and ₦85.

(i) Prepare the matrix of:
   ♦ Raw materials requirements (w)
   ♦ Unit raw material cost (x)
   ♦ Cost of other raw materials
   ♦ Unit selling price

(ii) Calculate, for a unit of each product the:
   ♦ Raw material cost
   ♦ Total cost
   ♦ Profit

(iii) Evaluate $Q = ZWX$, Where

$Z = (250, 300, 350)$ is the matrix of the product monthly sales.

Source: ICAN foundation examination May 29 2003 Quantitative Analysis as contained in pathfinder May 2003 foundation examination.

SUGGESTED SOLUTION TO QUESTION 22

(a) Let $U$ represent total number of samples taken, $Q$ represent no that passed Q/A, $L$ represent no that passed law 11, and $M$ represent no that passed management.

\[
n(U) = 35
\]

\[
\begin{align*}
n(Q) &= 21 \\
n(L) &= 18 \\
n(M) &= 11
\end{align*}
\]

\[
\begin{align*}
n(Q \cap L) &= 5 \\
n(Q \cap M) &= 7 \\
n(L \cap M) &= 3 \\
n(Q \cap L \cap M) &= 1
\end{align*}
\]

(i) Number that passed Q/A but not law11 = 4 + 5 = 9
(ii) Exactly two papers = 10 + 5 + 1 = 16
(iii) None of the 3 papers = \( n(U) - n(Q) \cup L \cup M \)

\[
= 35 - (4 + 5 + 3 + 10 + 5 + 1 + 2) = 5
\]

(b) (i) Raw materials requirement
\[
W = \begin{pmatrix}
4 & 3 & 2 & 5 \\
3 & 2 & 1 & 3 \\
5 & 3 & 2 & 6
\end{pmatrix}
\]

\[
X = \begin{pmatrix}
2 \\
3 \\
5 \\
6
\end{pmatrix}
\]

\[\text{Cost of raw material}\]

\[
Y = \begin{pmatrix}
16 \\
50 \\
85
\end{pmatrix}
\]

(iii) \quad \text{Raw materials cost } R = WX

\[
= \begin{pmatrix}
4 & 3 & 2 & 5 \\
3 & 2 & 1 & 3 \\
5 & 3 & 2 & 6
\end{pmatrix}
\begin{pmatrix}
2 \\
3 \\
5 \\
6
\end{pmatrix}
= \begin{pmatrix}
8 + 9 + 10 + 30 \\
6 + 6 + 5 + 18 \\
10 + 9 + 10 + 36
\end{pmatrix}
\]

\[
= \begin{pmatrix}
57 \\
35 \\
65
\end{pmatrix}
\]

(iv) \quad \text{Monthly cost raw materials } = \text{₦ 47,500}

\[\text{Total cost } T = Y + R\]

\[
\begin{pmatrix}
16 \\
14 \\
11
\end{pmatrix}
+ \begin{pmatrix}
57 \\
65 \\
35
\end{pmatrix}
= \begin{pmatrix}
73 \\
79 \\
46
\end{pmatrix}
\]

\[\text{Profit } P = \text{vector of unit selling price} - \text{vector of unit cost price}\]

\[
= \begin{pmatrix}
75 \\
79 \\
85
\end{pmatrix}
- \begin{pmatrix}
50 \\
46 \\
2
\end{pmatrix}
= \begin{pmatrix}
25 \\
33 \\
83
\end{pmatrix}
\]

(v) \quad Q = Z(WX) = ZR, where \[Z = \begin{pmatrix}
250 & 300 & 350
\end{pmatrix}\]

\[
\therefore Q = \begin{pmatrix}
\text{₦ } 50 \\
\\
300 \\
350 \\
\text{₦ } 35 \\
65
\end{pmatrix}
= \text{₦ 47,500}\]


**QUESTION 23**

Each one of the demand function $D(x)$ as well as the supply function $S(x)$ is linearly related to the quantity $x$ of a product by the equations:

\[
D(x) = a + bx, \quad \text{and} \\
S(x) = c + dx, \quad \text{where} \ a, \ b, \ c, \ d \ \text{are constants.}
\]

(a) Given that consumers demand 6 units of the product at a price of N40 while consumers’ demand of 2 units is at a price of N80. Determine the demand function.

(b) Given that the producer can supply 5 units of the products at N50 and 12 units at N71. Determine the supply function.

(c) Use (a) and (b) above to determine:

(i) The equilibrium quantity and price;
(ii) The consumers’ surplus; and
(iii) The producers’ surplus.

**SUGGESTED SOLUTION TO QUESTION 23**

(a) Demand curve: $D = a + bx$  
When $x = 6, p = 40$ and when $x = 2, p = 80$  
Substituting these into (i), we have  
\[a + 6b = 40\]
\[a + 2b = 80\]
Whose solution is $b = -10$ and $a = 100$  
The demand curve is $D(x) = 100 - 10x$

(b) The supply curve is $S = c + dx$  
When $x = 5, p = 50$ while $x = 12, p = 71$  
Substituting these in (ii) we get:  
\[c + 5d = 50\]
\[c + 12d = 71\]
With solution $d = 3$ and $c = 35$.  
The supply function is $S(x) = 35 + 3x$

(c) (i) At equilibrium condition $D(x) = S(x)$ that is,  
\[100 - 10x = 35 + 3x\]
\[13x = 65\]
\[x_e = \frac{5}{3}\]
as the equilibrium quantity
and $P_e = 100 - 10(5) = 100 - 50 = N50$ as the equilibrium price.
(ii) Consumers’ surplus CS is:

\[ CS = \int_{0}^{5} (P(x) - D(x)) \, dx \]

\[ = \int_{0}^{5} (100 - 10x - 50) \, dx \]

\[ = \int_{0}^{5} 50 - 10x \, dx \]

\[ = \left[ 50x - 5x^2 \right]_{0}^{5} \]

\[ = 50(5) - 5(25) \]

\[ = 250 - 125 \]

\[ = \text{₦125} \]

(iii) Producers’ surplus PS is:

\[ PS = \int_{0}^{5} (P(x) - S(x)) \, dx \]

\[ = \int_{0}^{5} (50 - 3x) \, dx \]

\[ = \left[ 50x - 3x^2 \right]_{0}^{5} \]

\[ = 75 - 3\left(\frac{25}{2}\right) \]

\[ = 75 - \frac{75}{2} = \frac{75}{2} \]

\[ = \text{₦37.50} \]

QUESTION 24

(a) Solve the following equations by the appropriate methods:

(i) \[ 3x + 2y = 13 \] ................. (1)

(ii) \[ 9x - y = 4 \] ................. (2)

(b) Evaluate \( \log_{a} 3125 \), if \( x^2 = 5 \)

(c) If the production units of a company’s product increases from 3000 to 5000, total cost of production also increases from ₦15,000 to ₦20,000 and assuming the revenue function of \( R = 8x - 700 \), determine:
(i) the cost function in order to show the relationship between the cost \( y \) and the number of units \( x \) produced (assuming linear function);

(ii) the profit function for the product and its profit at \( x = 1500 \).

**SUGGESTED SOLUTION TO QUESTION 24**

(a)(i) \[ 3x + 2y = 13 \] (1)
\[ 9x - y = 4 \] (2)

From (2), \[ y = 9x - 4 \] (3)

Using (3) in (1), we have
\[ 3x + 2(9x - 4) = 13 \]
\[ 3x + 18x - 8 = 13 \]

Collecting the like-terms:
\[ 21x = 21 \]

\[ x = 1 \]

Recall equation (3) and substitute the value of \( x \)

i.e. \[ y = 9x - 4 \]
\[ y = 9(1) - 4 \]
\[ y = 9 - 4 \]
\[ y = 5 \]

Answer: \( x = 1 \) and \( y = 5 \)

(ii) \[ x^2 - 6x + 5 = 0 \]

By factorization method:

\[ x^2 - x - 5x + 5 = 0 \]
\[ x(x - 1) - 5(x - 1) = 0 \]
\[ (x - 1)(x - 5) = 0 \]

\[ \Rightarrow x = 1 \text{ or } 5 \] (as answer).

(b) Given: \( \log_3 3125 \), if \( x^2 = 5 \)

Let \[ y = \log_5 3125 \]

\[ \Rightarrow x^2 = 3125 \]

ALITER \( x^2 = 5 \Rightarrow \)

\[ \Rightarrow x = 5 \times 625 \]
\[ \Rightarrow x = x^2 \times 625 \]

let \( \log 3125 = M \)
\[
\Rightarrow \frac{x^y}{x^t} = 625 \quad \therefore m^{\frac{m}{2}} = \sqrt{125} = 5^5
\]
\[
\Rightarrow x^{m^2} = 5^4 \quad \therefore m = 0
\]
\[
\Rightarrow x^{n^2} = (x^2)^4 \quad \therefore g^{-3125} = 0 \text{ if } x^2 = 1
\]
\[
\Rightarrow x^{n^2} = x^8
\]
\[
\Rightarrow y - 2 = 8
\]
\[
\Rightarrow y = 8 + 2
\]
\[
y = 10
\]
\[
\therefore \log 3125 = 10
\]

(c). Let the Linear function be \( ax + by + c = 0 \) where \( a, b \) and \( c \) are constant; \( x \) and \( y \) represent units of production and cost of production respectively.
\[
\therefore x = 3000, \quad y = 15000, \text{ we have }
\]
\[
3000a + 15000b + c = 0 \quad \text{(i)}
\]
Also, if \( x = 5000, \quad y = 2000 \), then we have
\[
5000a + 20000b + c = 0 \quad \text{(ii)}
\]
Multiply equation (i) by 4 and equation (ii) by 3, we have
\[
12000a + 60000b + 4c = 0 \quad \text{(iii)}
\]
\[
15000a + 6000b + 3c = 0 \quad \text{(iv)}
\]
Equation (iv) - Equation (iii)
\[
3000a + c = 0
\]
\[
a = -\frac{c}{3000} \quad \text{(v)}
\]
Using (v) in (i) gives:
\[
3000\left(-\frac{c}{3000}\right) + 15000b + c = 0
\]
\[
-c + 15000b + c = 0
\]
\[
15000b = 0
\]
\[
\therefore b = 0
\]
the linear relationship is:
\[
-\frac{c}{3000}x + 0y + c = 0
\]
Divide through by \(-\frac{c}{3000}\), we have \(-x + 3000 = 0\)

or \(x - 3000 = 0\) as required cost function.
(ii) Profit = Total Revenue – Total Cost
   i.e. \( \pi = 8x - 700 - (x - 3000) \)
   = \( 8x - 700 - x + 3000 \)
   = \( 7x + 2300 \)
   At \( x = 1500, \) \( \pi = 7(1500) + 2300 \)
   = \( 10500 + 2300 \)
   = \( \text{₦12,800} \).

**QUESTION 25**

(a) A machine which costs \( \text{₦100,000} \) depreciates each year by 10%. Determine the worth or value of the machine at the end of five years.

(b) Find the Interest to be paid on \( \text{₦50,000} \) for 4 years at 6% per annum if the interest is to be paid quarterly.

(c) A project with an initial cost of \( \text{₦100,000} \) generates the cash flows of \( \text{₦35,000, ₦32,000, ₦30,000} \) and \( \text{₦28,000} \) respectively for the four-year life span of the project. Calculate the NPV of the project if an annual rate of return of 10 percent is allowed.

**SUGGESTED SOLUTION TO QUESTION 25**

(a) At the end of the first year, the value of the machine
   \[
   = 100,000 \times (100 - 10)\% \\
   = 100,000 \times 90/100 \\
   = 100,000 \times 9/10
   \]
   At the end of 2\textsuperscript{nd} year, we have
   \[
   100,000 \times (9/10 \times 9/10) = 100,000 \times (9/10)^2
   \]
   so that at the end of 5\textsuperscript{th} year, the value of the machine is equal to
   \[
   100,000 \times (9/10)^5 = \text{₦59,049}.
   \]

(b) At the end of each quarter, the interest will be \( (6/4)\% = 1.5\% \)
   Therefore, the amount at the end of q quarter is equal to:
   \[
   A = P(1 + \frac{r}{100})^q, \text{ where } q = \text{quarter.}
   \]
   i.e. \[
   A = 50000\left(1 + \frac{0.015}{100}\right)^{16}, \text{ (4 years = 20 quarters)}
   \]
   \[
   = 50000(1 + 0.015)^{16}
   \]
   \[
   = 50000(1.015)^{16}
   \]
   \[
   = \text{₦63,449.28}
   \]
Required Interest = Amount – Principal
= ₦(63,449.28 – 50,000)
= ₦13,449.28

(c) Calculation of NPV

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Inflows(₦)</th>
<th>Discounting factor at 10% (from table)</th>
<th>Present Value of Cash inflows</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35,000</td>
<td>0.909</td>
<td>31815</td>
</tr>
<tr>
<td>2</td>
<td>32,000</td>
<td>0.826</td>
<td>26432</td>
</tr>
<tr>
<td>3</td>
<td>30,000</td>
<td>0.751</td>
<td>22530</td>
</tr>
<tr>
<td>4</td>
<td>28,000</td>
<td>0.621</td>
<td>17360</td>
</tr>
<tr>
<td></td>
<td>TOTAL</td>
<td></td>
<td>98,137</td>
</tr>
<tr>
<td></td>
<td>Less Cash Outlay</td>
<td></td>
<td>100,000</td>
</tr>
<tr>
<td></td>
<td>NPV</td>
<td></td>
<td>₦1863</td>
</tr>
</tbody>
</table>

Since NPV = ₦1863, which is lesser than zero, the investment on the project will not be gainful.

QUESTION 26

(a) Obtain the derivatives of the following function at x = 2:

(i) \( y = 2x^3 + 3x^2 + 6x + 5 \)

(ii) \( y = (2x + 3)(x^2 + 1) \)

(iii) \( y = \frac{2x^2 + x}{x^2 + x + 1} \)

(iv) \( y = 4(2x + x^2)^4 \)

(b) Determine the maximum profit of a company with revenue function \( R = 50q - 3q^2 \) and the total cost function \( T = q^3 - 43q^2 \)

SUGGESTED SOLUTION TO QUESTION 26

(a) (i) \( y = 2x^3 + 3x^2 + 6x + 5 \)

\[
\frac{dy}{dx} = 3(2x^2) + 3(2x^2) + 6 \times x^{-1} + 0 = 6x^2 + 6x + 6
\]

(ii) \( (2x + 3)(x^2 + 1) \)

Let \( U = 2x + 3 \), \( \frac{du}{dx} = 2 \)
and \( V = x^2 + 1 \), \( \frac{dv}{dx} = 2x \)

But \( \frac{dy}{dx} = U \frac{dv}{dx} + \frac{du}{dx} \)

\[
= 2x(2x + 3) + 2(x^2 + 1)
\]

\( = 4x^3 + 6x + 2x^2 + 2 \)

\( \frac{dx}{V} = 2x + 3 \)

(iii) \( y = \frac{2x^2}{x^2 + 1} \)

Let \( U = 2x^2 + 3 \), \( \frac{dx}{du} = 4x \)

\( V = x^2 + 3x \), \( \frac{dx}{V} = 2x + 3 \)

\[
\frac{dy}{dx} = \frac{V \frac{dx}{du} - \frac{dv}{dx}}{V^2}
\]

\[
= \frac{x^2 + x + \frac{4x}{x^2 + x} + \frac{2x}{x^2 + x}}{x^2 + x + \frac{2}{x^2 + x}}
\]

\[= \frac{4x^3 + 2x + 4x - 2x - 2x^2}{(x^2 + x)^2} \]

\[
= \frac{-ix^2 + 6x - ix}{(x^2 + ix + 1)^2}
\]

\[= \frac{-ix^2 + 0x + i}{(x^2 + ix + 1)^2} \]

i.e. \( \frac{dy}{dx} = \frac{-6x^2 - 0x + i)}{(x^2 + ix + 1)^2} \)

(b) Given that \( R = 50q - 3q^2 \) and \( T = 5q^3 - 43q^2 \)

Profit \( (\pi) = R - T \)

\( = 50q - 3q^2 - 5q^3 + 43q^2 \)

i.e. \( \pi = 50q + 40q^2 - 5q^3 \)

At the turning, \( \frac{d\pi}{dq} = 0 \)

\[
\frac{d\pi}{dq} = 50 + 80q - 15q^2 = 0
\]

\[\Rightarrow 15q^2 - 80q - 50 = 0\]
COMPREHENSIVE QUESTIONS AND SUGGESTED SOLUTIONS

3q^2 - 16q - 10 = 0

Using Quadratic formula

\[ q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

where \( a = 3 \), \( b = -16 \) and \( c = -10 \)

\[ q = \frac{16 \pm \sqrt{16^2 + (3)(10)}}{2(3)} = \frac{16 \pm \sqrt{376}}{6} \]

\[ q = \frac{16 \pm 9.4}{6} \]

\[ q = \frac{16 + 9.4}{6} \quad \text{or} \quad \frac{16 - 9.4}{6} \]

\[ q = \frac{35.4}{6} \quad \text{or} \quad \frac{6.6}{6} \]

\[ q = 5.9 \quad \text{or} \quad -0.567 \]

But \( \frac{d^2\pi}{dq^2} = 80 - 30q \)

\[ \therefore \text{At } q = 5.9, \quad \frac{d^2\pi}{dq} = 80 - 30(5.9) \]

\[ = -97 < 0 \]

Hence, at \( q = 5.9 \), we have the maximum profit.

\[ \therefore \text{The maximum profit } = 50(5.9) + 40(5.9)^2 - 5(5.9)^3 \]

\[ = 295 + 1392.4 - 1026.9 \]

\[ = 660.51 \]

QUESTION 27

The research unit of the Marketing department of a company came up with estimated payoffs in terms of monthly profit for the 3 products (X, Y, Z) that the company intends to launch as follows:

<table>
<thead>
<tr>
<th>Products</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000 (S₁)</td>
<td>50</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>16,000 (S₂)</td>
<td>60</td>
<td>35</td>
<td>25</td>
</tr>
<tr>
<td>12,000 (S₃)</td>
<td>75</td>
<td>40</td>
<td>23</td>
</tr>
</tbody>
</table>

What decision should the company take if:
(a) Minimax, (b) Maximin, (c) Regret Criterion is applied?
SUGGESTED SOLUTION TO QUESTION 27

(a) Minimax

<table>
<thead>
<tr>
<th></th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>50</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>Y</td>
<td>30</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>Z</td>
<td>30</td>
<td>25</td>
<td>23</td>
</tr>
<tr>
<td>Column Max</td>
<td>50</td>
<td>60</td>
<td>75</td>
</tr>
</tbody>
</table>

Hence, it is advisable for the company to adopt S₁.

(b) Maximin Criteria

<table>
<thead>
<tr>
<th></th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>50</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>Y</td>
<td>30</td>
<td>35</td>
<td>40</td>
</tr>
<tr>
<td>Z</td>
<td>30</td>
<td>25</td>
<td>23</td>
</tr>
<tr>
<td>Maximin</td>
<td>30</td>
<td>25</td>
<td>23</td>
</tr>
</tbody>
</table>

The Maximin of column Maximin is 30, hence, it is advisable for the company to adopt strategy S₁.

(c) Regret Criterion

<table>
<thead>
<tr>
<th>State of Nature</th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>75 – 50 = 25</td>
<td>75 – 60 = 5</td>
<td>75 – 75 = 0</td>
</tr>
<tr>
<td>Y</td>
<td>40 – 30 = 10</td>
<td>40 – 35 = 5</td>
<td>40 – 40 = 0</td>
</tr>
<tr>
<td>Z</td>
<td>30 – 30 = 0</td>
<td>30 – 25 = 5</td>
<td>30 – 23 = 7</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Hence, it is advisable for the company to adopt the opportunity loss strategy S₃.

QUESTION 28

Given the L.P. problem

\[
\text{Min } Z = 4x₁ + 5x₂ \\
\text{s.t. } \\
3x₁ + 6x₂ \geq 80 \\
4x₁ + 3x₂ \geq 100 \\
x₁, x₂ \geq 0
\]

Obtain the optimal value by Graphical Method.

SUGGESTED SOLUTION TO QUESTION 28

\[
Z = 4x₁ + 5x₂ \\
\text{s.t. } \\
3x₁ + 6x₂ = 80 \\
4x₁ + 3x₂ = 100
\] (i) (ii)

from equ. (i) when \(x₁ = 0\),
$$3(0) + 6(x_2) = 80$$
$$6x_2 = 80$$
$$x_2 = \frac{80}{6}$$
$$\therefore x_2 = 13.33$$

when $$x_2 = 0$$

$$3x_1 + 6(0) = 80$$
$$3x_1 = 80$$
$$x_1 = \frac{80}{3}$$

$$= 26.67$$

from equ. (ii)

when $$x_1 = 0$$

$$4(0) + 3x_2 = 100$$
$$3x_2 = 100$$
$$\therefore x_2 = \frac{100}{3} = 33.33$$

when $$x_1 = 0$$

$$4x_1 + 3(0) = 100$$
$$4x_1 = 100$$
$$\therefore x_1 = \frac{100}{4} = 25.$$ 

The values of $$x_1$$ and $$x_2$$ are:

$$(x_1 , x_2) = (13.33 , 26.67)$$

and

$$(x_1 , x_2) = (33.33 , 25)$$
Solving for B, requires using simultaneous equation on equation (i) and (ii)

\[3x_1 + 6x_2 = 80 \quad \text{(i)}\]

\[4x_1 + 3x_2 = 100 \quad \text{(ii)}\]

\[12x_1 + 24x_2 = 320\]

\[12x_1 + 9x_2 = 300\]

\[15x_2 = 20\]

\[x_2 = 1.33\]

To solve for \(x_1\):

\[3x_1 + 1.33(6) = 80\]

\[3x_1 + 8 = 80\]

\[3x_1 = 80 - 8\]

\[3x_1 = 72\]

\[x_1 = 24\]

\[\therefore \text{ Point B = 24, 1.33}\]
**Extreme Point** | **Coordinates \((x_1, x_2)\)** | **Objective function \(Z = 4x_1 + 5x_2\)**
---|---|---
A | \((0, 33.3)\) | \(4(0) + 5(33.3) = 166.5\)
B | \((24, 1.33)\) | \(4(24) + 5(1.33) = 102.65\)
C | \((26.7, 0)\) | \(4(26.7) + 5(0) = 106.8\)

\[\text{The optimal value is } Z = 102 \text{ (the minimum } Z) \text{. with } x_1 = 23, x_2 = 2.\]

**QUESTION 29**

A Company whose monthly supply of 2000 units to its customers has the monthly holding and setup costs of ₹50 and ₹200 respectively.

(a) If shortage is not allowed, determine the 
(i) E.O.Q. 
(ii) Optimum Scheduling Period \((t)\)

(b) If shortage of ₹20 is allowed; determine the 
(i) E.O.Q. and 
(ii) Optimal stock level; 
(iii) Optimum Shortage level. 
(iv) Total cycle.

**SUGGESTED SOLUTION TO QUESTION 29**

(a) \(D = 2000\)

\(\text{K} = \text{₹}200\)

\(h = \text{₹}50\)

(i) \(\text{EOQ} = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2(200)(2000)}{50}} = \sqrt{16000} = 126.5\) units per month.

(ii) \(t = \sqrt{\frac{2K}{Dh}} = \sqrt{\frac{2(200)}{2000(50)}} = \sqrt{0.004} = 0.063\) per month.

(b) Recall that \(D = 2000, \ K = \text{₹}200\) and \(h = \text{₹}50\). In addition, also we have \(S_c = \text{₹}20\)

(i) \(Q = \sqrt{\frac{2KD}{h} \left(\frac{h+S_c}{S_c}\right)} = \sqrt{\frac{2(200)(2000)}{50} \left(\frac{50 + 20}{20}\right)} = \sqrt{1600(3.5)} = \sqrt{56000} = 236.64\) units.
(ii) \( M = \sqrt{\frac{2KD}{h \left( \frac{S_c}{h + S_c} \right)}} = \sqrt{\frac{2(200)(2000)}{50 \left( \frac{20}{50 + 20} \right)}} = \sqrt{16000(0.2857)} = \sqrt{4571.43} = 67.61 \)

(iii) \( S(\text{Shortage}) = Q - M \)

\( = 236.64 - 67.61 \)

\( = 169.02 \approx 169 \text{ units.} \)

(iv) \( t = \sqrt{\frac{2K}{Dh} \left( \frac{h + S_c}{S_c} \right)} = \sqrt{\frac{2(200)}{2000(50)} \left( \frac{50 + 20}{50} \right)} = \sqrt{0.004 \times 1.4} = \sqrt{0.0056} = 0.07483 \text{ per month.} \)

**QUESTION 30**

A project consists of 8 activities and their three time estimates are given below:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Activity Name</th>
<th>Time Estimates (Weeks)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( t_0 )</td>
</tr>
<tr>
<td>1 – 2</td>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td>2 – 3</td>
<td>B</td>
<td>10</td>
</tr>
<tr>
<td>2 – 4</td>
<td>C</td>
<td>5</td>
</tr>
<tr>
<td>2 – 5</td>
<td>D</td>
<td>5</td>
</tr>
<tr>
<td>3 – 6</td>
<td>E</td>
<td>6</td>
</tr>
<tr>
<td>4 – 6</td>
<td>F</td>
<td>9</td>
</tr>
<tr>
<td>5 – 7</td>
<td>G</td>
<td>8</td>
</tr>
<tr>
<td>6 – 7</td>
<td>H</td>
<td>3</td>
</tr>
</tbody>
</table>

(a) Determine the expected duration and variance for each activity;

(b) Draw a network diagram;

(c) What is the expected project length (duration) using slack/float approach; and

(d) Calculate the variance and standard deviation of the project.
SUGGESTED SOLUTION TO QUESTION 30

(a) The $t_e$ and $\sigma^2$ are computed in the following table.

<table>
<thead>
<tr>
<th>Activity Name</th>
<th>Time Estimates (Weeks)</th>
<th>$t_e = \frac{t_0 + \frac{1}{2}t_s + \frac{1}{6}t_p}{2}$</th>
<th>$\sigma^2 = \frac{1}{6}(t_p - t_0)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3 4 5</td>
<td>4</td>
<td>0.667</td>
</tr>
<tr>
<td>B</td>
<td>10 12 14</td>
<td>12</td>
<td>2.667</td>
</tr>
<tr>
<td>C</td>
<td>5 6 7</td>
<td>6</td>
<td>0.667</td>
</tr>
<tr>
<td>D</td>
<td>5 7 9</td>
<td>7</td>
<td>2.667</td>
</tr>
<tr>
<td>E</td>
<td>6 7 8</td>
<td>7</td>
<td>0.667</td>
</tr>
<tr>
<td>F</td>
<td>9 10 11</td>
<td>10</td>
<td>0.667</td>
</tr>
<tr>
<td>G</td>
<td>8 10 12</td>
<td>10</td>
<td>2.667</td>
</tr>
<tr>
<td>H</td>
<td>3 6 9</td>
<td>6</td>
<td>6.000</td>
</tr>
</tbody>
</table>

(b) The network diagram of the problem.

![Network Diagram]

(c) The critical path through slack/float.

Here, we need to obtain the Earliest Expected time ($ET_i$) and Latest time ($LT_i$).
Computation of Forward Pass ($ET_j$)

$ET_j = \text{Max} \left[ ET_i + t_{ij} \right]$  

$ET_1 = 0$  

$ET_2 = \text{Max}[0, 0+4] = 4 \text{ weeks}$  

$ET_3 = \text{Max}[4, 4+12] = 16 \text{ weeks}$  

$ET_4 = \text{Max}[4, 4+6] = 10 \text{ weeks}$  

$ET_5 = \text{Max}[4, 4+7] = 11 \text{ weeks}$  

$ET_6 = \text{Max}[10, 10+10, 16, 16+7] = 23 \text{ weeks}$  

$ET_7 = \text{Max}[23, 23+6, 11, 11+10] = 29 \text{ weeks}$

Computation of Backward Pass ($LT_j$)

$LT_j = \text{Max} \left[ LT_i + t_{ij} \right]$  

$LT_7 = ET_7 = 29 \text{ weeks}$  

$LT_6 = \text{Min}[29, 29-6] = 23 \text{ weeks}$  

$LT_5 = \text{Min}[29, 29-10] = 19 \text{ weeks}$  

$LT_4 = \text{Min}[23, 23-10] = 13 \text{ weeks}$  

$LT_3 = \text{Min}[23, 23-7] = 16 \text{ weeks}$  

$LT_2 = \text{Min}[16, 16-12, 13, 13-6, 19, 19-7]$  

$= 4 \text{ weeks}$  

$LT_1 = \text{Min}[4, 4-4]$  

$= 0 \text{ weeks}$

Float Table

<table>
<thead>
<tr>
<th>Event</th>
<th>$LT_j$</th>
<th>$ET_j$</th>
<th>Float/Slack $(LT_j - ET_j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
<td>19</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>29</td>
<td>29</td>
<td>0</td>
</tr>
</tbody>
</table>
The critical path is 1–2–3–6–7 with the activities A–B–E–H.

The critical length is 4+12+7+6 = 29 weeks.

(d) The variance of the project length is the
var. of A + var. of B + var. of E + var. of H
= 0.667+2.667+0.667+6.000=10.001

The standard deviation of the project length is \(\sqrt{10.001} = 3.162\) weeks.

**QUESTION 31**

For the following cost table of transportation model:

<table>
<thead>
<tr>
<th>Destination</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>2</td>
<td>11</td>
<td>10</td>
<td>3</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Y</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Z</td>
<td>3</td>
<td>9</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>Demand</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Determine the initial basic feasible solution by
(b) North-west corner rule.
(b) Least cost method.
(c) Vogel’s approximation method.

**SOLUTION TO QUESTION 31**

(a) North-West Corner Rule.

Minimum Total Cost:
\((3\times2)+(1\times11)+(0\times10)+(0\times3)+(0\times7)+(0\times1)+(2\times4)+(4\times7)
+(2\times2)+(0\times1)+(0\times3)+(0\times9)+(0\times4)+(3\times8)+(6\times12)\)
= 6+11+0+0+8+28+4+0+0+24+72 = 153.

(b) Least Cost Method.

<table>
<thead>
<tr>
<th>Source</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1</td>
<td>2</td>
<td>11</td>
<td>10</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Y</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Z</td>
<td>3</td>
<td>9</td>
<td>4</td>
<td>2</td>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

Minimum Cost: 85

(c) Vogel’s Approximation Method.

Penalty Table:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>$X_{25}$, $X_{32}$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>$X_{33}$</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>*</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>$X_{34}$</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>*</td>
<td>1</td>
<td>1</td>
<td>*</td>
<td>3</td>
<td>1</td>
<td>*</td>
<td>$X_{31}$</td>
</tr>
</tbody>
</table>

Minimum Cost: $(4 \times 3) + (2 \times 4) + (6 \times 1) + (3 \times 3) + (1 \times 9) + (4 \times 4) + (1 \times 8)

= 12 + 8 + 6 + 9 + 9 + 16 + 8

= 68
**QUESTION 32**

(a) The frequency of the resistors' failures which have life span of 4 months, are given below:

<table>
<thead>
<tr>
<th>Months</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Of failure</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>30</td>
</tr>
</tbody>
</table>

Given that 800 resistors are fixed for use at a time, each resistor costs ₦15 if group replacement and ₦50 if replacement is done individually, calculate the cost of monthly replacement done individually.

(b) A town has two supermarkets (P, Q) and transition matrix of retainment and loss of the customer is given below:

\[
\begin{pmatrix}
P & Q \\
P & 0.6 & 0.4 \\
Q & 0.3 & 0.7
\end{pmatrix}
\]

Obtain the equilibrium condition for the supermarkets.

**SUGGESTED SOLUTION TO QUESTION 32**

(a) We first calculate the average life span of the resistor as follows:

<table>
<thead>
<tr>
<th>Month (X_i)</th>
<th>Percentage Failure</th>
<th>Probabilities</th>
<th>P, X_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>0.4</td>
<td>1.2</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>0.3</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1.0</td>
<td>2.9</td>
</tr>
</tbody>
</table>

From the table, the average life span (t) of resistor

\[
\Rightarrow t = \frac{\sum P, X_i}{\sum P_i} = \frac{2.9}{1.0} = 2.90 \text{ months.}
\]

But average number of monthly replacement equals
\[ \frac{R}{1} = \frac{N}{t} = \frac{300}{2.90} = 102.86 \approx 103 \]

\[ C = RK = 276 \times 50 = N13,800. \]

(b) For equilibrium condition, we have

\[ I = MSn - 1 \times Pij \text{ (as defined in equation 23.4.1)} \]

i.e. \( (P, Q) = (P, Q) \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix} \)

\[ = \begin{pmatrix} 0.6P + 0.3Q \\ 0.4P + 0.7Q \end{pmatrix} \]

\[ \Rightarrow P = 0.6P + 0.3Q \quad \text{............... (i)} \]
\[ Q = 0.4P + 0.7Q \quad \text{............... (ii)} \]
\[ 1 = P + Q \quad \text{............... (iii)} \]

Solving equations (i) and (ii) simultaneously,

i.e. \( P = 0.6P + 0.3Q \quad \text{....... (i)} \)
\[ 1 = P + Q \quad \text{....... (iii)} \]

(i) \[ \Rightarrow 0 = -0.4P + 0.3Q \]
\[ \Rightarrow 0.4P = 0.3Q \]
\[ \Rightarrow P = \frac{0.3}{0.4}Q = 0.75Q \quad \text{....... (iv)} \]

Use (iv) in (iii), we have

\[ 1 = 0.75Q + Q \]
\[ 1 = 1.57Q \quad \Rightarrow = 1/1.57 = 0.57 \]

and \( P = 0.75 \times 0.57 = 0.43 \)

Therefore, the solution for equilibrium market shares are supermarket \( P = 43\% \) and supermarket \( Q = 57\% \).
APPENDIX III

NORMAL DISTRIBUTION TABLE

The table gives the area under the normal curve between the mean and a point \( z \) standard deviations above the mean.

<table>
<thead>
<tr>
<th>( z )</th>
<th>.00</th>
<th>.01</th>
<th>.02</th>
<th>.03</th>
<th>.04</th>
<th>.05</th>
<th>.06</th>
<th>.07</th>
<th>.08</th>
<th>.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>.000</td>
<td>.004</td>
<td>.008</td>
<td>.012</td>
<td>.016</td>
<td>.019</td>
<td>.023</td>
<td>.027</td>
<td>.031</td>
<td>.035</td>
</tr>
<tr>
<td>0.1</td>
<td>.039</td>
<td>.043</td>
<td>.048</td>
<td>.052</td>
<td>.057</td>
<td>.060</td>
<td>.064</td>
<td>.067</td>
<td>.071</td>
<td>.075</td>
</tr>
<tr>
<td>0.2</td>
<td>.079</td>
<td>.083</td>
<td>.087</td>
<td>.091</td>
<td>.094</td>
<td>.098</td>
<td>.102</td>
<td>.106</td>
<td>.110</td>
<td>.114</td>
</tr>
<tr>
<td>0.3</td>
<td>.117</td>
<td>.121</td>
<td>.125</td>
<td>.129</td>
<td>.133</td>
<td>.136</td>
<td>.140</td>
<td>.144</td>
<td>.148</td>
<td>.151</td>
</tr>
<tr>
<td>0.4</td>
<td>.155</td>
<td>.159</td>
<td>.163</td>
<td>.166</td>
<td>.170</td>
<td>.173</td>
<td>.177</td>
<td>.180</td>
<td>.184</td>
<td>.187</td>
</tr>
<tr>
<td>0.5</td>
<td>.191</td>
<td>.195</td>
<td>.198</td>
<td>.202</td>
<td>.205</td>
<td>.208</td>
<td>.213</td>
<td>.215</td>
<td>.219</td>
<td>.224</td>
</tr>
<tr>
<td>0.6</td>
<td>.225</td>
<td>.229</td>
<td>.232</td>
<td>.235</td>
<td>.238</td>
<td>.242</td>
<td>.245</td>
<td>.248</td>
<td>.251</td>
<td>.254</td>
</tr>
<tr>
<td>0.7</td>
<td>.258</td>
<td>.261</td>
<td>.264</td>
<td>.267</td>
<td>.270</td>
<td>.273</td>
<td>.276</td>
<td>.279</td>
<td>.282</td>
<td>.285</td>
</tr>
<tr>
<td>0.8</td>
<td>.288</td>
<td>.291</td>
<td>.293</td>
<td>.296</td>
<td>.299</td>
<td>.302</td>
<td>.305</td>
<td>.308</td>
<td>.310</td>
<td>.313</td>
</tr>
<tr>
<td>0.9</td>
<td>.315</td>
<td>.318</td>
<td>.321</td>
<td>.323</td>
<td>.326</td>
<td>.329</td>
<td>.331</td>
<td>.334</td>
<td>.336</td>
<td>.339</td>
</tr>
<tr>
<td>1.0</td>
<td>.341</td>
<td>.343</td>
<td>.346</td>
<td>.348</td>
<td>.350</td>
<td>.353</td>
<td>.355</td>
<td>.357</td>
<td>.359</td>
<td>.362</td>
</tr>
<tr>
<td>1.1</td>
<td>.364</td>
<td>.366</td>
<td>.368</td>
<td>.370</td>
<td>.372</td>
<td>.374</td>
<td>.377</td>
<td>.379</td>
<td>.381</td>
<td>.383</td>
</tr>
<tr>
<td>1.2</td>
<td>.384</td>
<td>.386</td>
<td>.388</td>
<td>.390</td>
<td>.392</td>
<td>.394</td>
<td>.396</td>
<td>.398</td>
<td>.399</td>
<td>.401</td>
</tr>
<tr>
<td>1.3</td>
<td>.403</td>
<td>.404</td>
<td>.406</td>
<td>.408</td>
<td>.409</td>
<td>.411</td>
<td>.413</td>
<td>.414</td>
<td>.416</td>
<td>.417</td>
</tr>
<tr>
<td>1.4</td>
<td>.419</td>
<td>.420</td>
<td>.422</td>
<td>.423</td>
<td>.425</td>
<td>.426</td>
<td>.427</td>
<td>.429</td>
<td>.430</td>
<td>.431</td>
</tr>
<tr>
<td>1.5</td>
<td>.433</td>
<td>.434</td>
<td>.435</td>
<td>.437</td>
<td>.438</td>
<td>.439</td>
<td>.440</td>
<td>.441</td>
<td>.442</td>
<td>.444</td>
</tr>
<tr>
<td>1.6</td>
<td>.445</td>
<td>.446</td>
<td>.447</td>
<td>.447</td>
<td>.448</td>
<td>.449</td>
<td>.450</td>
<td>.451</td>
<td>.452</td>
<td>.453</td>
</tr>
<tr>
<td>1.7</td>
<td>.455</td>
<td>.456</td>
<td>.457</td>
<td>.458</td>
<td>.459</td>
<td>.460</td>
<td>.460</td>
<td>.461</td>
<td>.462</td>
<td>.463</td>
</tr>
<tr>
<td>1.8</td>
<td>.464</td>
<td>.464</td>
<td>.465</td>
<td>.466</td>
<td>.466</td>
<td>.467</td>
<td>.467</td>
<td>.468</td>
<td>.469</td>
<td>.470</td>
</tr>
<tr>
<td>1.9</td>
<td>.471</td>
<td>.471</td>
<td>.472</td>
<td>.472</td>
<td>.473</td>
<td>.473</td>
<td>.474</td>
<td>.475</td>
<td>.476</td>
<td>.477</td>
</tr>
<tr>
<td>2.0</td>
<td>.477</td>
<td>.478</td>
<td>.478</td>
<td>.478</td>
<td>.479</td>
<td>.479</td>
<td>.480</td>
<td>.480</td>
<td>.481</td>
<td>.481</td>
</tr>
<tr>
<td>2.1</td>
<td>.482</td>
<td>.482</td>
<td>.483</td>
<td>.483</td>
<td>.484</td>
<td>.484</td>
<td>.484</td>
<td>.485</td>
<td>.485</td>
<td>.485</td>
</tr>
<tr>
<td>2.2</td>
<td>.486</td>
<td>.486</td>
<td>.486</td>
<td>.487</td>
<td>.487</td>
<td>.487</td>
<td>.487</td>
<td>.487</td>
<td>.488</td>
<td>.488</td>
</tr>
<tr>
<td>2.3</td>
<td>.489</td>
<td>.489</td>
<td>.489</td>
<td>.490</td>
<td>.490</td>
<td>.491</td>
<td>.490</td>
<td>.491</td>
<td>.491</td>
<td>.491</td>
</tr>
<tr>
<td>2.4</td>
<td>.491</td>
<td>.492</td>
<td>.492</td>
<td>.492</td>
<td>.492</td>
<td>.492</td>
<td>.492</td>
<td>.493</td>
<td>.493</td>
<td>.493</td>
</tr>
<tr>
<td>2.5</td>
<td>.493</td>
<td>.494</td>
<td>.494</td>
<td>.494</td>
<td>.494</td>
<td>.494</td>
<td>.494</td>
<td>.494</td>
<td>.495</td>
<td>.495</td>
</tr>
<tr>
<td>2.6</td>
<td>.495</td>
<td>.495</td>
<td>.495</td>
<td>.496</td>
<td>.496</td>
<td>.496</td>
<td>.496</td>
<td>.496</td>
<td>.496</td>
<td>.496</td>
</tr>
<tr>
<td>2.7</td>
<td>.496</td>
<td>.496</td>
<td>.496</td>
<td>.496</td>
<td>.496</td>
<td>.497</td>
<td>.497</td>
<td>.497</td>
<td>.497</td>
<td>.497</td>
</tr>
<tr>
<td>2.8</td>
<td>.497</td>
<td>.497</td>
<td>.497</td>
<td>.497</td>
<td>.497</td>
<td>.497</td>
<td>.497</td>
<td>.497</td>
<td>.498</td>
<td>.498</td>
</tr>
</tbody>
</table>
### Student - Distribution

The values of $t_\alpha$ given in the table has a probability $\alpha$ of being exceeded.

<table>
<thead>
<tr>
<th>d.f</th>
<th>$t_{100}$</th>
<th>$t_{050}$</th>
<th>$t_{025}$</th>
<th>$t_{010}$</th>
<th>$t_{005}$</th>
<th>d.f</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.078</td>
<td>6.314</td>
<td>12.706</td>
<td>31.821</td>
<td>63.657</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1.886</td>
<td>2.920</td>
<td>4.303</td>
<td>6.965</td>
<td>9.925</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1.638</td>
<td>2.353</td>
<td>3.182</td>
<td>4.541</td>
<td>5.841</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1.533</td>
<td>2.132</td>
<td>2.776</td>
<td>3.747</td>
<td>4.604</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1.476</td>
<td>2.015</td>
<td>2.571</td>
<td>3.365</td>
<td>4.032</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>1.440</td>
<td>1.943</td>
<td>2.447</td>
<td>3.143</td>
<td>3.707</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>1.415</td>
<td>1.895</td>
<td>2.365</td>
<td>2.998</td>
<td>3.499</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1.397</td>
<td>1.860</td>
<td>2.306</td>
<td>2.896</td>
<td>3.335</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>1.383</td>
<td>1.833</td>
<td>2.262</td>
<td>2.821</td>
<td>3.250</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>1.372</td>
<td>1.812</td>
<td>2.228</td>
<td>2.764</td>
<td>3.169</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>1.363</td>
<td>1.796</td>
<td>2.201</td>
<td>2.718</td>
<td>3.106</td>
<td>11</td>
</tr>
<tr>
<td>12</td>
<td>1.356</td>
<td>1.782</td>
<td>2.179</td>
<td>2.681</td>
<td>3.055</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>1.350</td>
<td>1.771</td>
<td>2.160</td>
<td>2.650</td>
<td>3.012</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>1.345</td>
<td>1.761</td>
<td>2.145</td>
<td>2.624</td>
<td>2.977</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>1.341</td>
<td>1.753</td>
<td>2.131</td>
<td>2.602</td>
<td>2.947</td>
<td>15</td>
</tr>
<tr>
<td>16</td>
<td>1.337</td>
<td>1.746</td>
<td>2.120</td>
<td>2.583</td>
<td>2.921</td>
<td>16</td>
</tr>
<tr>
<td>17</td>
<td>1.333</td>
<td>1.740</td>
<td>2.110</td>
<td>2.567</td>
<td>2.898</td>
<td>17</td>
</tr>
<tr>
<td>18</td>
<td>1.330</td>
<td>1.734</td>
<td>2.101</td>
<td>2.552</td>
<td>2.878</td>
<td>18</td>
</tr>
<tr>
<td>19</td>
<td>1.328</td>
<td>1.729</td>
<td>2.093</td>
<td>2.539</td>
<td>2.861</td>
<td>19</td>
</tr>
<tr>
<td>20</td>
<td>1.325</td>
<td>1.725</td>
<td>2.086</td>
<td>2.528</td>
<td>2.845</td>
<td>20</td>
</tr>
<tr>
<td>21</td>
<td>1.323</td>
<td>1.721</td>
<td>2.080</td>
<td>2.518</td>
<td>.831</td>
<td>21</td>
</tr>
<tr>
<td>22</td>
<td>1.321</td>
<td>1.717</td>
<td>2.074</td>
<td>2.508</td>
<td>2.819</td>
<td>22</td>
</tr>
<tr>
<td>23</td>
<td>1.319</td>
<td>1.714</td>
<td>2.069</td>
<td>2.500</td>
<td>2.807</td>
<td>23</td>
</tr>
<tr>
<td>24</td>
<td>1.318</td>
<td>1.711</td>
<td>2.064</td>
<td>2.492</td>
<td>2.797</td>
<td>24</td>
</tr>
<tr>
<td>25</td>
<td>1.316</td>
<td>1.708</td>
<td>2.060</td>
<td>2.485</td>
<td>2.787</td>
<td>25</td>
</tr>
<tr>
<td>26</td>
<td>1.315</td>
<td>1.706</td>
<td>2.056</td>
<td>2.479</td>
<td>2.779</td>
<td>26</td>
</tr>
<tr>
<td>27</td>
<td>1.314</td>
<td>1.703</td>
<td>2.052</td>
<td>2.473</td>
<td>2.771</td>
<td>27</td>
</tr>
<tr>
<td>28</td>
<td>1.313</td>
<td>1.701</td>
<td>2.048</td>
<td>2.467</td>
<td>2.763</td>
<td>28</td>
</tr>
<tr>
<td>29</td>
<td>1.311</td>
<td>1.699</td>
<td>2.045</td>
<td>2.462</td>
<td>2.756</td>
<td>29</td>
</tr>
<tr>
<td>$\infty$</td>
<td>1.282</td>
<td>1.645</td>
<td>1.960</td>
<td>2.326</td>
<td>2.57</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
APPENDIX IV

GLOSSARY OF TERMS

A Diagonal Matrix
A matrix which has all its elements zero except those in the leading diagonal.

A Matrix
An array of numbers arranged into rows and columns.

A random experiment
Is one whose outcome cannot be predetermined.

A set
A collection of objects.

A Triangular Matrix
A matrix with zero entries above or below the leading diagonal.

Activity
This represents an action, project operation or task which is a time and resource required effort to complete a particular path or overall project.

Amortization
Is the difference between the amount of instalment and interest.

Amount of Annuity
Is the sum total of the accumulated values of all the payments made during the term of annuity.

An event
A experiment of the sample space.

Annuity
Is a series of equal annual payments extending over a specified number of years.

Artificial Variable
Is a variable represented by a letter to make a standard form of L.P. problem which is not in canonical form.

Asymmetric form
Is an L.P problem in which the decision variable is unrestricted in sign and the inequality constraints are of the mixed type.

Bar Chart
Pictorial representation of data by bars such that the length of a bar is proportional to the magnitude of the data of the category of data.

Base Year
A year when the value of a quantity is relatively stable thereby making the comparison of this value with those existing in other years.
**Basic Solution**
is a solution set which has at most \( m \) zero values in a L.P. problem that has \( n \) variables and \( m \) constraints.

**Binomial Distribution**
The distribution of the number of S’s in a \( n \) trials of a dicotomous trials with the outcomes S or F.

**Binominal Distributor**
The distribution of a random variable with only two possible outcomes S or F.

**Break-Even Analysis**
Establishes the relationship between costs and revenues.

**Break-Even Point**
Is the point of intersection of the total cost and total sales lines in a break-even chart.

**Census**
A listing of the units in a population including the collection of some key information on each unit.

**Coefficient of Skewness:**
A measure of the degree of departure from symmetry.

**Coefficient of Variation**
The percentage variation of the data points from their mean.

**Combination**
Selection or counting technique where order of selection is unimportant.

**Completion Coefficient**
This measures the magnitude and direction (direct or indirect) association between two variables.

**Complement of a Set**
The points that are in the universal but which are not in the set.

**Components of a Time Series**
Irregular variation, cyclinical variation, trend and seasonal variation.

**Compound Interest**
When interest is computed on the previous year’s amount.

**Cardinality of a Set**
The number of all possible points in the set.

**Constraint**
Is the limit of achievement of an objective function.

**Continuous Review Policy**
Is a policy in which order is placed whenever the stock level falls below the prescribed reorder point or pre-specified level.
**Correlation**
This deals with spotting a linear association between variables contained in paired data. This includes evaluating the magnitude of such association.

**Data**
A collection of pieces of information

**Decision Alternatives**
Refers to all possible and viable alternatives that can be considered in decision making.

**Decision Pay-Off**
Refers to a numerical value resulting from each possible combination of alternative courses of action and states of nature.

**Decision Theory**
Is a body of methods that provides an analytical and systematic approach in order to select wisely or appropriately a course of action among several available actions.

**Decision Variables**
These are unknown variables that we seek to determine.

**Deterministic Model**
Is a model in which it is assumed that the demand rate is known.

**Differential Calculus**
Is the process of finding the derivative of a function with respect to an independent variable.

**Differentiation**
Is a mathematical concept that deals with the rate of change.

**Dummy Activity**
Is an activity which carries zero duration because it does not consume either time or resource.

**Economic Order Quantity (EOQ)**
Is the amount to be ordered so that inventory decision will be optimal.

**Empty set**
A set that has no element.

**Event**
Represents the start or completion of an activity or task and as such does not consume time or resource.

**Expected Monetary Value (EMV)**
Is the sum of weighted payoffs.

**Expected Opportunity Loss (EOL)**
Is the difference between the actual profit obtained for the particular course of action taken and the highest profit (payoff) for a state of nature.
**Expected Value of Perfect Information (EVPI)**
Is the difference between expected profit under perfect information (EPPI) and expected monetary value (EMV).

**First-Order Markov Chain**
A Markov Chain is said to be first-order if the probability of occurrence of each state depends only upon the immediate preceding state.

**Forecasting**
Using regression line to predict a future value of the dependent variable in a regression problem.

**Holding Cost**
This is the cost of keeping items until they are disposed off.

**Hurwicz Criterion**
Is the criterion in which the decision maker needs to make a compromise between the optimistic (Maximax) and pessimistic (Maximin) criteria.

**Hypothesis**
A conjectured value of an unknown population parameter.

**Independent Event**
Two events are independent when the occurrence of one does not prevent the occurrence of the other.

**Index Number**
Representation of a current value as a percentage of its corresponding value in a base period.

**Interest**
This refers to certain amount a man or organization that lends money to another man or organization expects to receive in return for the usage of his money.

**Internal Estimator or Confidence Interval**
The statistical construction of an interval using data from a sample. A large number of such intervals are expected to contain a given percentage of the value of a population parameter.

**Intersection of Two Sets**
The points that are common to both sets.

**Inventory**
Is the list of usable but idle resources such as men, machines, materials or money.

**Lead Time**
Is the time required before a delivery is received when an order is placed.

**Linear Function**
Is an expression of first degree in the variable(s) involved.

**Linear Programming**
Is a technique that involves modeling and solving problems (stated in functional forms) mathematically.
**Markov Process**
Is the movement of a system from one state to another depending upon the immediate processing state with a constant probability.

**Maximin (or Minimax) Criterion**
Is a criterion that enables the decision to make the best out of the worst possible conditions.

**Mean**
The average of some numbers.

**Median**
The middle observation of an ordered data set.

**Minimax Regret Criterion:**
Is the criterion that aims at moderating conservatism in the Maximin (minimax) criterion by reducing the regrets.

**Minimin (Maximax) Criterion**
Is an optimistic criterion that provides the decision maker the opportunity to achieve the largest possible profit (Maximax) or lowest possible cost (Minimin).

**Mode**
The observation that occurs most frequently.

**Model**
Is an abstract representation of reality.

**Mutually Exclusive Events**
Two or more events that are disjoint and have no elements in common.

**Network Analysis**
Is a set of operations research techniques needed and useful for planning, scheduling and controlling large and complex projects.

**Non-Deterministic Model**
Is a stochastic model in which the demand rate is probabilistic.

**Normal Distribution**
Most commonly used distribution in real life. It is symmetrical about its mean.

**Objective Function**
Is a function that states the goal that needs to be optimized.

**Ogive**
A graph of cumulative frequencies.

**Operations Research**
Is the application of methods of science to complex problems in the direction and management of large systems of men, machine, materials and money in industry, business, government and defence.
Optimization
Is the process of either minimizing cost or maximizing profit.

Order Cycle
Is the time period between two successive orders.

Ordering Cost
Is the cost incurred in connection with ordering and procurement.

Parameter
An unknown characteristic of a population needed to understand a given population.

Periodic Review Policy
Is an inventory policy that places orders at equal intervals of time.

Permutation
Arrangement or counting technique where order is unimportant.

Perpetuity
Is an annuity that is payable forever.

Pie Chart
Pictorial representation of data by angles such that the magnitude of each angle is proportional to the magnitude of the data in the category.

Point Estimator
A single reliable value of an unknown population parameter.

Poisson Distribution
Mostly deals with occurrence of (rare) events like first class, death etc.

Population
A collection of all the units (individuals) in a target universe. Any data collected is to be used to plan for this population.

Price Index
The correct price of a commodity as a percentage of the price in the base period.

Primary Data
Data collected directly from informants

Principal
Refers to the amount lent out by the lender.

Probability
The ratio of the numbers of points in an event to those in the sample space.

Project
Is a well-defined set of jobs, tasks or activities, which must be completed in a specified time sequence order.
Quadratic Function
Is a polynomial of degree two.

Quantity Index
Similarly defined as price index.

Quartiles
The points that divide an ordered data set into four equal parts.

Questionnaire
A sequence of questions passed to a respondent to capture some basic information needed to answer the questions in the aims and objectives of a study.

Random Variable
A function which operates on a point in the sample space to produce a real number.

Random Variable
A function, which operates on every point in a sample space to provide a real number.

Regression
This is the representation of the relationship between two variables using the least squares method.

Salvage Cost
Is the cost incurred due to perishable or expired items.

Sample Space
The set of all possible outcomes of a random.

Sample
A part of the population selected for a study. It must be as representative of the population as possible.

Sampling Distribution of the Mean
The shape of the frequency polygon of all possible sample means of size n from the population.

Sampling Distribution
The shape of the frequency polygon of all sample proportions calculated from a population of size n from the population.

Scatter Plot
A plot showing the individual of data that are given in pairs. The points are not joined by any line or curve in the graph.

Seasonal Index
Ratio (%) of the observed value of a time series and its trend estimate.

Seasonal Variation
Difference between an observed value of time series and its trend estimate.
**Secondary Data**  
Data collected from an organization or agency.

**Second-Order Derivative**  
Is a derivative in which a function is differentiated more than once.

**Second-Order Markov Chain:**  
A Markov chain is said to be second-order if the probability of occurrence in the forthcoming periods depends upon the state in the last three periods.

**Sequence**  
Is an ordered list of terms in which the terms are governed by a certain rule.

**Series**  
Refers to the resulting expression obtained from the addition of sequence terms.

**Shortage Cost**  
Is the cost incurred because the material is not stocked at that particular time.

**Significance Level of a Test**  
The probability of committing type I error or the probability of rejecting a true hypothesis.

**Simple Interest**  
Means when interest is payable only on principal.

**Sinking Fund**  
Is the sum of the amount that accumulates at compound interest by setting aside a fixed amount at regular intervals.

**Skewness**  
Tendency of a data set to depart from symmetry. Skewness can be positive or negative.

**Slack Variable**  
Refers to unused resource.

**Smoothing of Time Series**  
An attempt to reduce flections in a time series.

**Square Matrix**  
A matrix with the same number of rows and columns.

**Standard Deviation**  
The square root of the variance. This is in the same scale of measurement as the data points.

**Standard Error of an Estimator**  
Numerical value of the standard deviation of the estimator evaluated from a sample.

**Standard Form**  
Is the conversion of the original given problem of L.P. to the form that is amenable to algebraic computations and manipulations.
**States of nature**
Refers to expected future events that may occur.

**Storage Cost**
Is the cost of renting a space for storing items.

**Surplus Variable**
Refers to the amount by which solution values exceed a resource.

**Symmetric Form**
Is an L.P. problem in which all the variables are restricted to be non-negative and all the constraints are inequality of same type.

**Symmetric Matrix**
A matrix such that if its rows are turned to column, the matrix remains unchanged.

**Third-Order Markov Chain**
A Markov chain is said to be third-order if the probability of occurrence in the forthcoming periods depends upon the state in the last three periods.

**Time series Models**
Additive and multiplicative models.

**Time Series**
A sequence of data observed at regular intervals of time.

**Total Amount (or Amount)**
Is the sum of interest and principal.

**Transition Matrix**
Is the collection of all probabilistic measures which are arranged in rows and columns.

**Type I Error**
An error committed when a true hypothesis is rejected.

**Type II Error**
An error committed when a false hypothesis is accepted.

**Union of Two Sets**
The points that are in at least one of the sets.

**Universal Set**
A collection of all units in a given situation.

**Variance**
Measures how disperse the data points are from the center of the data (Mean).

**Volume Index**
Similarly defined as price or quantity index but using volume = quantity X price delete with respect to quantity of sales.
APPENDIX V

BIBLIOGRAPHY


Evans, J. (1991): “Creative Thinking in Decision and Management Sciences.” Cincinnati, Ohio; South-Western Publishing.


New Age Newspaper (2005): “Turn over of share holders’ funds and capital employed of Triple Gee,” February 14, New Age Newspaper publishers.


APPENDIX VI

STUDY AND EXAMINATION TECHNIQUES

This appendix contains notes on:

a. Using the questions and answers provided in the manual
b. Effective study
c. Examination techniques.

6.1 Questions and answers

Introduction

1. Two types of question are provided in this manual
   a. Questions set at the ends of chapters with answers provided in Appendix 1
   b. Questions with answers set in Appendix 2.

Questions with answers

2. These questions are either
   a. questions intended to test the understanding of the points arising out of the particular chapter; or
   b. examination questions inserted at a stage where it is considered the student will be best able to give a reasonable answer.

3. Most answers are given in outline but some examination answers go a little further in order to provide greater guidance and provide students with the basis for study.

4. When answers are comprehensive you could not be expected to write them in the time allowed. Do not worry if you feel you could not write such answers; you are not expected to. But you must grasp the main points or principles involved which will form the basis for good marks in an examination.

5. Do not worry if your answer differs, there is often more than one approach. You must satisfy yourself however, that it is only the approach that differs, and that you have not missed the fundamental principles.

6. Authors' Comments. These have been included to give additional points or elaborate on matters arising out of the subject covered by the question to which it is felt you should give some thought.

Using the answers

7. Have a shot at each question yourself before consulting the answer, you will achieve nothing if you do not do this. Write your answer out in full or jot down the main points. Do not hurry to the answer.
8. Look at the answer. (See para 5 in the case of examination answers). Study the particular area thoroughly now making sure of your understanding. Repeat the process outlined in para 7 and this paragraph after a suitable interval. You must do this to get any benefit at all. Make sure the main points stick.

9. Just browsing through the answers will really get you nowhere. You must test yourself by writing down your version of the answer.

6.2 Effective study

Introduction
1. These notes are intended for those who are new to studying for examination subjects, although those who are not may also benefit. They have been written in relation to study involving the reading of textbooks, and they apply to all subjects. It is often very difficult to pick out the important principles from such books. Careful reading of these notes will be of benefit even in studying the manual.

General
2. Study means more than just reading a piece of literature. It means close concentrated reading with a notebook at your side. Unless you are one of a few people do not kid yourself you can absorb material by just one general read through it, you cannot!

3. Read a small area, making notes as you go along. Then ask yourself – what have I just learnt? Write down what you think it was all about. Then look again and you may be surprised to find you have missed a key point or points – they must be down in your notebook and eventually in your head.

Compilation of notebook
4. A well-compiled NOTEBOOK is a must. Use block capitals or different colour inks to headline the main areas and subdivisions of those areas. Notes made during lectures or private study should not go straight into your NOTEBOOK. Take them down on a “rough” paper and write them in your NOTEBOOK as soon as possible after the lecture or study period, thinking about what you are writing.

Memory aids
5. Mnemonics are very useful – if the sequence of points in the textbook is not significant, change it if it makes for a better mnemonic.

6. Association of the points with familiar objects which will serve to recall them is also useful.

7. Some people memorise things by saying them over and over out loud, others have to write them down time after time.

8. Many students have small blank cards and using one side of each card for each study area, put down the main points. They carry the cards everywhere with them and use every opportunity to study them. As they are small they are easily carried. It is surprising how much of your day can be utilized this way.

Programme
9. Map out a programme for yourself; set targets and achieve them. One thing is certain, studying is not easy but it is not too difficult if you go about it in an orderly purposeful way. Many students fail their examinations through bad
**preparation.** Tackle your studies as you would a project at work, systematically. Allocate a number of hours each week to each subject. Try fixing specific times for each subject, then keep to them by refusing to let anything keep you from your planned task.

**Revision**

10. Revise periodically. The nearer the examination gets, the more you should concentrate on the major headlines in your notebook and less with the supporting details.

**6.3 Examination technique**

**First impressions**

1. However well prepared you may be, you are still likely to look at the paper on the day and say to yourself, after a quick look at the questions, “There’s not much there I can do”.

2. The atmosphere of the exam room has something to do with this. Try to blot everything from your mind other than the job in hand. Concentrate hard. If you feel a bit panicky (most people do – despite the apparent looks of serenity around you) grip the table, take a deep breath, and get on with it. Remember things are never as bad as they seem!

**Time allocation**

3. Allocate each question time appropriate to the number of marks. At the end of the allotted time for a question go on to the next – remember, the first 5 or 10 marks on the new question are more readily picked up than the last 1 or 2 on the previous question.

4. The temptation will be to say “I’ll write just one more sentence”, but before you know where you are you would have written several more and probably just managed to scrape another mark, whereas the same time on the next question could have earned 5 or 6 marks. TIME ALLOCATION IS IMPORTANT.

5. If you are running out of time write down the main headings first, leaving a few lines between each – at least the examiner will see that you had the overall picture. Then go back putting in as much supporting detail as you can.

**General approach**

6. Read the instructions at the top of the paper

7. Read the question paper once through. Make your choice of questions quickly. Pick the easiest (if one appears so) and get on with it.

**Individual question**

8. Read the question again carefully. The question will involve a key principle or set of principles. What are they? It is so easy to make the wrong decision at this stage, so read the question, underlining what appear to be the key words. This should help you. Irrelevancy has been heavily criticised by examiners.

9. Do not rush into action with your pen yet. Jot down on a piece of scrap paper the main headings you will use in your answer. All this will take time – about 5 minutes or more, but the careful thought and outline answer represents marks already earned.

10. If the question is set out in a particular sequence, that is:

   a. .......................
b. ……………………
c. ……………………  etc.
then answer it in that sequence or you’ll have a hostile examiner to cope with.

11. Use the particular terminology used in the question, the examiner can then link the points in your answer to the relevant parts of the question.
12. Assumptions are sometimes required (for example because of the lack of standardization of terminology in this subject). Having stated your assumptions, make sure that what you write is consistent with them. Do ensure, however, that your assumptions are valid and are not just a device for changing the meaning of the question to suit your knowledge!

Layout of answer
13. Tabulate where appropriate, using block capitals for your main headings and underline subheadings. Underline words or phrases which require emphasis. Use a ruler.
14. Leave a line between your paragraphs and subparagraphs. This makes for a good layout. However, do not write one very other line within paragraphs, or on one side of the paper only – examiners are waste conscious!
15. The use of different colour pens, where appropriate, is useful but do not overdo it. In fact one black and red felt-tip pen would be sufficient (use the felt-tip pens which have a fine point).

Charts and diagrams
16. A descriptive heading or title must be given to each diagram (using the one in the question if indicated).
17. Do not squeeze a diagram into a corner – spread it out.
18. Do not clutter your diagram up with too much detail – this defeats the object, which should be clarity.
19. Give a key to the symbols and the different lines you’ve used, and again – use a ruler.

End of examination procedure
20. Have a quick look at each answer, checking for grammatical errors and badly formed letters.
21. Ensure each answer sheet has your number on it and do not leave any lying on the table.

Conclusion
22. Good technique plays a large part in examination success; this is a fact. Refuse to be panicked, keep your head, and with reasonable preparation you should make it.
23. Remember – you do not have to score 100% to pass.
24. A final point; once you’re in the examination room stay there and make use of every minute at your disposal.
25. Practice your technique when answering the questions set in the manual.
INDEX

Adjoint, 332, 335
Aggregate Index, 277, 280, 283
Aggregate Price Relative, 278
Amortization, 579
Annual Rate, 561
Annuity, 579
Anticipated, 146, 164, 519
Artificial Variable, 387, 579
Association, 2, 544, 592
Assumed Mean, 536
Bar Chart, 472, 538, 539
Base, 209, 229, 232, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 493, 526, 582, 584
Base Period, 283
Base Year, 274, 579
Basic Solution, 387, 580
Break Even, 269, 270, 525, 526
Buffers, 32
Canonical Form, 387
Cardinality, 580
Census, 580
Certain, 10, 25, 28, 43, 48, 49, 78, 79, 84, 90, 111, 127, 139, 147, 182, 219, 232, 252, 260, 288, 437, 582, 586, 592
Charts, 470, 594
Class, 34, 72, 439, 440, 458, 534, 584

Class Mark, 534
Coefficient of Determination, 509, 510
Coefficient of Variation, 543
Co-factors, 329, 334, 335, 336, 338, 345
Combination, 95, 157, 251, 329, 375, 379, 380, 462, 476, 581
Complement of a Set, 309, 311, 580
Compound Interest, 580
Confidence Interval, 582
Continuous Review, 580
Correlation, 183, 581
Correlation Coefficient, 509, 510, 549
Cost, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 21, 22, 23, 24, 28, 29, 30, 31, 33, 34, 35, 37, 38, 264, 265, 266, 267, 268, 269, 270, 501, 505, 506, 507, 508, 511, 514, 515, 516, 517, 518, 519, 523, 524, 525, 526, 527, 528, 530, 533, 546, 547, 548, 549, 550, 582, 584, 585, 586, 587
Critical Path Method, 411
Decision Theory, 581
Decision Variables, 376, 406
Determinants, 321, 328, 329, 350
Deterministic Model, 581, 583
Diagonal Matrix, 323, 579
Difference of Two Matrices, 326
Differentiation, 353, 356, 365, 371
Direct, 5, 47, 49, 57, 61, 69, 71, 72, 77, 78, 84, 121, 122, 143, 156, 157, 163, 168, 190, 205, 206, 219, 264, 265, 266, 267, 269, 278, 340, 342, 350, 462, 487, 507, 515, 516, 517, 518, 523, 524, 527, 531, 580
Duality Concept, 404
Dummy Activities, 414, 425
Economic Order Quantity, 581
Equal Matrices, 321
Estimate, 36, 75, 79, 90, 98, 103, 157, 235, 421, 505, 507, 508, 585
Expectation, 113, 234
Expected Opportunity, 581
Exponential Function, 363
Finite Set, 310
Fisher’s Index, 271, 279, 280, 285
Forecasting, 466, 528
Frequency, 304, 433, 534, 535, 574, 585
Function of function, 361, 373

Functions, 3, 56, 88, 210, 353, 356, 358, 359, 377, 427, 468, 478, 529
Fundamental Principles, 591
Geometric Mean, 280
Histogram, 534
Holding Cost, 582
Hypothesis, 582
Identity Matrix, 321, 323, 335, 350, 351
Independence, 297, 378
Index Number, 582
Indirect, 14, 52, 57, 72, 84, 85, 109, 110, 111, 164, 167, 174, 175, 264, 504, 505, 515
Information, 10, 133, 149, 184, 462, 582
Intersection of Sets, 307, 318
Interval, 435, 438, 472, 543, 544, 582, 592
Interview, 36, 37, 143
Introduction, 1, 51, 135, 209, 478, 589, 590, 591, 592
Inventory, 24, 25, 28, 30, 31, 34, 37, 122, 131, 145, 159, 164, 166, 168, 214, 215, 250, 252, 484, 581, 584
Irregular Variation, 580
Laspeyre’s Index, 271, 279
Lead Time, 24, 582
least Squares Line, 585
Linear Programming, 375, 376, 582, 589, 590
Lorenz Curve, 538
Markov Process, 583
Matrix, 323, 324, 325, 329, 333, 335, 342, 343, 455, 579, 586, 587
Matrix Inverse, 340
Median, 583
Minor, 332
Mode, 463
Moving Averages, 551, 552
Net Present Value, 138
Network Analysis, 411, 416
Normal Approximation, 547
Normal Distribution, 577
Null Set, 310
Objective Functions, 407
Ogive, 534, 536
Operations Research, 375, 478, 583, 590
Ordering Cost, 584
Paache's Index, 271
Pearson's, 537, 547
Perfect, 582
Periodic Review, 584
Permutation, 584
Perpetuity, 584
Pie Chart, 584
Poisson Distribution, 584
Price Relative, 271, 275, 278, 279
Primal, 404, 406
Primary, 57, 209, 231, 348, 350, 375, 461
Primary Data, 584
Probability of an Event, 290, 304
Probability Tree, 301
Quadratic, 564, 585
Quantity Relative, 275
Quartiles, 585
Random Experiment, 288, 305, 541, 542, 579
Random Variable, 580
Regression, 585
Regression Line, 582
<table>
<thead>
<tr>
<th>Relative Frequency, 291</th>
<th>Skewness, 580, 586</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replacement Analysis, 427</td>
<td>Standard Deviation, 424, 586</td>
</tr>
<tr>
<td>Salvage Cost, 585</td>
<td>Standard Error, 586</td>
</tr>
<tr>
<td>Sample, 541, 542, 585</td>
<td>Standard Form, 385, 586</td>
</tr>
<tr>
<td>Sampling, 585, 589</td>
<td>Storage Cost, 587</td>
</tr>
<tr>
<td>Scaling Factor, 536</td>
<td>Subset, 289, 304, 305, 309</td>
</tr>
<tr>
<td>Scatter Plot, 585</td>
<td>Tally, 534</td>
</tr>
<tr>
<td>Secondary Data, 586</td>
<td>Time series, 587</td>
</tr>
<tr>
<td>Sequence, 586</td>
<td>Transition Matrix, 574</td>
</tr>
<tr>
<td>Series, 473, 474, 580, 586, 587</td>
<td>Triangular Matrix, 323, 324, 351</td>
</tr>
<tr>
<td>Set Difference, 312</td>
<td>Value Relative, 275</td>
</tr>
<tr>
<td>Shortage Cost, 586</td>
<td>Variance, 4, 5, 142, 146, 173, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 206, 207, 210, 421, 422, 423, 490, 518, 527, 528, 530, 569, 572, 586</td>
</tr>
<tr>
<td>Significance Level, 586</td>
<td>Venn Diagram, 307, 312, 314, 315, 317, 318, 554</td>
</tr>
</tbody>
</table>